Learning Optical Flow

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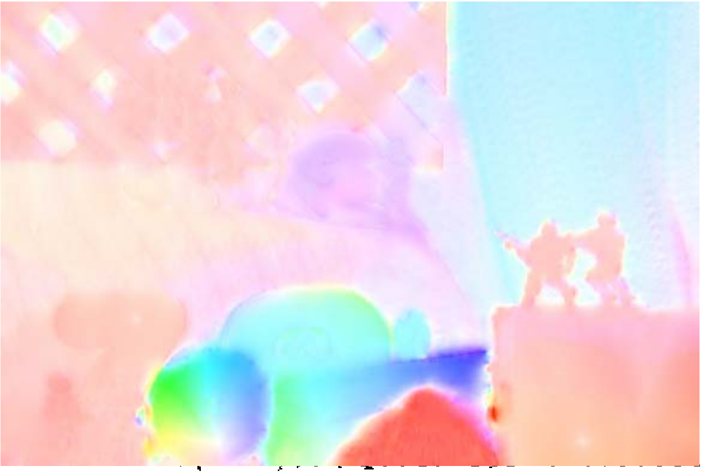
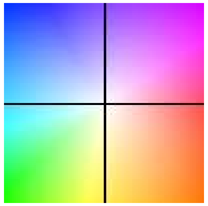
Optical flow

Motion (displacement) of image pixels

“Army”

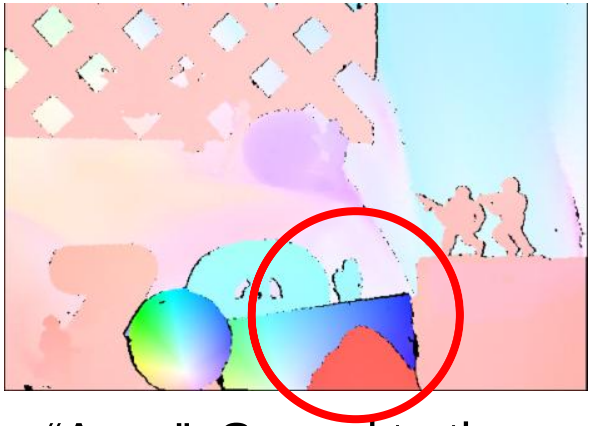


Horn & Schunck 1981



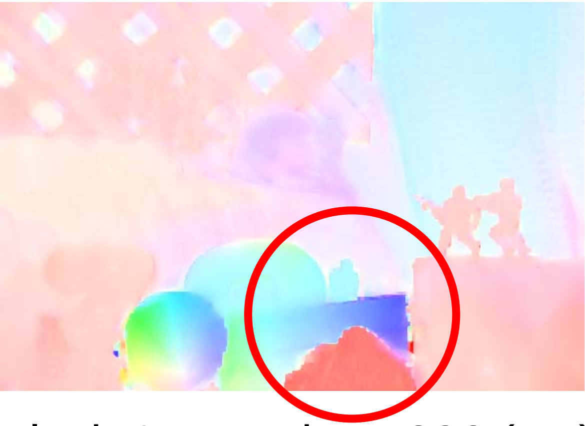
Key

Two standard methods

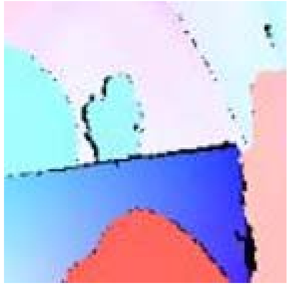


“Army”: Ground truth

Horn & Schunck 1981 (HS)



Black & Anandan 1996 (BA)

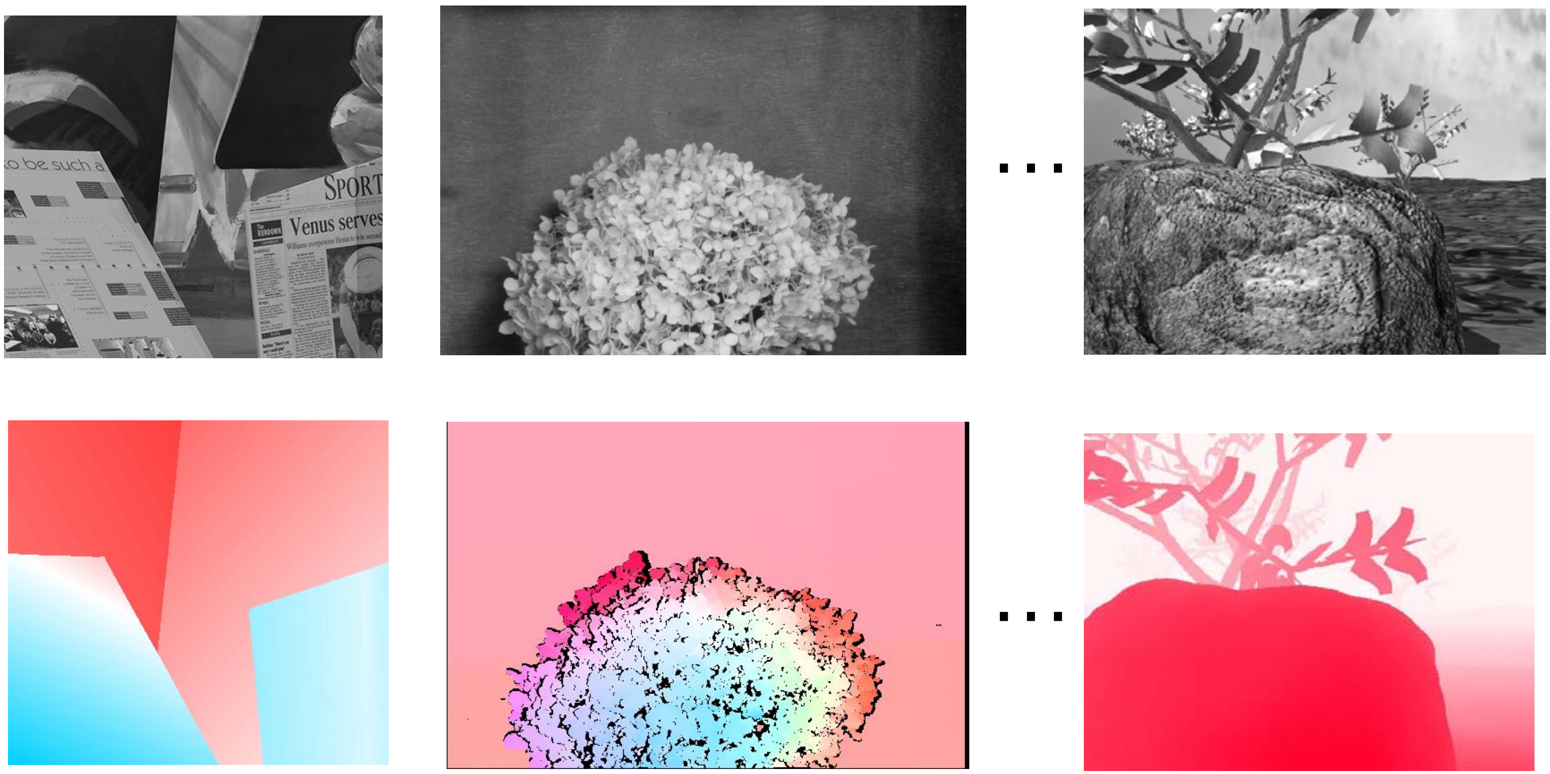


Problems

- Ad-hoc choices, hand-tuned parameters
- Can we learn models from training data?

