

transient and steady-state analysis. A "proof" of $L = \lambda W$ is written in a way that requires service to be in order of arrival, even though restriction can be avoided with a little explanation. The explanation of PASTA is serviceable.

Chapter Three is devoted to birth-and-death queues. A nice feature of the presentation is that the conservation of flow principle (called the rate-equality principle here) is given at the start of the chapter. It is used to get the familiar balance equations for the M/M/1 queue, $\lambda p_n + \mu p_n = \lambda p_{n-1} + \mu p_{n+1}$; a more refined application would produce the rate-up equals rate-down equations, $\lambda p_n = \mu p_{n+1}$, which are easier to work with. Waiting times, output processes, and busy period analysis are given for the usual models. Significant attention is paid to transient analysis, and the waiting time distribution for the finite-source FIFO model is derived. There are 22 exercises that explore variations of the basic models.

The next chapter covers Markovian models that are not birth-and-death processes. This includes the method of stages for queues with Erlang arrivals or services and some bulk queues. The latter is treated extensively, and is a distinguishing feature of this book.

Chapter Five is about networks of queues. Open and closed Jackson networks are described, and a proof of Jackson's theorem is given. The interpretation of this theorem is not stated carefully enough: "It is implied that the states n_i of individual nodes i , ($i = 1, 2, \dots, k$) in steady state are independent random variables." BCMP networks are defined, and the product form result is stated but not proven.

The sixth chapter covers the embedded Markov chain models, M/G/1, GI/M/1, GI/M/c, and the insensitivity property of the M/G/c/c model. The penultimate chapter covers the G/G/1 models. This includes the Lindley equations, Marshall's characterization of the moments of the waiting time in terms of moments of the idle times, and some bounds. The final chapter contains Kingman's heavy traffic approximation for the waiting time in the GI/G/1 queue, diffusion approximations for several non-Markovian queues, queueing systems with vacations, and design and control models.

In summary, this is a good book for learning about the important formulas in queueing

theory. The presentation is clean and basically correct. The exercises contain some interesting results, and the references are extensive.

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Oriented Projective Geometry: A Framework for Geometric Computations. By Jorge Stolfi. Academic Press, Boston, MA, 1991. vii + 237 pp. \$39.95, cloth. ISBN 0-412-672025-8.

This book presents *oriented projective geometry*, which is the author's name for spherical geometry, of which unoriented projective geometry is the quotient by the action of \mathbb{Z}_2 . As the author promises in the introduction, this monograph does not push the frontiers of mathematics—none of the theorems is much harder than a difficult exercise for an advanced undergraduate course.

Taken as a whole, however, the presentation gives a framework for understanding and addressing the problems that arise when traditional homogeneous coordinates (in which (x_1, \dots, x_n) is the same as $(\lambda x_1, \dots, \lambda x_n)$ for any nonzero λ) are used for doing the sorts of geometric computations that arise in computer graphics. The solution is to restrict the equivalence by requiring that λ be positive. This effectively yields two copies of the affine plane glued together at infinity, yielding a spherical topology. One can then provide oriented versions of almost anything imaginable, including operations like oriented intersections of oriented lines, oriented intersections of oriented planes, etc. More remarkably, one can extend to this new geometry many of the constructions of Euclidean geometry, including notions of convexity. Furthermore, the consistency of this geometry makes it ideal for computer-based use because of the perils of writing programs that handle special cases.

These ideas are interesting, and I certainly want to have this book in my laboratory's library. I will also recommend that my students look at it closely and try to understand the subtleties involved in working in oriented projective geometry. In fact, implementing a package for manipulating the objects of oriented projective geometry, and devising an application program for experimenting with it, would

be an excellent project for a strong computer science undergraduate.

Unfortunately, however, this book has some real drawbacks. First, the book is substantially derived from the author's Ph.D. dissertation, and suffers from this close connection to the original. Dissertations tend to be the last paper one ever writes in which absolutely everything is spelled out, as proof to the rest of the world that everything said is actually true. This can be an effective way to convince the rest of the world, but it is not necessarily an effective way to communicate. The result, in this case, is a book that fails to present its material adequately. If the audience consists of mathematicians, then one can assume a familiarity with (or at least an ability to grasp) the notions of homogeneous coordinates, orientation, etc., and the ability to deduce generalizations from a few particular examples. If the audience consists of people studying computer graphics, one can be confident of a working familiarity with homogeneous coordinates, but perhaps not with their correspondence to models of projective geometry. In either case, the reader who can make it through the first chapter (whose explanations of the correspondences among the various models of projective geometry I found confusing rather than enlightening) should have little trouble in establishing many of the later results independently, *and should be allowed to*. As I read the book, I felt as if I were constantly seeing multiple versions of the same statement: property X is defined, and then the author examines its meaning for points, for lines, for planes, and so on... (cf. §4.2, or §§7.2 and 7.3). It seems that many of these things, along with the typical proofs of commutativity and associativity of operations, etc., should be left to the reader as exercises. Thus, for example, when "meet" and "join" are defined, there is no real reason to go through the proofs of the properties of both these operations, particularly since the principle of duality will soon show exactly how to convert from one proof to the other. Going through the details merely slows the development and puts distance between the introduction of the two ideas and their unification.

