Algorithms for Large-Scale Prescriptive Evacuations

by

Julia Romanski

Submitted to the Department of Computer Science
in partial fulfillment of the requirements for the degree of

Master of Science in Computer Science

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Abstract

This thesis considers prescriptive evacuation planning for a region threatened by a natural disaster such as a flood, hurricane, or bushfire. The evacuation planning problem is modeled as a dynamic network flow problem on an evacuation graph, where the objective is to maximize the number of people evacuated within a given time horizon. In particular, this work considers convergent evacuation planning, which means that the chosen evacuation routes must be free of forks. Then two types of decision variables are required: binary variables that select road segments and continuous variables that track vehicle flows. Vehicle dynamics are modelled macroscopically. Experimental results show that maximizing the flow of evacuees while also selecting the road segments for the evacuation routes does not scale to realistic problem instances. Therefore, an alternative procedure using the Benders decomposition is proposed. The results show that the Benders decomposition approach leads to provably optimal solutions in many cases. The model can be extended to include the possibility of infrastructure improvements, which is an important consideration for many areas of the world where road networks are ill-equipped for evacuations, due to limited vehicle capacity and minimal protection from the effects of natural disasters. Recognizing the limitations of macroscopic simulation, this work additionally considers mesoscopic simulation through an implementation of the Cell Transmission Model.

Thesis Supervisor: Pascal Van Hentenryck
Title: Seth Bonder Collegiate Professor, University of Michigan

Reader: John F. Hughes
Title: Professor, Brown University

Reader: Thomas W. Doeppner Jr.
Title: Professor, Brown University

Vita

Julia Romanski was born in Toronto, Canada. In 2012, she graduated from the University of Toronto Schools. That fall, she came to Brown intending to pursue a concentration in biology or chemistry. After taking *Computational Probability and Statistics* from the Applied Mathematics department in the fall of her junior year, Julia switched her concentration to Applied Mathematics, and was later admitted to the Concurrent Bachelor's/ Master's Program. She will graduate with a Bachelor of Science degree in Applied Mathematics and a Master of Science degree in Computer Science in May 2016. Since her first year, Julia has been tutoring math at nearby Hope High School through Swearer Tutoring and Enrichment in Math and Sciences, a program which she co-cordinated during her last year. Next year, Julia will begin a PhD at the MIT Operations Research Center.

Acknowledgments

I would like to thank my advisor, Pascal Van Hentenryck, for his guidance over the past year. I am grateful for allowing me to work with him at National Information and Communications Technology (NICTA) last summer, and for everything he taught me while I was there. This work is largely the product of our frequent discussions at NICTA. Thank you, Pascal, for introducing me to the academic world. Several others from NICTA were involved in evacuation planning research. This work builds upon work by Caroline Even and Victor Pillac on the evacuation planning problem, and Kunal Kumar's research on the evacuation network design problem. As well, test instances were created by Caroline and Victor, while flooding was modelled by Tim Murphy. Thank you also to Professor John Hughes, who was my on-campus advisor. My family and friends have been a wonderful source of support throughout this project. I owe a huge thank you to my parents, who always encouraged me to pursue the opportunities available to me.

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Chapter 1

Introduction

Controlled evacuation plans are essential to minimize injury and loss of life during a natural disaster such as a flood, hurricane, or bushfire. The aim of an evacuation planning algorithm is to produce an operationally viable set of instructions to authorities, who will need to close roads and manage traffic, as well as clear directions to evacuees for when to leave their homes and where to go. This task is computationally challenging because many factors must be taken into account, such as the nature of the natural disaster, the layout of the road network, the locations of evacuees, and human behavior. Most automated evacuation planning systems only control the road management component, leaving evacuees to choose their own departure times and routes. However, this approach often leads to congestion when many people choose to use the same roads. An additional significant source of delays, observed during the Hurricane Katrina evacuation, is driver hesitation at forks [25]. In order to eliminate the delays associated with forks, [6] introduced the Convergent Evacuation Planning Problem (CEPP). When designing evacuation plans, it is important to keep their real-world execution in mind. From the operational point of view, convergent paths are easy to enforce because once the necessary roads are blocked, minimal vehicle guidance is required. This thesis builds upon the work by [6] using a Benders decomposition approach. The Benders decomposition is an iterative method for solving complex MIP problems. During an iteration of the algorithm, the complicating variables are temporarily fixed, the subproblem is solved, and a cut is generated for the master problem.

Contributions

1. The Benders decomposition is applied to a large-scale evacuation planning problem (see [22]).

2. The Benders decomposition is applied to the problem of designing an evacuation-ready road network (see [26]).

3. The Cell Transmission Model is used to model and simulate evacuations.

The remainder of the thesis is organized as follows: Chapter 2 outlines a method where the master problem selects a set of convergent paths for evacuation and the subproblem schedules the flow of evacuees on those paths, mirroring the two-stage approach of [6]. Experimental results on a real case study show that major improvements can be made by applying the Benders decomposition to the two-stage approach within reasonable computation time. Chapter 3 extends the evacuation planning problem to include the possibility of infrastructure upgrades. Chapter 4 gives an implementation of the Cell Transmission Model for finer evacuation simulation, and Chapter 5 concludes the thesis.

1.1 Literature Review

Evacuation planning studies are diverse in the approaches they take to model evacuation scenarios and solve the evacuation problem. An evacuation scenario is typically modeled as a graph, where nodes represent junctions, such as road intersections, and arcs represent physical connections between the nodes, such as roads, in the case of a city-level evacuation. Evacuation models differ in the components that are included such as contraflow lanes, and the factors considered, such as evacuee behavior. They may be time-dependent, and may take uncertainty into account. Approaches to evacuation planning can broadly be classified as microscopic or macroscopic [10].