Image pair

Ground truth flow



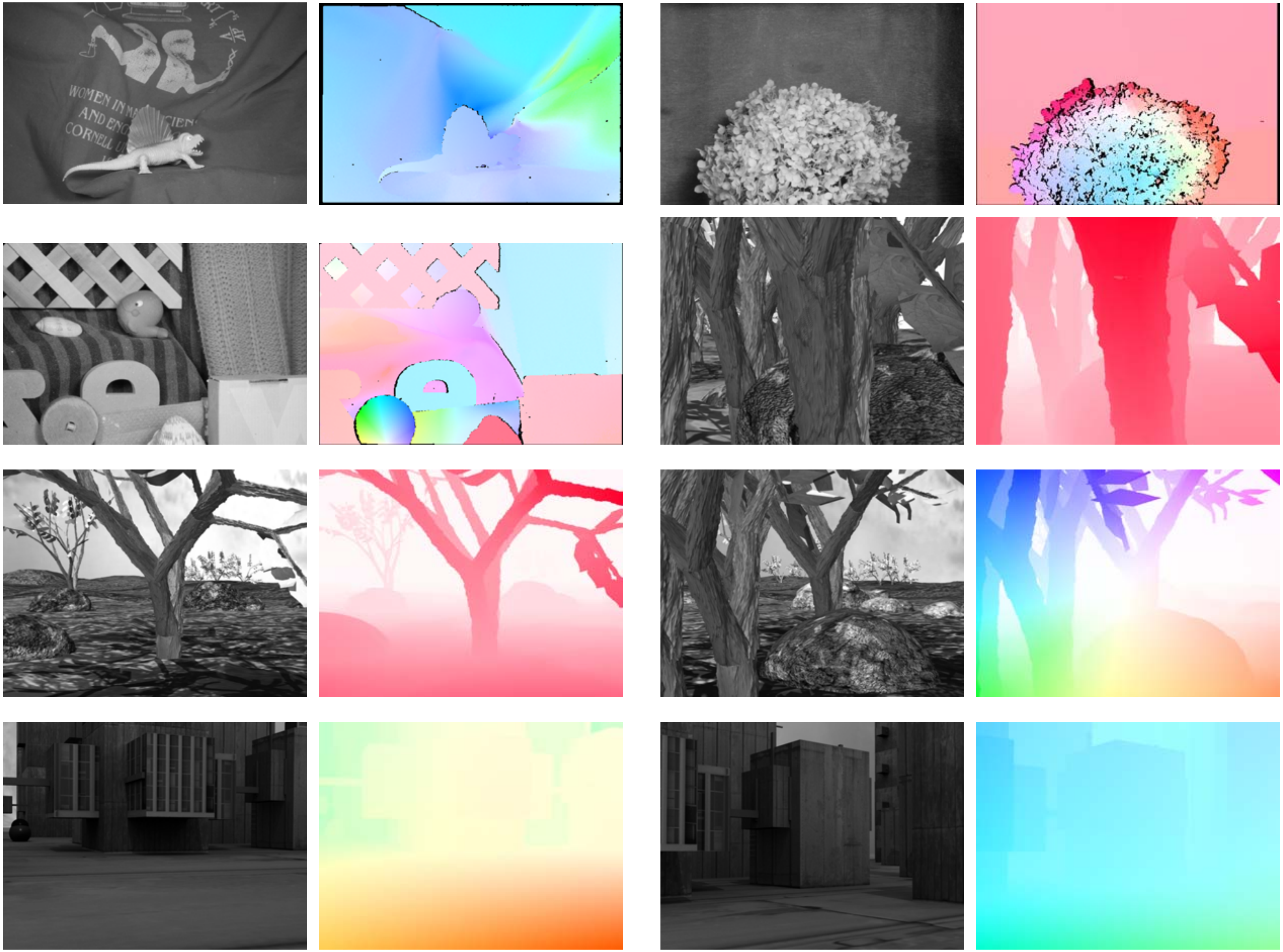
Middlebury optical flow benchmark (Baker et al. 2007)

Approach

- Ad-hoc choices, hand-tuned parameters
- Can we learn models from training data?

- Learn standard MRF model
  - Spatial smoothness (Roth & Black 2007)
  - Analyze and exploit statistics of brightness (in)constancy
- Generalize to more advanced data and spatial terms (more parameters)

Selected training sequences (out of 45)



Standard Bayesian formulation

$\mathbf{u}$: Horizontal flow $\mathbf{v}$: Vertical flow $\mathbf{I}_1$: First image $\mathbf{I}_2$: Second Image

$$p(\mathbf{u},\mathbf{v}|\mathbf{I}_1,\mathbf{I}_2) \propto p(\mathbf{I}_2|\mathbf{u},\mathbf{v},\mathbf{I}_1)p(\mathbf{u},\mathbf{v})$$

Data term

How second image can be generated from first image and flow field

Spatial term

Prior knowledge of flow field

$$E(\mathbf{u},\mathbf{v}) = E_{\mathrm{D}}(\mathbf{u},\mathbf{v}) + \lambda E_{\mathrm{S}}(\mathbf{u},\mathbf{v})$$

Brightness constancy (BC)

$$E_{\mathrm{D}}(\mathbf{u},\mathbf{v}) = \sum_{(i,j)} \rho(I_1(i,j) - I_2(i+u_{ij}, j+v_{ij}))$$

- Penalty functions ρ (for linearized BC)
  - Quadratic (Horn & Schunck)
  - Charbonnier (Bruhn et al.)
  - Lorentzian (Black & Anandan)
- Which one should we use?

Brightness constancy

$$E_{\mathrm{D}}(\mathbf{u},\mathbf{v}) = \sum_{(i,j)} \rho(I_1(i,j) - I_2(i+u_{ij}, j+v_{ij}))$$

Probabilistic interpretation

$$p_{\mathrm{BC}}(\mathbf{I}_2|\mathbf{u},\mathbf{v},\mathbf{I}_1) \propto \exp\{-\sum_{(i,j)} \rho(I_1(i,j) - I_2(i+u_{ij}, j+v_{ij}))\}$$

$$\downarrow$$

$$p_{\mathrm{BC}}(\mathbf{I}_2|\mathbf{u},\mathbf{v},\mathbf{I}_1) \propto \prod_{(i,j)} \phi(I_1(i,j) - I_2(i+u_{ij}, j+v_{ij}))$$

ϕ = potential function

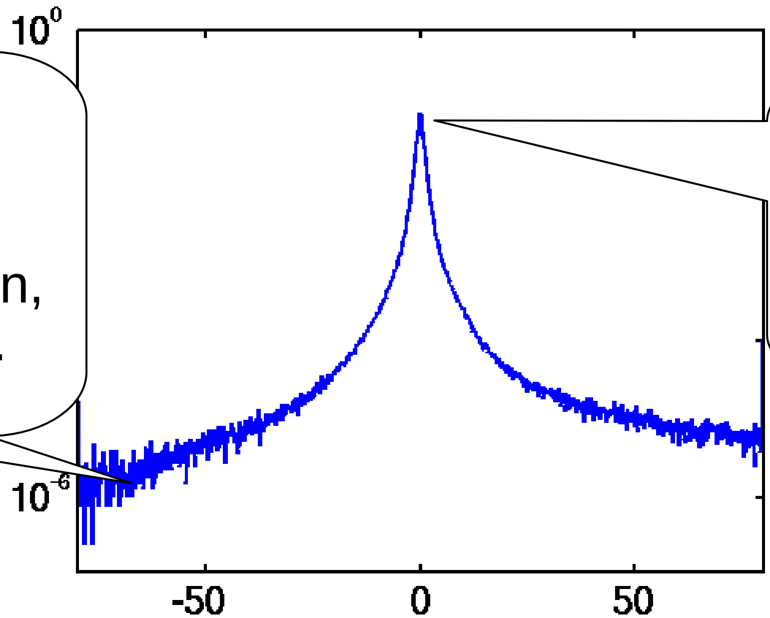
Brightness constancy

$$p_{\mathrm{BC}}(\mathbf{I}_2|\mathbf{u},\mathbf{v},\mathbf{I}_1) \propto \prod_{(i,j)} \phi(I_1(i,j) - I_2(i+u_{ij}, j+v_{ij}))$$

Histogram from training data (images + ground truth flow)

Heavy tails

violations due to reflection, occlusion, transparency etc.



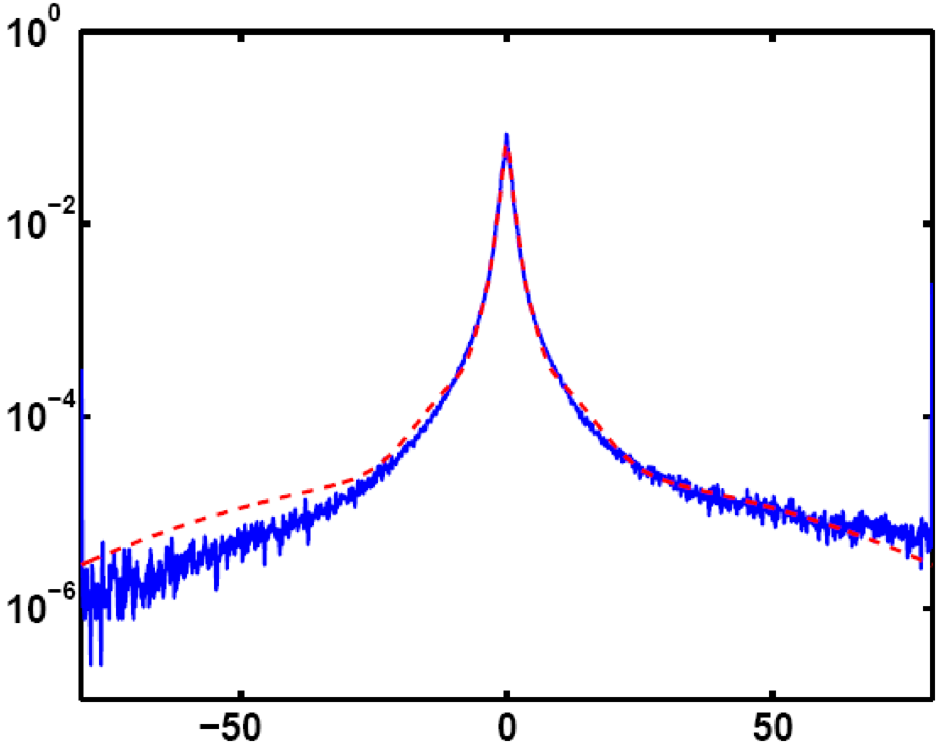
$$I_1(i,j) - I_2(i+u_{ij}, j+v_{ij})$$

Sharp peak

brightness constancy usually holds

Brightness constancy

Idea: Fit the histogram



$$I_1(i,j) - I_2(i+u_{ij}, j+v_{ij})$$

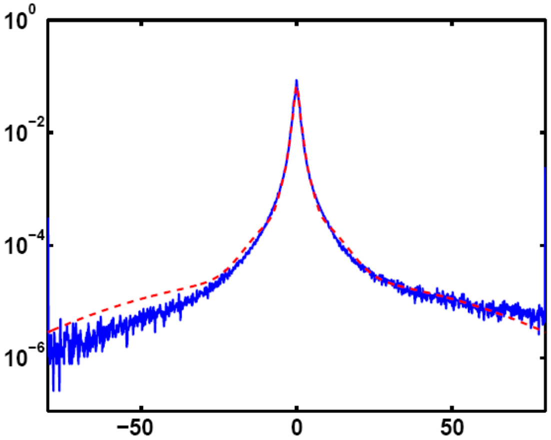
Brightness constancy

$$p_{\mathrm{BC}}(\mathbf{I}_2 | \mathbf{u}, \mathbf{v}, \mathbf{I}_1) \propto \prod_{(i,j)} \phi(I_1(i,j) - I_2(i + u_{ij}, j + v_{ij}))$$