Since nearly everything is actually spoon-fed to the reader (e.g., the meaning of a definition in zero, one, two, and three dimensions), one can only infer that the audi-

ence is not deemed particularly mathematically sophisticated. But readers who are *users* of mathematics rather than mathematicians themselves (e.g., the typical reader from the world of computer graphics) are bound to be disappointed as well: there is occasional mention of a topic like how best to store a projective map in a computer, but there are no class definitions for objects like "oriented point," "oriented line," etc., or corresponding definitions for the operations among these classes. The reader, who apparently cannot work out simple proofs or derive examples from definitions, seems nonetheless expected to derive data types that effectively model the mathematics. I am skeptical about this.

The cost of working out every detail for the reader is that there is no space to address many of the interesting issues that arise from the use of homogeneous coordinates. For example, in computer graphics, it is conventional to store the coordinates of points in 3-space as a list of four numbers (x, y, z, w) , where one usually divides by the homogeneous coordinate w to get the briefer representation $(x/w, y/w, z/w, 1)$, which can be stored using just three memory locations, since the fourth coordinate is implicitly "known" to be 1. At the same time, it's conventional to use vectors, also stored as lists of three numbers. These "vectors" represent two different things: (a) displacements between points ("ordinary vectors") and (b) things with certain inner-product relations to ordinary vectors (e.g., "a vector u perpendicular to the vectors v and w "), which can be called "covectors." If a projective map, represented by multiplication by a 4×4 matrix M is applied to the points of 3-space, the ordinary vectors can also be transformed by this projective map by matrix multiplication, provided the projective map fixes the points at infinity, i.e., provided its restriction to the finite points is an affine transformation. If this condition is not met, the explicit "division by w ," i.e., the choice of affine specialization, must be included in computing the differential of the transformation, and there is no longer a single matrix that can be applied to every vector at every point in a consistent fashion. The underlying reason for this, and for the more complicated changes in covectors under projective transformations, is a fascinating subject and one that has caused frequent consternation in

computer graphics. The author is in an ideal position to explain the subtleties, but alas does not.

There is another failure in presentation that is typographical: the author uses \bar{F} to denote the inverse of a function F . One should adopt new notation soberly and only when its value is clear. Here, I found it merely obscured my understanding, particularly when the author chose to make a pun with the notation and use it to denote a pseudoinverse as well. Also, on page 69, the author uses the letter M set in two different but extremely similar fonts, to mean quite different things. It took me some time to determine what was going on.

The book suffers from another sort of problem as well, best described as failures in scholarship. First, the earliest reference in the bibliography is dated 1949, and it cites neither Plücker nor Grassman, although both are mentioned in the text. Second, there are numerous typographical errors. Being the author of another book with numerous typos, I am sympathetic. The author should, however, make available a list of corrections. On page 5, for example, the sentence "Note that $\langle W, X, Y \rangle$ and $\langle \lambda X, \lambda Y, \lambda Z \rangle$ are the same line for all $\lambda \neq 0$ " should be fixed. Third, and most troublesome, were certain lapses in mathematical accuracy. On page 7, the author says that with homogeneous coordinates, "the absence of division steps also makes it possible to do exact geometric computations with all-integer arithmetic." This statement is fine provided one restricts one's geometric computations to operations on flats. If one wishes to also admit circles into the domain of discourse (as the author later does, speaking in §17.4 of distance in the two-sided plane), then one must extend one's coordinates to surds. On page 30, the author defines equivalence of bases of a vector space in terms of "continuous deformations," which he leaves undefined. This is probably a wise idea, since he has also not restricted his vector spaces to have nice scalar fields. One need only examine vector spaces over $\mathbb{Z}/2\mathbb{Z}$ to see the difficulties with this definition.

Finally, any author who thanks someone in his introduction for transforming his syntax and style "from truly atrocious to, I hope, merely dreadful," runs the risk of having people agree that the transformation was success-

ful. In this case one can be more charitable: the writing is not dreadful, but neither is it very good. This is not so much the author's fault, for he is evidently aware of his limitations as a writer, and has made some effort to address them. The publisher, however, should have considered editing the book once more before publication.

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Mathematical Ecology of Plant Species Competition: A Class of Deterministic Models for Binary Mixtures of Plant Genotypes. By Anthony G. Pakes and R. A. Maller. Cambridge University Press, New York, 1990, xiii + 193 pp. \$49.50. ISBN 0-521-37388-3.

This work is a specialized monograph dealing with mathematical extensions of the de Wit "replacement series" approach of analyzing competition between plant species. The model developed specifically addresses competition between two strains of the subterranean clover (*Trifolium subterranean* L.), a species native to southern Europe and widely grown in Australian pasturelands. There are a number of features of the life cycle of this annual species which make modeling of its population dynamics and competitive interactions both interesting and challenging. Specifically, the seeds are released in packets or "burrs" which break apart over time. Seeds will not germinate initially, as they are hard and water-impermeable. However, in a given growing season a proportion of this hard seed softens, and it is softened seed which germinates the following year. The proportion of seed which softens is age-dependent, making modeling of the population dynamics even more difficult.

Much of the book is concerned with the development of a model incorporating these interesting (and unusual) features of the life cycle of subterranean clover. It is for this reason that the monograph is highly specialized—it is difficult to see how the model could be readily extended to other plant species. Given that the book assumes a strong background in advanced calculus, it is likely to appeal mainly to mathematical biologists.

Some important details of the biology of the species, and data on which the model is