Microscopic evacuation planning seeks to accurately model individual evacuee behavior, while macroscopic approaches, such as the one used in this thesis, treat evacuees collectively as flows. Objectives for evacuation planning algorithms include finding the maximum flow, the quickest flow, or the minimum cost flow. The work by [7] on the Maximum Dynamic Network Flow Problem (MDFP) for shipping goods is the foundation for much of later work on maximum flow evacuation problems. The same authors also defined the notion of a time-expanded graph for solving dynamic network flow problems [8]. [3] introduced the quickest (single) path problem for data transmission, which was generalized to include multiple paths by [1] in the quickest flow problem (QFP). The QFP was extended by [11] to include multiple sources and sinks in the quickest transshipment problem (QTP).

The evacuation planning problem has several key requirements not shared by other dynamic network flow problems. For example, evacuees living near each other (part of the same evacuation node) should follow the same path in order to avoid confusion and increase compliance. Also, any traffic controls should be simple for authorities to enact. Few papers have proposed plans consistent with these requirements. [12] developed a two-stage approach for giving instructions to evacuees, where the first stage creates possible paths and departure times and the second stage assigns them to evacuees. [6] proposed another two-stage approach. In the first stage, which they call the Tree Design Problem (TDP), they solve a relaxation of the maximum flow problem with aggregated arc capacities, generating an evacuation network with convergent paths. This tree is then passed into the second stage, the Flow Scheduling Problem (FSP), which schedules the flow of evacuees on the corresponding time-expanded network. They also use a dichotomic search to find the minimum clearance time.

Few studies have applied the Benders decomposition to evacuation planning. [2] introduced the Building Evacuation Problem with Shared Information (BEPSI), using the Benders decomposition to solve the quickest flow problem in a building evacuation. They showed that the BEPSI is NP-hard. Andreas and Smith (2009) solved a variant of the quickest flow problem, using arc traversal penalty functions in order to encourage earlier evacuation. The model includes a number of possible scenarios, each with a given probability of occurring, and the objective is to minimize the expected sum of arc traversal penalties. The master problem chooses an evacuation tree and the subproblem solves the corresponding flow problem. However, the master problem does not explicitly consider the flow variables, and thus some of the trees it generates contain subtours that make scheduling the evacuation infeasible. As a result, subtours have to be removed, either as they appear or with the addition of dummy flow variables at each node that force a spanning tree structure.

Very few studies on evacuation planning include the possibility of improving road infrastructure, yet existing infrastructure capacity is often well below that which is required for a large-scale evacuation [16]. Several studies use contraflow in order to increase road capacities ([27], [24], [5], [14]). However, [27] warns that the presence of contraflow lanes can lead to congestion due to drivers' unfamiliarity with lane reversal. [21] considered structural upgrades that would strengthen roads against earthquake damage. In their model, the upgrades increase the probability that a road will withstand an earthquake. This thesis applies the Benders decomposition to a two-stage evacuation planning problem that is based upon the work by [6], but with the addition of infrastructure upgrades.

1.2 Case Study: The Hawkesbury-Nepean Floodplain

The algorithms developed in this thesis will be tested on a case study of the Hawkesbury-Nepean Floodplain. The Hawkesbury-Nepean Floodplain is a low-lying region located near Sydney, Australia (Figure 1-1). Evacuation planners are concerned about the prospect of a 1 in 100 years flood that would require the evacuation of the entire region (Figure 1-2). For example, this could occur if the Warragamba Dam on the Hawkesbury River fails and spills over (Figure 1-3). As the population of the region is 80,000 people and quickly growing, traditional guideline-based evacuation planning will be insufficient to ensure the safety of the population, and robust computational methods must be designed.

Figure 1-1: Hawkesbury River Basin. (from [9]).