Gaussian scale mixture (GSM) model

(Wainwright & Simoncelli 1999)

$$\phi(x; \Omega) = \sum_{l=1}^{L} \omega_l \cdot \mathcal{N}(x; 0, \sigma^2 / s_l)$$



Standard Bayesian formulation

$$p(\mathbf{u}, \mathbf{v} | \mathbf{I}_1, \mathbf{I}_2) \propto p(\mathbf{I}_2 | \mathbf{u}, \mathbf{v}, \mathbf{I}_1) p(\mathbf{u}, \mathbf{v})$$

Data term

How second image can be generated from first image and flow field

Spatial term

Prior knowledge of flow field

Spatial term

Smoothness: Neighboring pixels usually belong to same surface

Spatial term

- Smoothness expressed by canonical derivative

$$E_{\mathrm{Su}}(\mathbf{u}) = \sum_{(i,j)} \rho(u_{i,j+1} - u_{ij}) + \rho(u_{i+1,j} - u_{ij})$$

Penalty functions and parameters?

- Probabilistic interpretation: Pairwise MRF (PW)

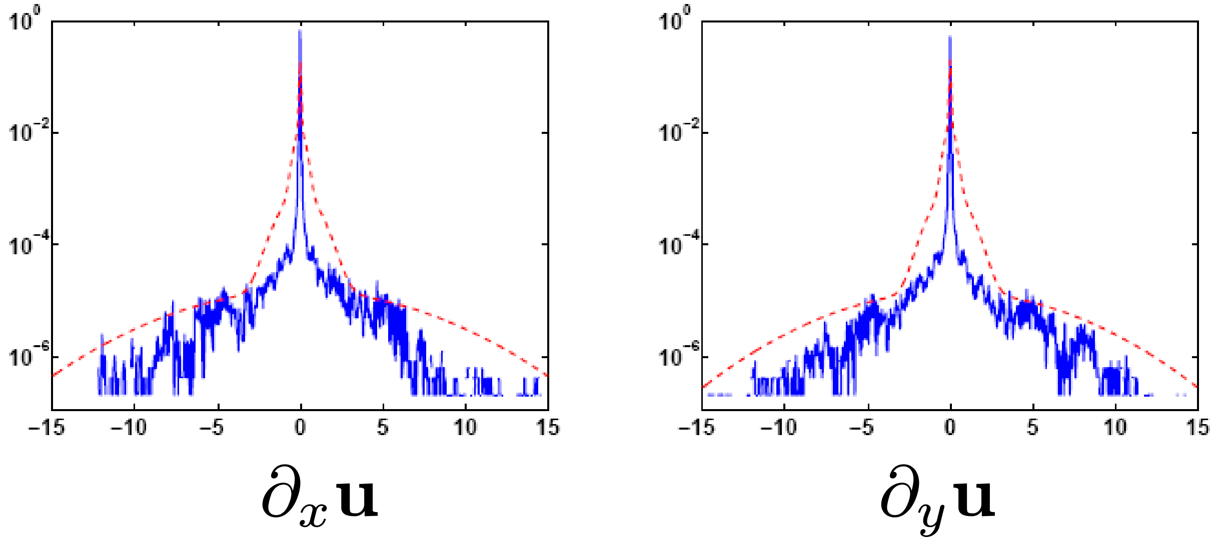
$$p_{\mathrm{PW}}(\mathbf{u}) = \frac{1}{Z} \prod_{(i,j)} \phi(u_{i,j+1} - u_{ij}) \cdot \phi(u_{i+1,j} - u_{ij})$$

Spatial term

Pairwise MRF (PW) model

$$p_{\mathrm{PW}}(\mathbf{u}) = \frac{1}{Z} \prod_{(i,j)} \phi(u_{i,j+1} - u_{ij}) \cdot \phi(u_{i+1,j} - u_{ij})$$

- Approximate ML learning by contrastive divergence (CD) algorithm (Hinton2000)

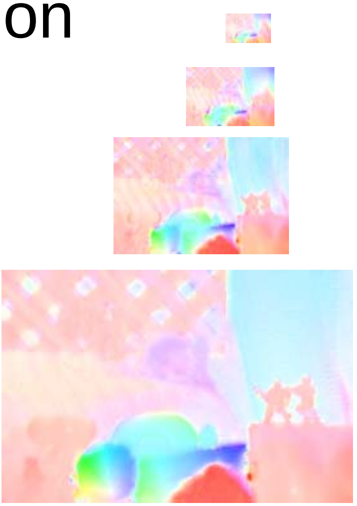


- Minimizing training loss (Li & Huttenlocher 2008)

Optical flow estimation

Maximizing posterior pdf, or minimizing energy function

- Nonlinear data term
  - Coarse-to-fine, warping-based flow estimation



Optical flow estimation

Maximizing posterior pdf, or minimizing energy function

- Nonlinear data term

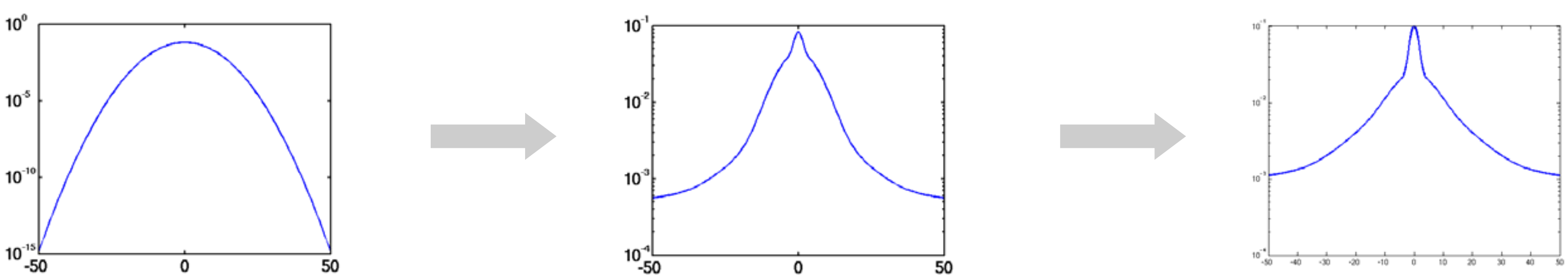
  Coarse-to-fine, warping-based flow estimation

- Non-convex potentials

  Graduated non-convexity (Blake & Zisserman 1987)

$$E_{\mathrm{C}}(\mathbf{u},\mathbf{v},\alpha) = \alpha E_{\mathrm{Q}}(\mathbf{u},\mathbf{v}) + (1-\alpha)E(\mathbf{u},\mathbf{v}),\ \ \alpha \in [0,1]$$

$\alpha = 1$ $\qquad\qquad$ $\alpha = 0.5$ $\qquad\qquad$ $\alpha = 0$



Optical flow estimation

Maximizing posterior pdf, or minimizing energy function

– Nonlinear data term

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$\alpha = 1$

$\alpha = 0.5$

$\alpha = 0$

Evaluation

Middlebury test set (Baker et al. 2007)

“Army”

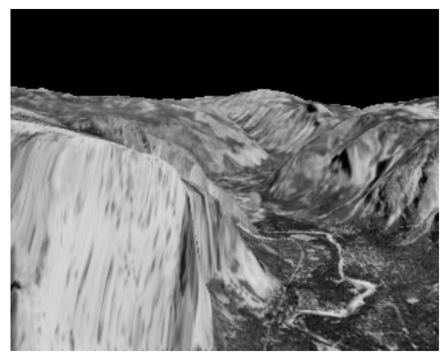
“Mequon”

“Schefflera”

“Wooden”

“Grove”

“Urban”

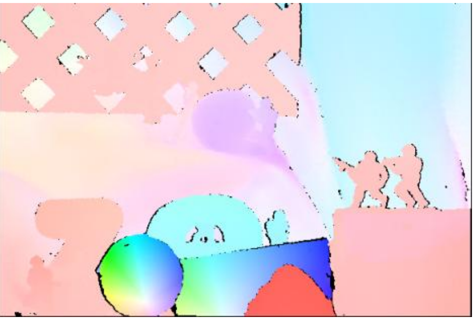


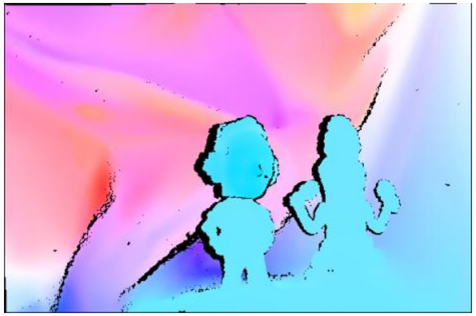
“Yosemite”

