What Communication Complexity Can Tell Us About Circuit Complexity

by

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ABSTRACT

In this thesis, I explore the relationship between different systems of representation—in particular, different monotone circuit models. As shown in [RM99], there are monotone boolean functions with significantly more efficient representations on monotone circuits than on monotone formulas. In other words, there are monotone boolean functions with small circuit representations when fanout is permitted but only large circuit representations when it is not. However, there are plenty of boolean functions that do not reap such benefits from the introduction of fanout, so we can ask—why are only a subset of boolean functions amenable to efficient representation via fanout? Here, I provide a communication-theoretic account as to why. That is, I give a characterization in terms of communication complexity of the boolean functions that are amenable to efficient representation via fanout. My ultimate hope for this thesis is to begin to demonstrate how the study of complexity theory can offer insight into the importance of linguistic diversity.

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