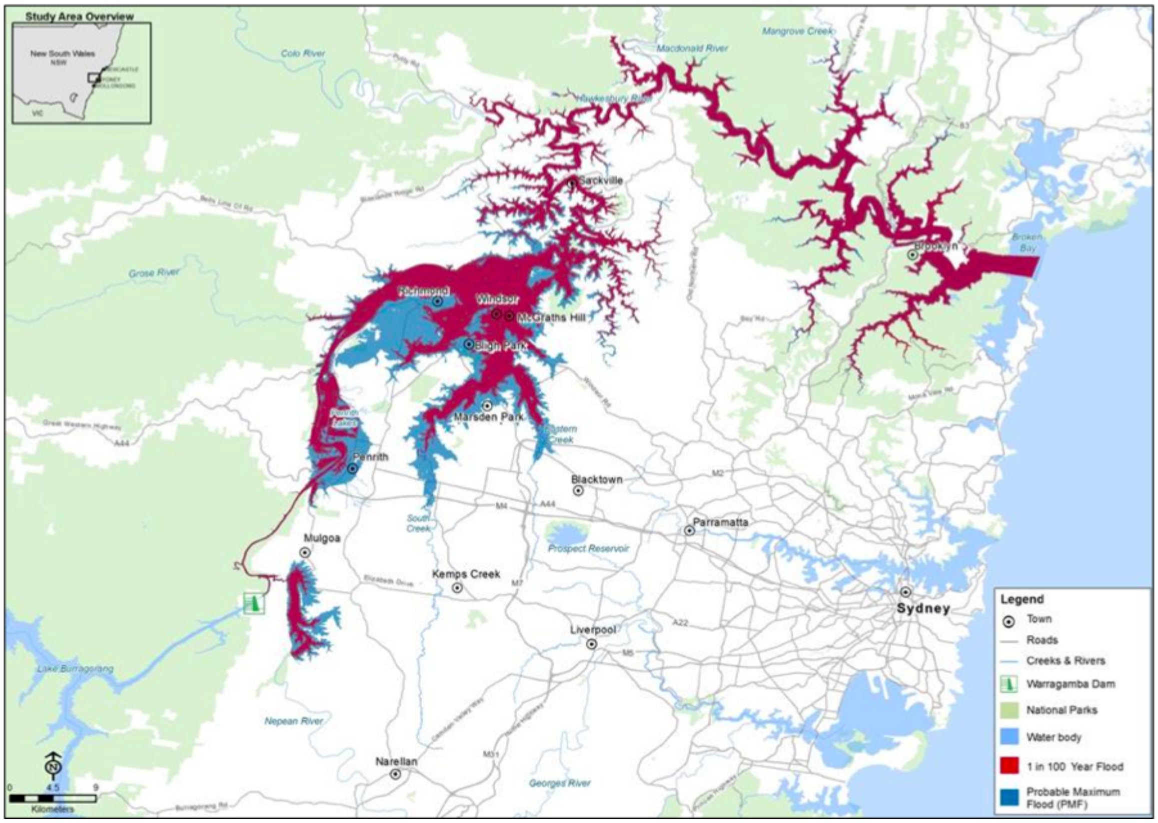


Figure 1-2: Areas at Risk of Heavy Flooding. (from [9]).

Figure 1-3: Warragamba Dam. (from [18]).

1.3 Convergent Evacuation Planning

Following [6], an evacuation scenario is represented by an *evacuation graph* $\mathcal{G} = (\mathcal{N} = \mathcal{E} \cup \mathcal{T} \cup \mathcal{S}, \mathcal{A})$, where $\mathcal{E}$, $\mathcal{T}$, and $\mathcal{S}$ are respectively the set of evacuation, transit, and safe nodes, and $\mathcal{A}$ is the set of arcs. Each evacuation node i has a demand d_i, and each arc e is characterized by its travel time s_e, its capacity u_e, and the time f_e at which it becomes unavailable due to flooding. Figure 1-4 offers an example of how evacuation instances are modeled. Figure 1-4a shows an evacuation scenario with one evacuation node, labeled "0", and two safe nodes, labeled "A" and "B." The times on each arc indicate when that arc will be flooded. Figure 1-4b is the corresponding evacuation graph. The evacuation node has a demand of 20 vehicles. Arc $(0, 1)$ has a travel time of 2 min and a capacity of 5 vehicles/min, and is flooded at 13:00.

In order to model the evolution of the evacuation over time, we discretize the time horizon and use a *time-expanded graph* $\mathcal{G}^x = (\mathcal{N}^x = \mathcal{E}^x \cup \mathcal{T}^x \cup \mathcal{S}^x, \mathcal{A}^x)$. To construct the time-expanded graph from the static graph, we create copies of nodes over time and replace each arc $e = (i, j)$ with arcs $e_t = (i_t, j_{t+s_{(i,j)}})$, for each time that e is available. Figure 1-4c illustrates the corresponding time-expanded graph.

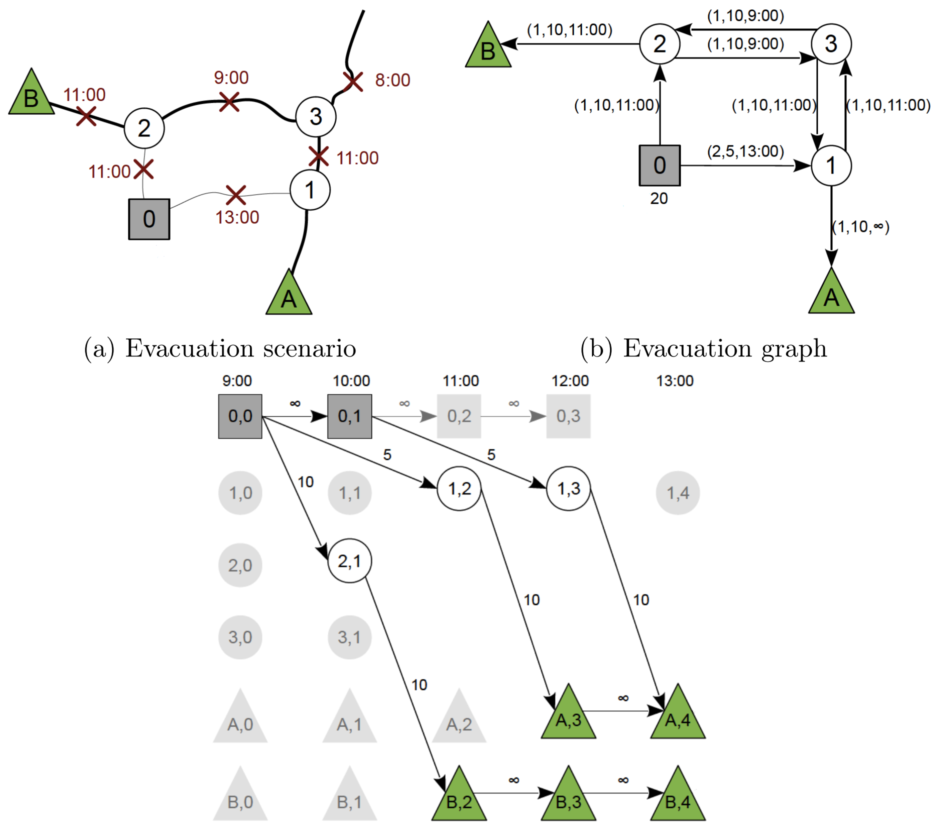
Definition 1. *A graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ is* connected *if for all $k \in \mathcal{E}$, there exists a path from k to a safe node.*

Definition 2. *A graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ is* convergent *if for all $i \in \mathcal{E} \cup \mathcal{T}$, the outdegree of i is* 1.