“Teddy”

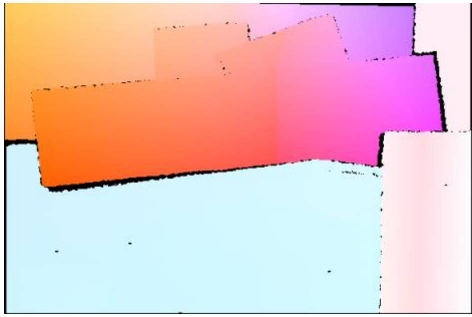
Evaluation

“Ground truth”


“Army”

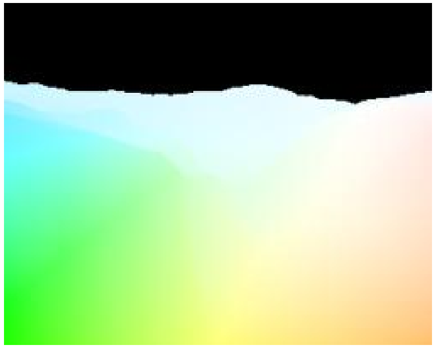

“Mequon”


“Schefflera”


“Wooden”


“Grove”


“Urban”

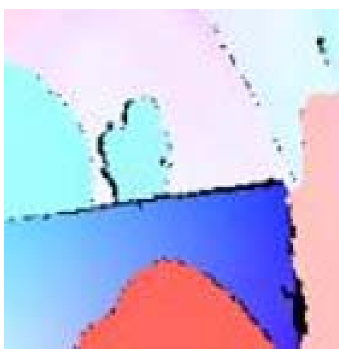
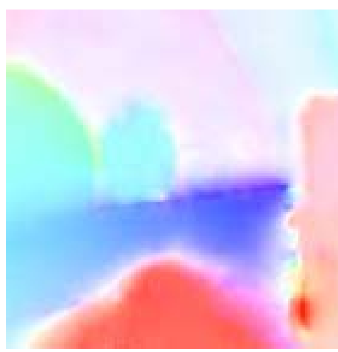

“Yosemite”


“Teddy”

Results I

AAE: Average angular error

	HS	BA	PW-BC
Average AAE	8.72	7.17	7.37
Rank (Middlebury)	18.2	12.1	13.0

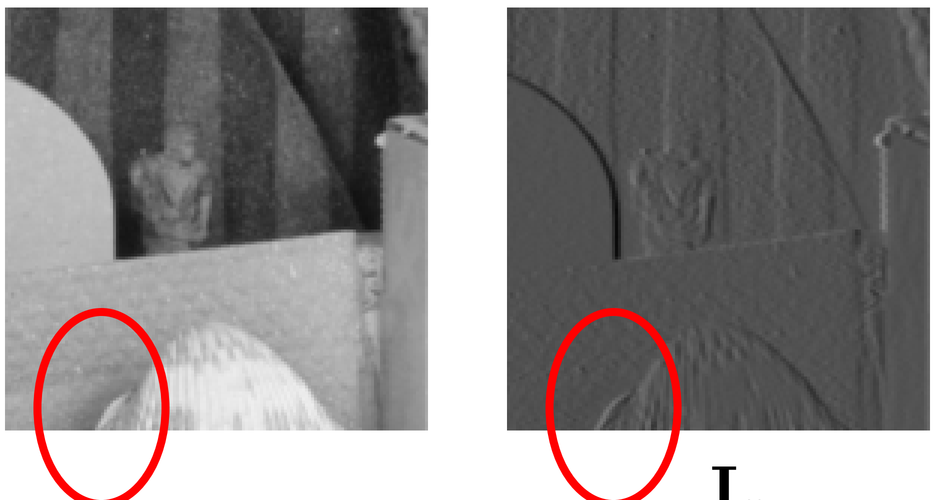
Ground truth

PW-BC = Pairwise MRF + brightness constancy

Brightness constancy

$$p_{\mathrm{BC}}(\mathbf{I}_2|\mathbf{u},\mathbf{v},\mathbf{I}_1) \propto \prod_{(i,j)} \phi(I_1(i,j) - I_2(i+u_{ij}, j+v_{ij}))$$

Independence assumption



$\mathbf{I}_x$

High order constancy (Adelson et al. 1984, Brox et al. 2004)

High order constancy

$$p_{\mathrm{BC}}(\mathbf{I}_2|\mathbf{u},\mathbf{v},\mathbf{I}_1) \propto \prod_{(i,j)} \phi\,(\quad I_1(i,j) \; - \quad I_2(i+u_{ij}, j+v_{ij}))$$

$$p(\mathbf{I}_2|\mathbf{u},\mathbf{v},\mathbf{I}_1) \propto \prod_{(i,j)} \phi\,((J * I_1)(i,j) - (J * I_2)(i+u_{ij}, j+v_{ij}))$$

$$p(\mathbf{I}_2|\mathbf{u},\mathbf{v},\mathbf{I}_1) \propto \prod_{(i,j)} \prod_{k} \phi_k((J_k * I_1)(i,j) - (J_k * I_2)(i+u_{ij}, j+v_{ij}))$$

High order random field

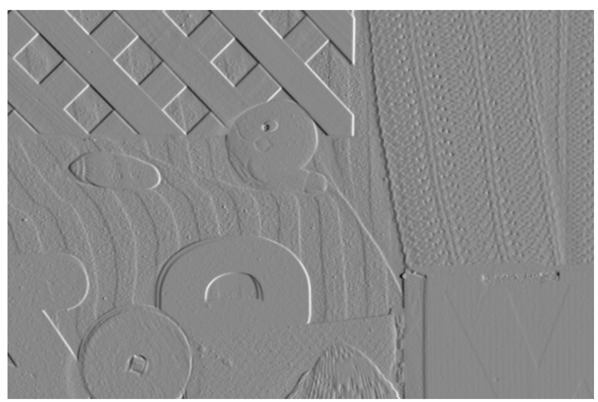
Fixed filter response constancy (FFC)

$$p_{\mathrm{FFC}}(\mathbf{I}_2|\mathbf{u},\mathbf{v},\mathbf{I}_1) \propto \prod_{(i,j)} \prod_{k} \phi_k((J_k * I_1)(i,j) - (J_k * I_2)(i+u_{ij}, j+v_{ij}))$$

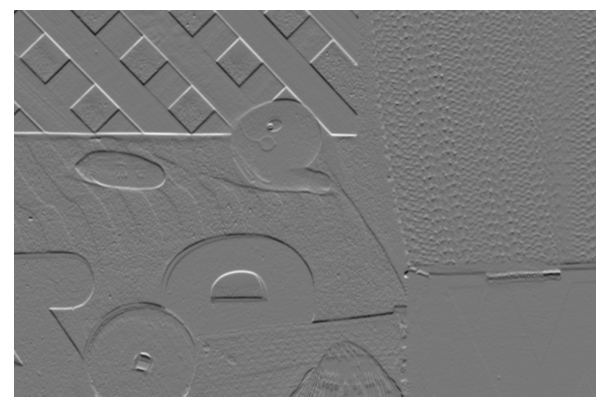
- High order random field: Learn experts ϕ_k from training data