As stated by [6], any connected evacuation graph $\mathcal{G}$ contains a connected and convergent subgraph $\mathcal{G}'$. If an evacuation graph is connected and convergent, each evacuation node will have a unique path to a safe node.

The Convergent Evacuation Planning Problem (CEPP) is defined as follows:

Definition 3. *Given a connected evacuation graph $\mathcal{G}$, the Convergent Evacuation Planning Problem (CEPP) consists of finding a convergent subgraph $\mathcal{G}'$ of $\mathcal{G}$ and a set of evacuee departure times that maximize the flow from evacuation nodes to safe nodes.*



(c) Time-expanded graph

Figure 1-4: Modeling of an Evacuation Planning Problem. (from [6]).

1.4 A First Attempt

This section presents a Mixed-Integer Programming (MIP) model for solving the CEPP. The model is adapted from [6]: Variable x_e is binary and indicates whether arc e is selected and variable φ_{e_t} is continuous and represents the flow on arc $e_t \in \mathcal{A}^x$. The evacuation is scheduled over a discretized time horizon $\mathcal{H}$.

$$\max \sum_{k \in \mathcal{E}} \sum_{e_t \in \delta^+(k)} \varphi_{e_t} \tag{1.1}$$

s.t.

$$\sum_{e_t \in \delta^-(i)} \varphi_{e_t} - \sum_{e_t \in \delta^+(i)} \varphi_{e_t} = 0 \qquad \forall i \in \mathcal{T}^x \tag{1.2}$$

$$\sum_{e \in \delta^+(i)} x_e \leq 1 \qquad \forall i \in \mathcal{E} \cup \mathcal{T} \tag{1.3}$$

$$\varphi_{e_t} \leq x_e \cdot u_{e_t} \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \tag{1.4}$$

$$\sum_{e_t \in \delta^+(k)} \varphi_{e_t} \leq d_k \qquad \forall k \in \mathcal{E} \tag{1.5}$$

$$\varphi_{e_t} \geq 0 \qquad \forall e_t \in \mathcal{A}^x \tag{1.6}$$

$$x_e \in \{0, 1\} \qquad \forall e \in \mathcal{A} \tag{1.7}$$

In the models, $\delta^-(i)$ and $\delta^+(i)$ denote the set of incoming and outgoing edges of node i respectively. Constraints (1.2) require flow conservation at each of the transit nodes, constraints (1.3) ensure that the output paths are convergent, constraints (1.4) are the capacity constraints, constraints (1.5) state that no more than the demand can be evacuated for each residential area, and the objective (1.1) maximizes the total evacuee flow. Unfortunately, [6] showed that this MIP model does not scale to the Hawkesbury-Nepean evacuation instances: After 24 hours of running time, the number of people evacuated in the MIP model was substantially smaller than in their two-stage approach.

1.5 Benders Decomposition: Theory

This section is adapted from the slides by [20]. The Benders decomposition is an iterative algorithm for solving complex mixed-integer programs (MIP). The general form of a linear MIP is

$$\begin{aligned}
&\max z(x, y) = c^T x + f(y) \\
&\text{s.t. } Ax + By \leq b \\
&x \geq \vec{0} \\
&y \in S
\end{aligned}$$

The idea behind the Benders decomposition is that maximizing z over x and y can be made easier by temporarily fixing the complicating y variables, since $\max_{x,y} z(x,y) = \max_y \max_x z(x,y)$. With y is fixed, the Benders subproblem is

$$\begin{aligned} &\max c^T x \\ &\text{s.t. } Ax \leq b - By \\ &x \geq 0^n \end{aligned}$$

The dual of the subproblem is

$$\min (b - By)^T u \tag{1.8}$$

$$\text{s.t. } A^T u \geq c \tag{1.9}$$

$$u \geq 0^m \tag{1.10}$$

When solving the master problem, one must look ahead to ensure that the chosen y leaves the subproblem feasible. Then y must be chosen from the set $F := \{y \in S : \exists x \geq 0^n \text{ s.t. } Ax \leq b - By\}$. To choose such a y, we apply Farkas' Lemma ([28]):

Lemma 1.5.1. *If $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$, then the system $\{Ax = b, x \geq 0^n\}$ has a feasible solution x iff the system $\{A^T u \geq 0^m, b^T u < 0\}$ has* no *feasible solution u.*

The subproblem is feasible (for fixed y) if

$$\begin{aligned} &Ax + s = b - By \\ &x \geq 0^n \\ &s \geq 0^m \end{aligned}$$

is feasible. For y to lie in F, Farkas' lemma tells us that $\{A^T u \geq 0^n, (b - By - s)^T u < 0\}$ cannot have a solution. Alternatively, we need

$$A^T u \geq 0^n \implies (b - By - s)^T u \geq 0$$

If we restrict $u \geq 0^m$, then an equivalent condition is

$$A^T u \geq 0^n \implies (b - By)^T u \geq 0 \tag{1.11}$$

Define C to be the polyhedral cone $\{u : u \geq 0^m, A^T u \geq 0^n\}$, and let R denote the set of extreme rays of C. Condition (1.11) holds for all $u \in C$ if and only if it holds for $u^r \in R$. Therefore $y \in F$ if and only if $(b - By)^T u^r \geq 0$ for all $u^r \in R$.

The original problem can be rewritten as

$$\begin{aligned} &\max f(y) + c^T x \\ &\text{s.t. } y \in F \\ &Ax \leq b - By \\ &x \geq 0^n \end{aligned}$$

Alternatively, we can use the dual of the subproblem and write

$$\begin{aligned} &\max f(y) + \min(b - By)^T u \\ &\text{s.t. } y \in F \\ &A^T u \geq c \\ &u \geq 0^n \end{aligned}$$

Letting E denote the extreme points of the feasible region of the subproblem, ranging over all $y \in F$, we write

$$\max_{y \in F} f(y) + \min_{u^e \in E} (b - By)^T u^e$$

This can be expressed as

$$\begin{aligned} &\max z \\ &\text{s.t. } z \leq f(y) + (b - By)^T u^e \qquad\qquad \forall u^e \in E \end{aligned}$$

$$(b - By)^T u^r \geq 0 \qquad \forall u^r \in R$$

In practice, the sets E and R would not be know a priori, so the extreme points and rays must be added one by one. Let E' and R' respectively denote the possibly incomplete set of extreme points and rays. The Restricted Master Problem is:

$$\begin{aligned}
&\max z \\
&\text{s.t. } z \leq f(y) + (b - By)^T u^e && \forall u^e \in E' \\
&(b - By)^T u^r \geq 0 && \forall u^r \in R'
\end{aligned}$$

Since not all constraints are necessarily included in the Restricted Master Problem, the solution z is an upper bound. After solving the Restricted Master Problem, the dual of the subproblem is solved. Note that the objective value of the subproblem is a lower bound. If the objective value of the subproblem solution is unbounded, then the solution u is added to R'. If the objective value is finite, then u is added to E'. We call the constraint $z \leq f(y) + (b - By)^T u$ a *Benders cut.* The optimal solution to the original problem is attained when the objective values of the Restricted Master Problem and subproblem match. Since the number of extreme points and directions in any polyhedron is finite, the Benders decomposition algorithm terminates in a finite number of iterations.

1.5.1 Magnanti-Wong Method

While the Benders decomposition is guaranteed to converge in a finite number of iterations, in practice it may not converge within a reasonable amount of time. One reason for slow convergence is that the subproblem may be degenerate: for any fixed y there can be multiple optimal subproblem solutions, and hence multiple possible Benders cuts. In order to generate stronger Benders cuts, one can use the Magnanti-Wong method [17]. At each iteration of the Benders decomposition, the Magnanti-Wong method generates the strongest possible cut, called a Pareto-optimal cut.

Algorithm 1 Benders decomposition [19]

Require: A tolerance ϵ.

lowestRMP $\leftarrow \infty$, highestSP $\leftarrow -\infty$.
while true **do**
  Solve the Restricted Master Problem $\rightarrow \overline{y}, z(RMP)$.
  if $z(RMP) < lowestRMP$ **then**
    $lowestRMP \leftarrow z(RMP)$
  end if.
  Solve the subproblem with $\overline{y}$ fixed $\rightarrow \overline{x}, z(SP)$.
  if $z(SP) > highestSP$ **then**
    $highestSP \leftarrow z(SP)$
  end if.
  if $lowestRMP - highestSP < \epsilon$ **then**
    return highestSP
  end if
  Generate a Benders cut from the subproblem dual variables and add it to the Restricted Master Problem.
end while

Definition 4. *A cut $z \leq (b - By)^T u^*$ is* Pareto-optimal *if it dominates all other cuts, i.e. $(b - By)^T u^* \geq (b - By)^T u$, for all y in S and for all u such that $u \geq 0^m, A^T u \geq c$. In that case we also say that u^* is Pareto-optimal.*

In order to generate a cut, the Magnanti-Wong method requires a *core point* of S.

Definition 5. *Core point: Let $ri(S)$ and S^c be the relative interior and convex hull of a set $S \subseteq \mathbb{R}^m$. Then a point $y \in ri(S^c)$ is called a* core point*.*

Let y_0 be a core point of S, let $\overline{y}$ be fixed from the Restricted Master Problem, and let $\overline{u}$ be a solution to the dual of the subproblem (1.8) - (1.10). Then the Magnanti-Wong Problem is given by

$$\begin{aligned}
&\min (b - By_0)^T u \\
&\text{s.t. } (b - B\overline{y})^T u = (b - B\overline{y})^T \overline{u} \\
&A^T u \geq c
\end{aligned}$$

[17] showed that the solution u to the Magnanti-Wong Problem is Pareto-optimal. [19] recommends using the dual of the Magnanti-Wong Problem instead in some

circumstances:

$$\max c^T(x + \xi \cdot \overline{x})$$
$$\text{s.t. } Ax + (b - B\overline{y})\xi \leq b - y^0$$
$$x \geq 0^n$$

Algorithm 2 Magnanti-Wong Method [19]

Require: A tolerance ϵ.