Gaussian
σ = 0.4

Horizontal
derivative

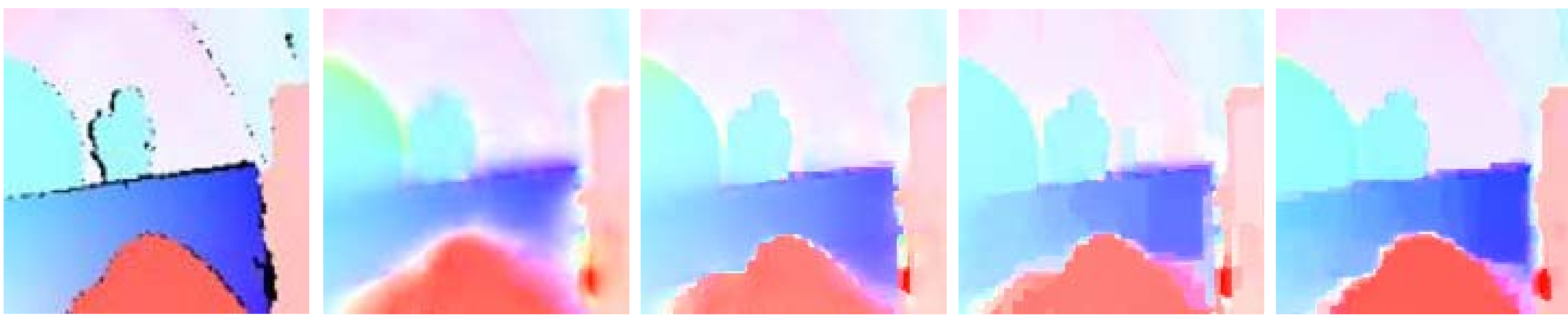


Vertical
derivative



Results II

	HS	BA	PW-BC	PW-FFC
Average AAE	8.72	7.17	7.37	5.96
Rank	18.2	12.1	13.0	12.3



Ground truth

PW-FFC = Pairwise MRF + fixed filter response constancy

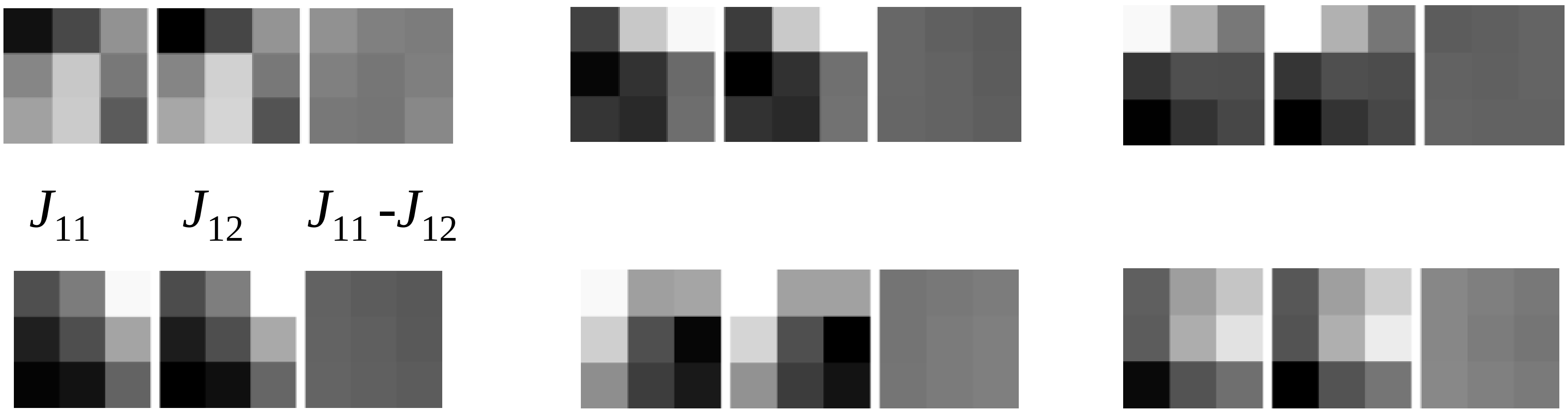
Learned filter response constancy (LFC)

What are appropriate filters?

$$p_{\mathrm{LFC}}(\mathbf{I}_2|\mathbf{u},\mathbf{v},\mathbf{I}_1) \propto \prod_{(i,j)}\prod_{k}\phi_k((J_{k1} * I_1)(i,j) - (J_{k2} * I_2)(i+u_{ij}, j+v_{ij}))$$

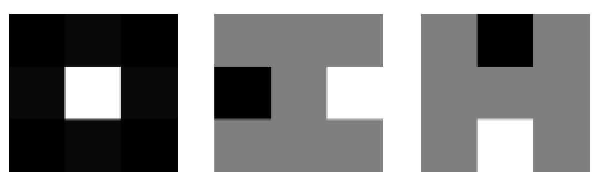
Can be viewed as a spatio-temporal *field of experts* (Roth & Black 2005)

Data term: learned filters



– J_{k1} and J_{k2} look similar, though not enforced to be same

– Unlike traditional filters



Results III

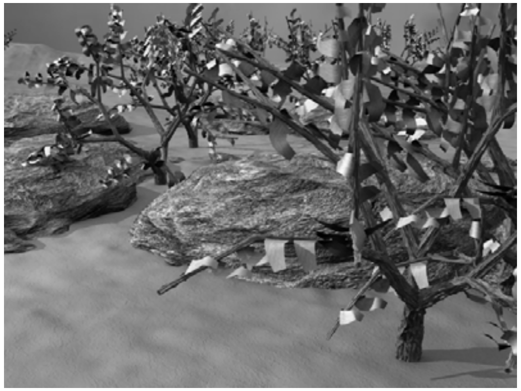
	BA	PW-BC	PW-FFC	PW-LFC
Average AAE	7.17	7.37	5.96	5.47
Rank	12.1	13.0	12.3	11.0

Ground truth

PW-LFC = Pairwise MRF + learned filter response constancy

Spatial smoothness

- Motion boundaries often correspond to image edges



First frame of “Grove3”

Ground truth flow

- Oriented smoothness of flow field (Nagel & Enkelmann 1986)

  Less smoothing to flow orthogonal to image edges
- How to formulate it statistically?

Bayesian formulation

$$p(\mathbf{u}, \mathbf{v} | \mathbf{I}_1, \mathbf{I}_2) \propto p(\mathbf{I}_2 | \mathbf{u}, \mathbf{v}, \mathbf{I}_1) p(\mathbf{u}, \mathbf{v})$$

$$p(\mathbf{u}, \mathbf{v} | \mathbf{I}_1, \mathbf{I}_2) \propto p(\mathbf{I}_2 | \mathbf{u}, \mathbf{v}, \mathbf{I}_1) p(\mathbf{u}, \mathbf{v} | \mathbf{I}_1)$$

Spatial term

Prior knowledge of flow field, given first image

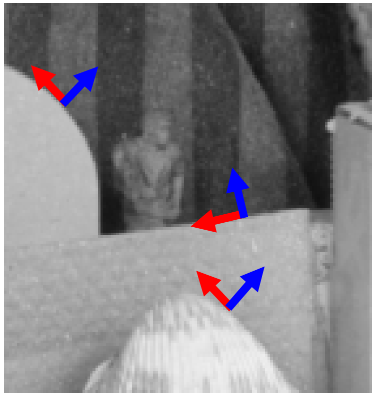
Steered models

Steerable random field (Roth & Black 2007)

$$p_{\mathrm{SRF}}(\mathbf{u}|\mathbf{I}_1) \propto \prod_{(i,j)} \phi((\partial_O^{\mathbf{I}_1} u)_{ij}) \cdot \phi((\partial_A^{\mathbf{I}_1} u)_{ij})$$

Smoothness expressed by steered derivatives

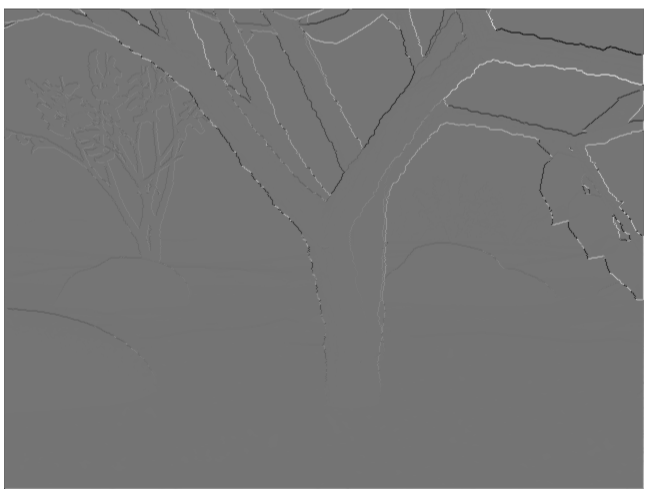
$$\begin{aligned} \partial_O^{\mathbf{I}_1} &= \cos\theta(\mathbf{I}_1)\cdot\partial_x + \sin\theta(\mathbf{I}_1)\cdot\partial_y \\ \partial_A^{\mathbf{I}_1} &= -\sin\theta(\mathbf{I}_1)\cdot\partial_x + \cos\theta(\mathbf{I}_1)\cdot\partial_y \end{aligned}$$