lowestRMP $\leftarrow \infty$, highestSP $\leftarrow -\infty$.
while true **do**
    Solve the Restricted Master Problem $\rightarrow \overline{y}, z(RMP)$.
    if $z(RMP) < lowestRMP$ **then**
        $lowestRMP \leftarrow z(RMP)$
    end if.
    Solve the subproblem with $\overline{y}$ fixed $\rightarrow \overline{x}, z(SP)$.
    if $z(SP) > highestSP$ **then**
        $highestSP \leftarrow z(SP)$
    end if.
    if $lowestRMP - highestSP < \epsilon$ **then**
        return highestSP
    end if
    Solve the dual Magnanti-Wong Problem with $\overline{y}$ and $\overline{x}$ fixed.
    Generate a Benders cut from the dual variables of the dual Magnanti-Wong Problem and add it to the Restricted Master Problem.
end while

Chapter 2

The Convergent Evacuation Planning Problem

This chapter presents a Benders decomposition approach for solving the Convergent Evacuation Planning Problem (CEPP), which separates the choice of the convergent paths from the flow scheduling. The Master Problem, called the Tree Design Problem (TDP), chooses the convergent evacuation paths. The subproblem, called the Flow Scheduling Problem (FSP), schedules the departure times of evacuees on those paths.

2.1 Benders Decomposition

2.1.1 Tree Design Problem (Master Problem)

The Master Problem for the Benders decomposition will be the TDP, as defined by [6]. The TDP is a relaxation of the CEPP where the evacuation flows and arc capacities are aggregated over time. The objective of the TDP is to maximize the flow of evacuees to safe nodes within the time horizon. The TDP is formulated as a MIP model with a binary arc selection variable x_e and a continuous flow variable ψ_e for each arc $e \in \mathcal{A}$:

$$\max \sum_{k \in \mathcal{E}} \sum_{e \in \delta^+(k)} \psi_e \tag{2.1}$$

subject to

$$\sum_{e \in \delta^-(i)} \psi_e - \sum_{e \in \delta^+(i)} \psi_e = 0 \qquad \forall i \in \mathcal{T} \tag{2.2}$$

$$\sum_{e \in \delta^+(i)} x_e \leq 1 \qquad \forall i \in \mathcal{E} \cup \mathcal{T} \tag{2.3}$$

$$\psi_e \leq x_e \sum_{t \in H} u_{e_t} \qquad \forall e \in \mathcal{A} \tag{2.4}$$

$$\sum_{e \in \delta^+(k)} \psi_e \leq d_k \qquad \forall k \in \mathcal{E} \tag{2.5}$$

$$\psi_e \geq 0 \qquad \forall e \in \mathcal{A} \tag{2.6}$$

$$x_e \in \{0, 1\} \qquad \forall e \in \mathcal{A} \tag{2.7}$$

Constraints (2.2) impose flow conservation at each transit node, constraints (2.3) ensure a convergent plan, constraints (2.4) and (2.5) enforce the capacity and demand constraints, and the objective (2.1) maximizes the total evacuee flow.

2.1.2 Flow Scheduling Problem (Subproblem)

The output of the TDP is a connected and convergent evacuation graph $\mathcal{G}$ encoded by the values $\overline{x}_e$. In the second stage, the Flow Scheduling Problem schedules the flow of evacuees on the associated time-expanded graph $\mathcal{G}^x$. The FSP can be formulated as follows:

$$\max \sum_{k \in \mathcal{E}} \sum_{e_t \in \delta^+(k)} \varphi_{e_t} \tag{2.8}$$

subject to

$$\sum_{e_t \in \delta^-(i)} \varphi_{e_t} - \sum_{e_t \in \delta^+(i)} \varphi_{e_t} = 0 \qquad \forall i \in \mathcal{T}^x \tag{2.9}$$

$$\varphi_{e_t} \leq \overline{x}_e \cdot u_{e_t} \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \tag{2.10}$$

$$\sum_{e_t \in \delta^+(k)} \varphi_{e_t} \leq d_k \qquad \forall k \in \mathcal{E} \tag{2.11}$$

$$\varphi_{e_t} \geq 0 \qquad \forall e_t \in \mathcal{A}^x \tag{2.12}$$

Constraints (2.9) are the flow conservation constraints, constraints (2.10) and (2.11) are the capacity and demand constraints, and the objective (2.8) maximizes the flow.

2.1.3 Restricted Master Problem

The Benders cuts are of the form:

$$z \leq \sum_{e \in \mathcal{A}} x_e \sum_{t \in \mathcal{H}} u_{e_t} \cdot y_{e_t} + \sum_{k \in \mathcal{E}} d_k \cdot y_k. \tag{2.13}$$

where $\{y_{e_t}\}$ and $\{y_k\}$ are the dual variables associated with constraints (2.10) and (2.11) respectively. Note that the subproblem is never infeasible, since it is always possible to send zero flow.