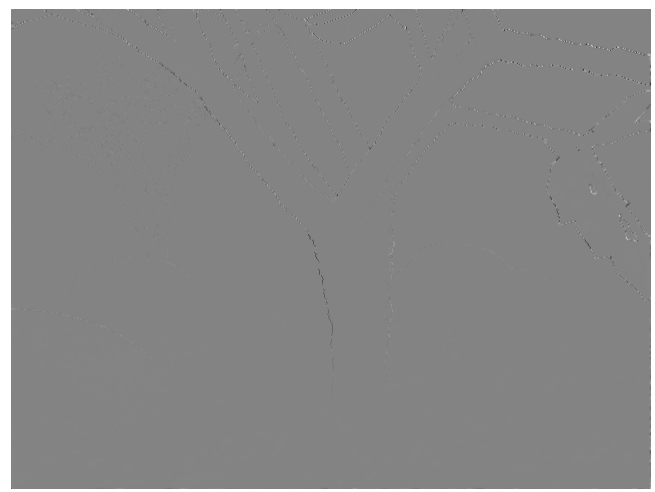
Steered derivatives

I_1

$\mathbf{u}$



$\partial_O^{\mathbf{I}_1}\mathbf{u}$

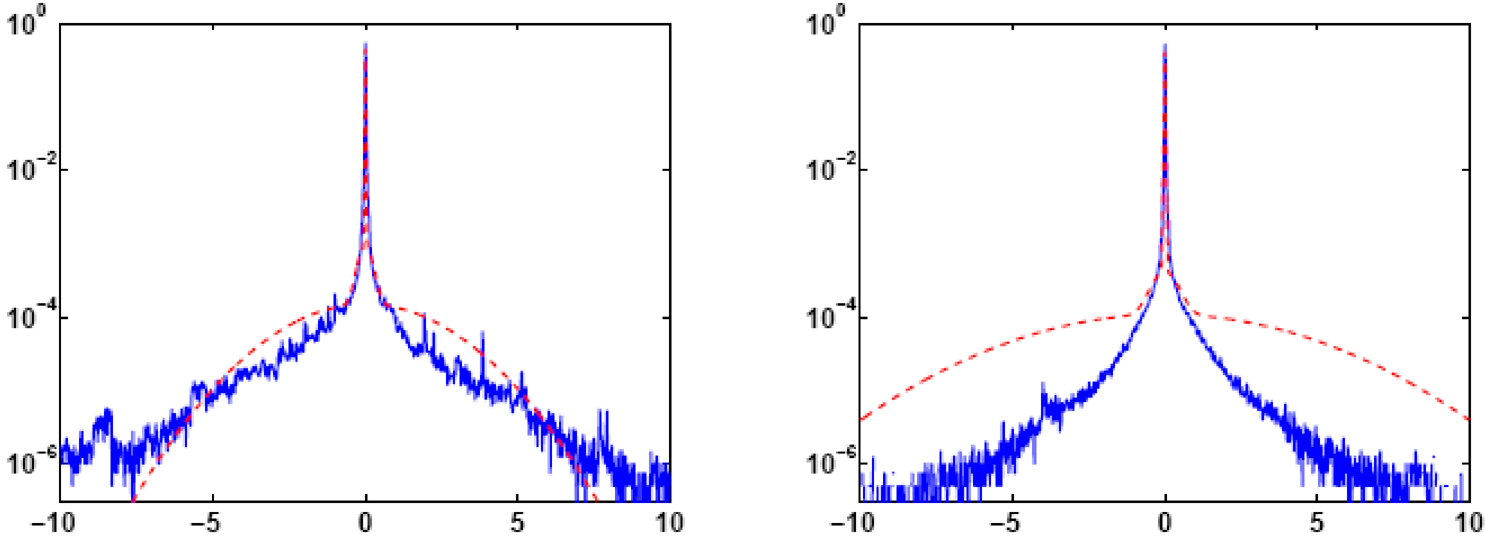


$\partial_A^{\mathbf{I}_1}\mathbf{u}$

Steered models

Steerable random field (SRF)

$$p_{\mathrm{SRF}}(\mathbf{u}|\mathbf{I}_1) \propto \prod_{(i,j)} \phi((\partial_O^{\mathbf{I}_1} u)_{ij}) \cdot \phi((\partial_A^{\mathbf{I}_1} u)_{ij})$$



$\partial_O^{\mathbf{I}_1} \mathbf{u}$ $\partial_A^{\mathbf{I}_1} \mathbf{u}$

Potential functions not necessarily the same as marginal (Zhu et al. 1997)

Results IV

	PW-FFC	SRF-FFC	PW-LFC	SRF-LFC
Average AAE	5.96	5.79	5.47	5.34
Rank	12.3	10.5	11.0	9.5