The RMP then becomes an extension of the TDP with the set of Benders cuts $\mathcal{C}$:

$$\begin{aligned}
&\max z \\
&\text{subject to} \\
&z \leq \sum_{k \in \mathcal{E}} \sum_{e \in \delta^+(k)} \psi_e \\
&z \leq \sum_{e \in \mathcal{A}} x_e \sum_{t \in \mathcal{H}} u_{e_t} \cdot y^c_{e_t} + \sum_{k \in \mathcal{E}} d_k \cdot y^c_k && \forall c \in \mathcal{C} \\
&\sum_{e \in \delta^-(i)} \psi_e - \sum_{e \in \delta^+(i)} \psi_e = 0 && \forall i \in \mathcal{T} \\
&\sum_{e \in \delta^+(i)} x_e \leq 1 && \forall i \in \mathcal{E} \cup \mathcal{T} \\
&\psi_e \leq x_e \sum_{t \in H} u_{e_t} && \forall e \in \mathcal{A} \\
&\sum_{e \in \delta^+(k)} \psi_e \leq d_k && \forall k \in \mathcal{E} \\
&\psi_e \geq 0 && \forall e \in \mathcal{A} \\
&x_e \in \{0, 1\} && \forall e \in \mathcal{A}
\end{aligned}$$

2.2 Magnanti-Wong Method

The Benders decomposition presented so far is guaranteed to converge in a finite number of iterations. However, in practice, the algorithm rarely converged within a reasonable amount of time, so the Magnanti-Wong Method was used to accelerate convergence. A single core point was used for all iterations: for each node i, $x_e^o = \frac{1}{|\delta^+(i)|+1}$ for each edge $e \in \delta^+(i)$. To obtain a Pareto-optimal Benders cut, the dual of the Magnanti-Wong Problem was solved, as suggested by [19]:

$$\begin{aligned}
&\max \sum_{k\in\mathcal{E}} \sum_{e_t\in\delta^+(k)} \varphi_{e_t} + \\
&\quad \xi \sum_{k\in\mathcal{E}} \sum_{e_t\in\delta^+(k)} \bar{\varphi}_{e_t} && && (2.14)\\
&\text{subject to} \\
&\sum_{e_t\in\delta^-(i)} \varphi_{e_t} - \sum_{e_t\in\delta^+(i)} \varphi_{e_t} = 0 && \forall i \in \mathcal{T}^x && (2.15)\\
&\varphi_{e_t} + \bar{x}_e \cdot u_{e_t} \cdot \xi \leq x_e^0 \cdot u_{e_t} && \forall e \in \mathcal{A}, \forall t \in \mathcal{H} && (2.16)\\
&\sum_{e_t\in\delta^+(k)} \varphi_{e_t} + d_k \cdot \xi \leq d_k && \forall k \in \mathcal{E} && (2.17)\\
&\varphi_{e_t} \geq 0 && && (2.18)
\end{aligned}$$

where $\{\bar{x}_e\}$ are from the optimal solution of the RMP and $\{\bar{\varphi}_{e_t}\}$ are from the optimal solution of the Benders subproblem. In order to generate a Pareto-optimal Benders cut, the values $\{y_{e_t}\}$ and $\{y_k\}$ in the cut come from constraints (2.16) and (2.17) respectively.

2.3 Results

This section presents experimental results for a case study of the evacuation of the Hawkesbury-Nepean (HN) floodplain, which is located near Sydney. The HN evacuation graph has 80 evacuation nodes, 184 transit nodes, 5 safe nodes, and 580 edges. A time horizon of 600 min is used for scenarios without flooding and 1000 min for scenarios with flooding, discretized into 5 minute time-steps. Several flood scenarios are considered for this region. The population is scaled by a factor $x \in [1.1, 3]$ to model projected population growth in the region. Each instance was run for ten hours, unless the algorithm converged earlier. The algorithms were implemented using JAVA 8 and GUROBI 6.0 and the results were obtained on a 64 bit machine with a 1.4 GHz Intel Core i5 processor and 4 GB of RAM. Algorithms based upon the work by [6] were reimplemented.

There are two main experimental settings: (1) The *deadline* setting used in [6] that requires the evacuation to be completed by a deadline (10 hours); (2) A *flood* setting in which the flood affects the road network at various times. In the deadline setting, the road network is available for the duration of the evacuation. In contrast, the flood setting takes into account the flood extent, the timing, and the height of the water produced by an hydro-dynamic simulation for a severe, 1 in 100 years flood event. The flood reaches the road network at 8, 9, 10, or 11 hours into the evacuation.

Table 2.1 displays the results for the deadline setting. The table reports the number of evacuees reaching safety (in percent) in the tree design problem (**TDP**), the flow scheduling problem (**FSP**), the last restricted master problem (**LRMP**), and the Benders decompositions (**BD**) (using the Magnanti-Wong method). The table also gives the CPU time and the duality gap. The duality gap is computed using the formula $\frac{z(LRMP))-z(\star)}{z(\star)}$ where $z(\star)$ is the total number of evacuees reaching safety in model $\star$.

The results show that the Benders decomposition closes all these instances in less than 5 minutes (1.5 minutes on average). The Benders decomposition improves the two-stage approach by an average of 0.4%. Notice that the TDP objective value is a good approximation for the true objective value in these scenarios, as the initial duality gap is very small. Figure 2-1 depicts the solution progress of the Benders decomposition over time for Instance HN-2.5.

Table 2.2 displays the results of the two-stage and Benders decomposition approaches for the flood setting. Three versions of the Benders decomposition are used: “Benders” denotes the standard decomposition, “Stationary M-W” denotes the

Instance	2-Stage Approach CPU (s)	TDP (%)	FSP (%)	Gap (%)	Benders Decomposition CPU (s)	LRMP(%)	BD (%)	Gap (%)
HN	1.8	100	99.1	1.0	34.3	100	100	0
HN-1.1	0.8	100	99.8	0.2	11.7	100	100	0
HN-1.2	1.3	100	100	0	1.3	100	100	0
HN-1.4	1.1	100	100	0	1.1	100	100	0
HN-1.7	1.4	100	100	0	1.4	100	100	0
HN-2.0	7.7	96.2	95.5	0.7	165.5	96.1	96.1	0
HN-2.5	3.5	81.1	80.3	1.0	292.8	80.8	80.8	0
HN-3.0	1.5	68.1	67.5	0.9	174.4	68.0	68.0	0
Average	*2.4*	*93.2*	*92.8*	*0.5*	*85.3*	*93.1*	*93.1*	*0*

Table 2.1: Results for the HN Instances in the Deadline Setting.