Ground truth

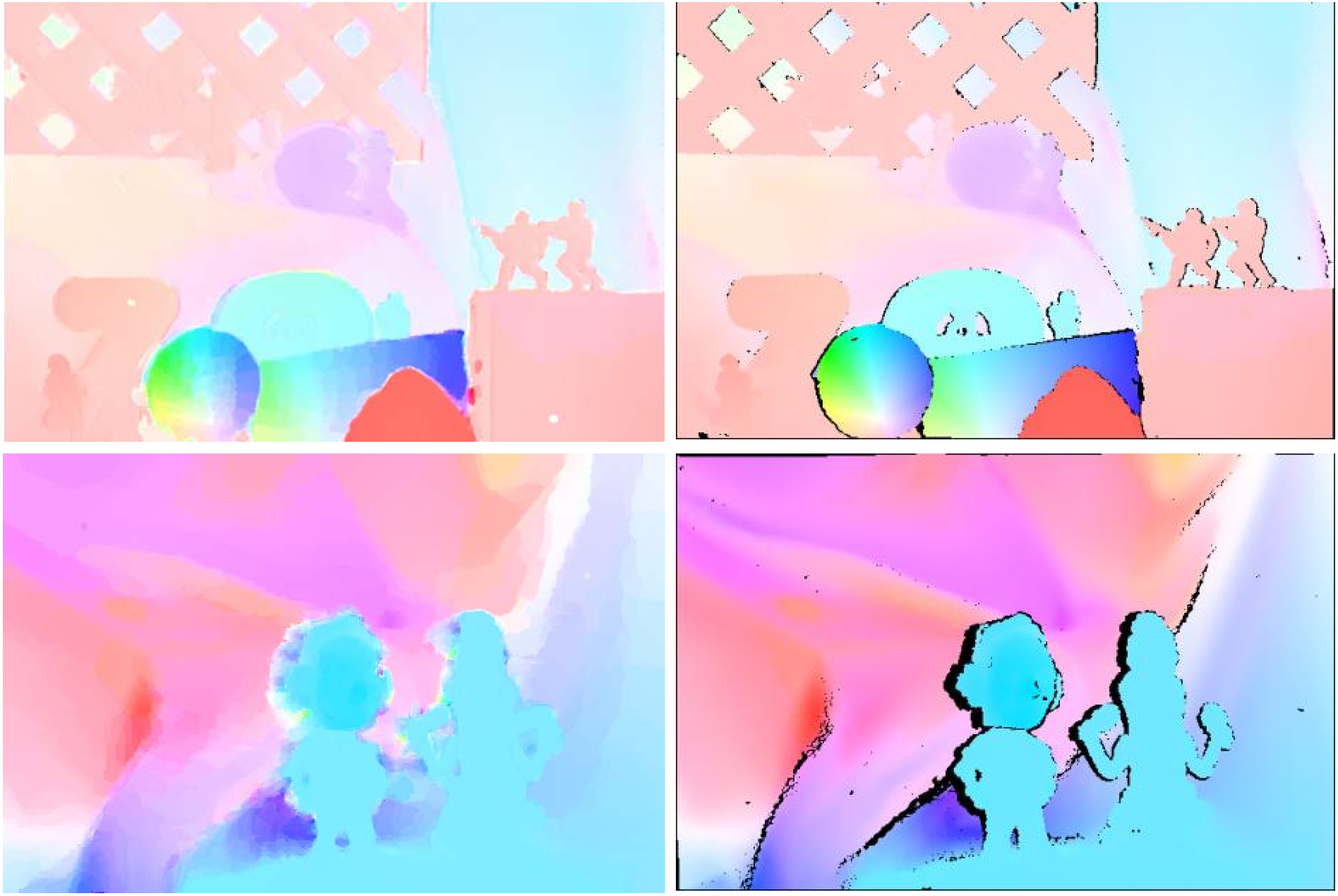
SRF = Steerable random field

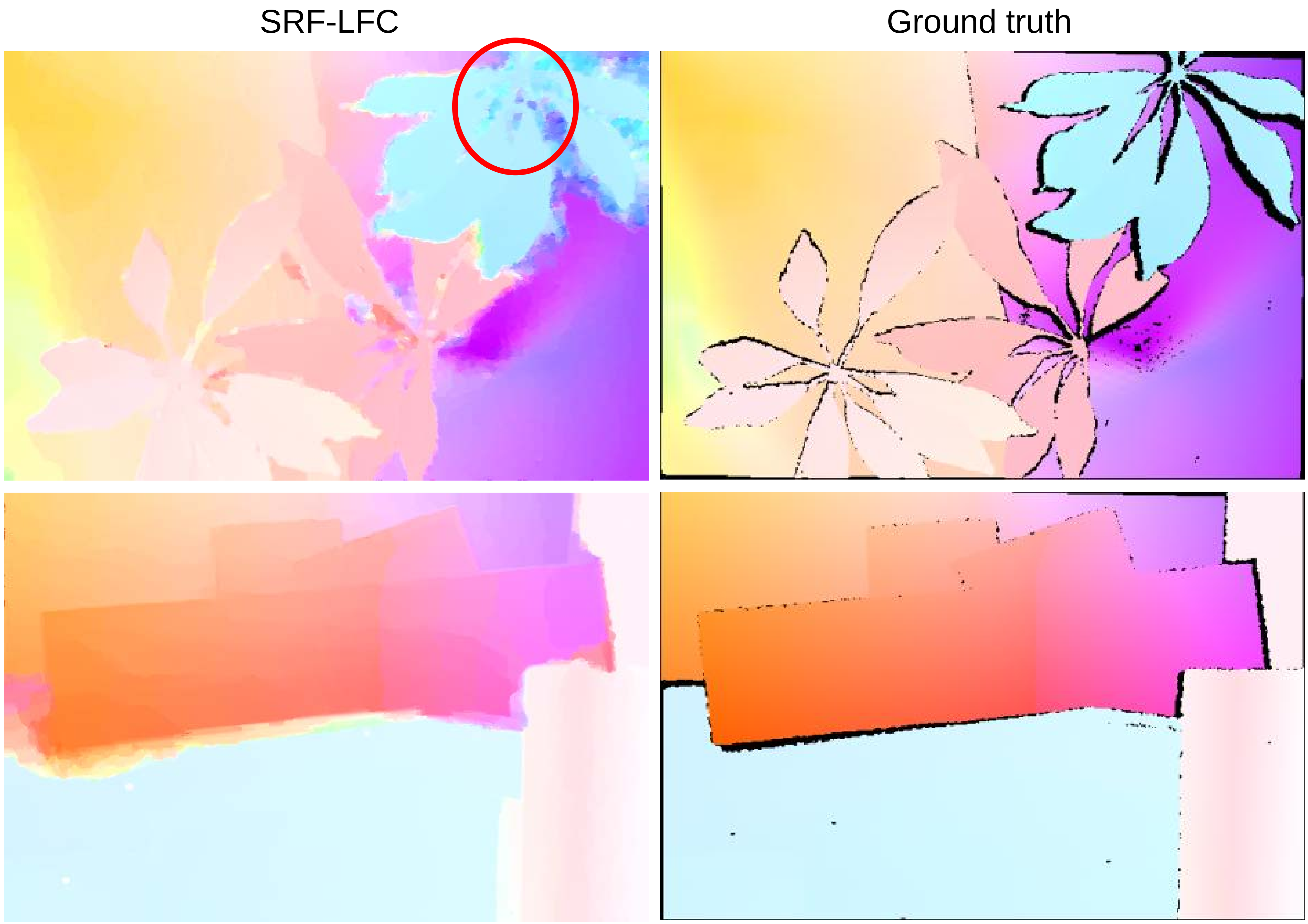
Results IV

Near motion boundaries (PW vs. SRF)

Model	Average AAE
PW-BC	15.81
SRF-BC	15.47
PW-FFC	15.37
SRF-FFC	14.74
PW-LFC	14.97
SRF-LFC	14.47

SRF-LFC Ground truth




SRF-LFC
Ground truth

Average angle error	avg.	Army (Hidden texture)			Mequon (Hidden texture)			Schefflera (Hidden texture)			Wooden (Hidden texture)			Grove (Synthetic)			Urban (Synthetic)			Yosemite (Synthetic)			Teddy (Stereo)		
		GT	im0	im1	GT	im0	im1	GT	im0	im1	GT	im0	im1	GT	im0	im1	GT	im0	im1	GT	im0	im1	GT	im0	im1
	rank	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext
F-TV-L1 [18]	4.7	5.44 7	12.5 5	5.69 9	5.46 8	15.0 8	4.03 8	7.48 8	16.3 5	3.42 5	5.08 6	23.3 9	2.81 5	**3.42** 1	**4.34** 1	**3.03** 1	4.05 2	**15.1** 1	**3.18** 1	2.43 6	3.92 6	1.87 5	3.90 2	9.35 2	**2.61** 1
Brox et al. [7]	5.5	4.80 6	14.4 9	4.29 7	4.05 6	13.5 4	3.71 6	6.63 4	16.0 4	7.26 8	5.22 7	22.7 8	3.22 7	4.56 10	6.09 15	3.40 2	**3.97** 1	17.9 5	3.41 2	2.07 3	3.76 4	1.18 2	5.14 3	11.9 5	4.28 3
Fusion [9]	5.9	4.43 4	13.7 7	4.08 5	**2.47** 1	**8.91** 1	2.24 2	**3.70** 1	**9.68** 1	3.12 3	**3.68** 1	19.8 3	2.54 4	4.26 7	5.16 8	4.31 10	6.32 4	16.8 3	6.15 6	4.55 14	5.78 14	3.10 10	7.12 10	13.6 10	7.86 12
Dynamic MRF [10]	6.3	4.58 5	12.4 4	4.14 6	3.25 3	13.9 6	2.27 3	6.02 3	16.8 6	**2.36** 1	4.39 4	22.6 7	2.51 3	3.61 2	4.55 3	3.46 3	6.81 8	22.2 14	6.78 8	2.41 5	3.48 2	3.69 11	9.26 15	17.8 14	10.2 15
SegOF [13]	6.5	5.85 8	13.5 6	3.98 4	7.40 9	14.9 7	8.13 12	8.55 10	17.3 9	9.01 9	6.50 11	**18.1** 1	5.14 11	3.90 6	4.53 2	4.81 13	6.57 7	21.7 12	6.81 9	**1.65** 1	3.49 3	**1.08** 1	**3.71** 1	**9.23** 1	3.63 2
CBF [15]	6.5	3.95 2	**10.1** 1	3.44 3	3.70 4	10.6 2	3.85 7	5.64 2	13.5 2	3.34 4	3.71 2	21.5 6	**1.99** 1	4.36 8	5.50 9	3.55 4	11.3 15	19.1 8	9.05 14	6.79 16	7.37 17	11.6 16	5.50 4	11.8 4	5.66 5
Second-order prior [11]	7.6	**3.84** 1	11.2 2	**3.11** 1	3.12 2	12.9 3	**2.17** 1	6.96 6	17.2 8	2.83 2	3.84 3	20.5 5	2.09 2	4.83 15	5.83 13	3.90 6	14.0 16	21.8 13	8.28 11	7.74 17	6.88 15	11.7 17	6.74 8	13.4 9	5.80 6
Learning Flow [14]	7.6	4.23 3	11.7 3	3.41 2	4.16 7	15.3 9	3.42 5	6.78 5	16.9 7	3.83 6	6.41 10	25.3 11	4.25 9	4.66 13	6.01 14	4.00 8	6.33 6	20.7 9	5.30 3	3.09 9	4.84 9	2.91 9	7.08 9	15.0 12	5.27 4
GraphCuts [17]	7.6	6.25 9	14.3 8	5.53 8	8.60 11	20.1 12	6.61 9	7.91 9	15.4 3	10.9 10	4.88 5	19.0 2	3.05 6	3.78 3	4.71 6	3.94 7	8.74 11	16.4 2	5.39 4	4.04 12	4.87 10	4.85 14	6.35 7	12.2 6	6.05 8
SPSA-learn [16]	9.0	6.84 10	16.7 10	6.74 12	8.47 10	19.4 11	7.49 10	12.5 11	23.1 11	13.1 12	8.40 12	25.8 12	7.08 12	3.87 5	4.66 5	4.10 9	6.32 4	18.8 7	6.89 10	2.56 7	3.85 5	1.79 4	7.29 11	12.5 7	7.47 10
2D-CLG [3]	10.1	10.1 16	22.6 17	7.59 13	9.84 14	16.9 10	11.1 15	16.9 15	28.2 16	18.8 15	14.1 16	31.1 15	13.1 16	3.86 4	4.62 4	4.53 11	5.98 3	21.2 10	5.97 5	1.76 2	**3.14** 1	1.46 3	6.29 6	12.9 8	5.81 7
LP Registration [8]	10.1	7.36 11	16.8 11	6.30 10	3.94 5	13.8 5	3.00 4	7.33 7	17.8 10	4.43 7	5.54 8	24.5 10	3.57 8	4.51 9	4.99 7	6.05 15	10.6 14	21.6 11	9.76 15	4.54 13	5.48 13	3.95 12	8.15 12	17.9 15	7.82 11
GroupFlow [12]	10.6	8.00 13	18.6 12	8.09 14	11.1 15	23.7 17	10.3 14	12.6 12	25.6 13	12.8 11	5.84 9	20.3 4	4.39 10	4.69 14	5.81 12	3.67 5	9.29 13	22.4 15	10.1 16	2.11 4	3.99 7	2.29 8	5.75 5	10.0 3	7.39 9
Black & Anandan 2 [2]	11.2	7.83 12	18.7 13	6.41 11	9.70 13	21.9 13	8.60 13	13.7 13	23.7 12	18.1 14	10.9 13	30.0 13	9.44 13	4.60 11	5.55 10	5.06 14	7.85 9	17.6 4	6.38 7	2.61 8	4.44 8	2.15 7	8.58 13	14.3 11	8.54 13
Horn & Schunck [6]	12.9	8.01 14	19.9 15	8.38 15	9.13 12	23.2 16	7.71 11	14.2 14	25.9 14	14.6 13	12.4 14	30.6 14	11.3 14	4.64 12	5.64 11	4.60 12	8.21 10	24.4 16	8.45 12	4.01 11	5.41 12	1.95 6	9.16 14	17.5 13	8.86 14
Black & Anandan [1]	14.3	9.32 15	19.4 14	10.0 16	13.5 16	22.5 14	14.3 16	17.2 16	27.4 15	18.9 16	14.0 15	32.0 16	12.9 15	5.89 16	6.74 16	8.03 16	8.99 12	17.9 5	8.77 13	3.10 10	4.88 11	3.96 13	13.2 16	18.9 16	15.2 16
Pyramid LK [4]	17.2	13.9 17	20.9 16	21.4 18	24.1 18	23.1 15	30.2 18	20.9 18	29.5 17	21.9 18	22.2 17	34.6 17	25.0 17	18.7 18	23.1 18	20.2 18	21.2 18	24.5 17	21.0 18	6.41 15	7.02 16	10.8 15	25.6 18	31.5 18	34.5 18
MediaPlayer^{TM} [5]	17.5	18.3 18	30.8 18	15.0 17	17.7 17	29.2 18	17.4 17	19.9 17	32.7 18	21.6 17	26.3 18	45.9 18	25.9 18	7.33 17	7.33 17	10.0 17	19.0 17	31.4 18	19.1 17	12.7 18	18.7 18	17.2 18	17.4 17	22.9 17	20.7 17

Concluding remarks

- A unified statistical framework for optical flow
- Rigorous learned models from training data
  - Steered model
  - Filter response constancy
  - Learning filters
  - Each with improved accuracy
- Hand-tuning vs. learning
- Optimization not a focus (Xu et al. 2008, Trobin et al. 2008, Lempitsky et al. 2008, Glocker et al. 2008)
- Still limited training data

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Average angle error	avg.	Army			Mequon			Schefflera			Wooden			Grove			Urban			Yosemite			Teddy		
		(Hidden texture) GT im0 im1			(Hidden texture) GT im0 im1			(Hidden texture) GT im0 im1			(Hidden texture) GT im0 im1			(Synthetic) GT im0 im1			(Synthetic) GT im0 im1			(Synthetic) GT im0 im1			(Stereo) GT im0 im1		
	rank	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext	all	disc	untext
Spatially variant [21]	4.1	3.73 2	10.2 3	3.33 3	3.02 3	11.0 4	2.67 5	5.36 3	13.8 4	**2.35** 1	**3.67** 1	19.3 4	1.84 2	3.81 5	4.81 9	3.69 8	4.48 4	16.0 3	3.90 4	2.11 4	3.26 2	2.12 8	4.66 6	9.41 4	4.35 7
TV-L1-improved [18]	5.0	**3.36** 1	**9.63** 1	**2.62** 1	2.82 2	10.7 3	2.23 2	6.50 7	15.8 7	2.73 3	3.80 4	21.3 8	**1.76** 1	**3.34** 1	4.38 2	**2.39** 1	5.97 5	18.1 10	5.67 7	3.57 13	4.92 14	3.43 13	4.01 5	9.84 5	3.44 3
F-TV-L1 [17]	6.6	5.44 10	12.5 8	5.69 13	5.46 11	15.0 11	4.03 11	7.48 11	16.3 9	3.42 8	5.08 10	23.3 12	2.81 8	3.42 2	**4.34** 1	3.03 2	4.05 2	**15.1** 1	**3.18** 1	2.43 7	3.92 7	1.87 6	3.90 4	9.35 3	**2.61** 1
JIF-Reg [20]	7.1	5.09 9	14.5 13	4.21 10	3.98 7	14.9 9	2.91 6	5.63 4	14.8 5	3.64 9	4.88 8	24.5 13	2.66 7	3.99 10	5.22 11	3.19 3	4.25 3	17.6 6	3.82 3	2.92 10	4.65 10	1.45 3	3.76 3	9.92 6	3.39 2
DPOF [19]	7.6	5.63 11	10.9 4	4.16 9	4.05 8	12.1 5	3.31 7	3.87 2	**8.82** 1	3.17 6	4.34 6	**16.2** 1	3.13 10	3.95 9	4.78 8	4.17 13	6.69 11	15.2 2	6.27 10	5.62 17	6.89 18	6.60 17	**2.44** 1	**4.83** 1	3.74 5
Fusion [8]	8.0	4.43 6	13.7 10	4.08 7	**2.47** 1	**8.91** 1	2.24 3	**3.70** 1	9.68 2	3.12 5	3.68 2	19.8 5	2.54 6	4.26 11	5.16 10	4.31 14	6.32 7	16.8 5	6.15 9	4.55 16	5.78 16	3.10 12	7.12 14	13.6 14	7.86 15
Brox et al. [7]	8.0	4.80 8	14.4 12	4.29 11	4.05 8	13.5 7	3.71 9	6.63 8	16.0 8	7.26 11	5.22 11	22.7 11	3.22 11	4.56 13	6.09 18	3.40 4	**3.97** 1	17.9 8	3.41 2	2.07 3	3.76 5	1.18 2	5.14 7	11.9 9	4.28 6
Dynamic MRF [9]	8.7	4.58 7	12.4 7	4.14 8	3.25 5	13.9 8	2.27 4	6.02 6	16.8 10	2.36 2	4.39 7	22.6 10	2.51 5	3.61 3	4.55 4	3.46 5	6.81 12	22.2 17	6.78 12	2.41 6	3.48 3	3.69 14	9.26 18	17.8 18	10.2 18
SegOF [12]	8.8	5.85 12	13.5 9	3.98 6	7.40 12	14.9 9	8.13 15	8.55 13	17.3 13	9.01 12	6.50 14	18.1 2	5.14 14	3.90 8	4.53 3	4.81 17	6.57 10	21.7 15	6.81 13	**1.65** 1	3.49 4	**1.08** 1	3.71 2	9.23 2	3.63 4
CBF [14]	9.2	3.95 4	10.1 2	3.44 5	3.70 6	10.6 2	3.85 10	5.64 5	13.5 3	3.34 7	3.71 3	21.5 9	1.99 3	4.36 12	5.50 12	3.55 6	11.3 18	19.1 12	9.05 18	6.79 19	7.37 20	11.6 19	5.50 8	11.8 8	5.66 9
Second-order prior [10]	10.3	3.84 3	11.2 5	3.11 2	3.12 4	12.9 6	**2.17** 1	6.96 10	17.2 12	2.83 4	3.84 5	20.5 7	2.09 4	4.83 18	5.83 16	3.90 9	14.0 19	21.8 16	8.28 15	7.74 20	6.88 17	11.7 20	6.74 12	13.4 13	5.80 10
GraphCuts [16]	10.4	6.25 13	14.3 11	5.53 12	8.60 14	20.1 15	6.61 12	7.91 12	15.4 6	10.9 13	4.88 8	19.0 3	3.05 9	3.78 4	4.71 7	3.94 10	8.74 15	16.4 4	5.39 6	4.04 15	4.87 12	4.85 16	6.35 11	12.2 10	6.05 12
Learning Flow [13]	10.6	4.23 5	11.7 6	3.41 4	4.16 10	15.3 12	3.42 8	6.78 9	16.9 11	3.83 10	6.41 13	25.3 14	4.25 12	4.66 16	6.01 17	4.00 11	6.33 9	20.7 13	5.30 5	3.09 11	4.84 11	2.91 11	7.08 13	15.0 16	5.27 8
SPSA-learn [15]	12.0	6.84 14	16.7 14	6.74 15	8.47 13	19.4 14	7.49 13	12.5 14	23.1 14	13.1 15	8.40 15	25.8 15	7.08 15	3.87 7	4.66 6	4.10 12	6.32 7	18.8 11	6.89 14	2.56 8	3.85 6	1.79 5	7.29 15	12.5 11	7.47 14
2D-CLG [3]	12.8	10.1 19	22.6 20	7.59 16	9.84 17	16.9 13	11.1 18	16.9 18	28.2 19	18.8 18	14.1 19	31.1 18	13.1 19	3.86 6	4.62 5	4.53 15	5.98 6	21.2 14	5.97 8	1.76 2	**3.14** 1	1.46 4	6.29 10	12.9 12	5.81 11
GroupFlow [11]	13.5	8.00 16	18.6 15	8.09 17	11.1 18	23.7 20	10.3 17	12.6 15	25.6 16	12.8 14	5.84 12	20.3 6	4.39 13	4.69 17	5.81 15	3.67 7	9.29 17	22.4 18	10.1 19	2.11 4	3.99 8	2.29 10	5.75 9	10.0 7	7.39 13
Black & Anandan 2 [2]	14.1	7.83 15	18.7 16	6.41 14	9.70 16	21.9 16	8.60 16	13.7 16	23.7 15	18.1 17	10.9 16	30.0 16	9.44 16	4.60 14	5.55 13	5.06 18	7.85 13	17.6 6	6.38 11	2.61 9	4.44 9	2.15 9	8.58 16	14.3 15	8.54 16
Horn & Schunck [6]	16.0	8.01 17	19.9 18	8.38 18	9.13 15	23.2 19	7.71 14	14.2 17	25.9 17	14.6 16	12.4 17	30.6 17	11.3 17	4.64 15	5.64 14	4.60 16	8.21 14	24.4 19	8.45 16	4.01 14	5.41 15	1.95 7	9.16 17	17.5 17	8.86 17
Black & Anandan [1]	17.3	9.32 18	19.4 17	10.0 19	13.5 19	22.5 17	14.3 19	17.2 19	27.4 18	18.9 19	14.0 18	32.0 19	12.9 18	5.89 19	6.74 19	8.03 19	8.99 16	17.9 8	8.77 17	3.10 12	4.88 13	3.96 15	13.2 19	18.9 19	15.2 19
Pyramid LK [4]	20.2	13.9 20	20.9 19	21.4 21	24.1 21	23.1 18	30.2 21	20.9 21	29.5 20	21.9 21	22.2 20	34.6 20	25.0 20	18.7 21	23.1 21	20.2 21	21.2 21	24.5 20	21.0 21	6.41 18	7.02 19	10.8 18	25.6 21	31.5 21	34.5 21
MediaPlayer™ [5]	20.5	18.3 21	30.8 21	15.0 20	17.7 20	29.2 21	17.4 20	19.9 20	32.7 21	21.6 20	26.3 21	45.9 21	25.9 21	7.33 20	7.33 20	10.0 20	19.0 20	31.4 21	19.1 20	12.7 21	18.7 21	17.2 21	17.4 20	22.9 20	20.7 20