Magnanti-Wong method with a stationary core point, and “Moving M-W” denotes the Magnanti-Wong method with a moving core point, where at each iteration the core point is taken to be the average of the previous core point and the current edge selection variable values. The results indicate that these instances are significantly harder since the Benders decomposition cannot always prove optimality in 10 hours and the duality gap can be as high as 35% (instance HN-2.0/ 8 h) initially. But the results also show that the Benders decomposition provides significant improvements in solution quality compared to the two-stage approach, bridging about half of the initial duality gaps. The Benders decompositions may improve the two-stage approach by more than 25% (instance HN-2.0/ 8 h). For the HN-1.7, HN-2.0, and HN-2.5 instances, the average improvements are 10.3%, 17.7%, and 10.6% for the standard decomposition, 0.3%, 16.9%, and 10.9% for the stationary Magnanti-Wong method, and 0.3%, 17.3%, and 11.1% for the standard decomposition. This is substantial in the context of evacuation planning for the HN region, since this corresponds to the evacuation of thousands more people. The duality gaps produced by the Benders decompositions are reasonably small: For the HN-1.7, HN-2.0, and HN-2.5 instances, they decrease from 10.7%, 19.1%, and 15.0% initially to 0%, 0.3%, and 3.5% for the standard decomposition, to 0%, 1.0%, and 3.2% for the stationary Magnanti-Wong method, and to 0%, 0.6%, and 3.0% for the moving Magnanti-Wong method. These results show that the three decomposition methods yield comparable results when run for the same amount of time.

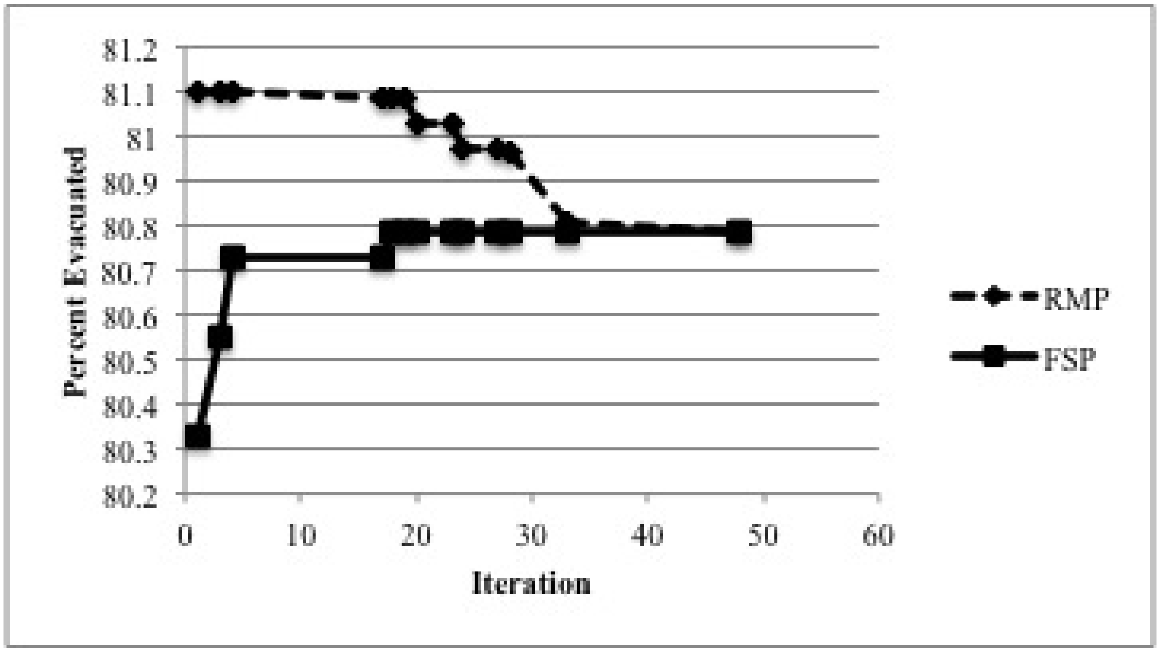


Figure 2-1: The Behavior of the Benders Decomposition in the Deadline Setting for Instance HN-2.5.

	2-Stage			Benders			Stationary M-W			Moving M-W		
Instance	**TDP** (%)	**FSP** (%)	**Gap** (%)	**LR** (%)	**MPBD** (%)	**Gap** (%)	**LR** (%)	**MPBD** (%)	**Gap** (%)	**LR** (%)	**MPBD** (%)	**Gap** (%)
HN-1.7												
8 h	100	88.4	13.1	100	100	0	100	100	0	100	100	0
9 h	100	83.0	20.5	100	100	0	100	100	0	100	100	0
10 h	100	93.3	7.2	100	100	0	100	100	0	100	100	0
11 h	100	98.1	2.0	100	100	0	100	100	0	100	100	0
Average	*100*	*90.7*	*10.7*	*100*	*100*	*0*	*100*	*100*	*0*	*100*	*100*	*0*
HN-2.0												
8 h	98.8	73.1	35.1	98.8	97.8	1.0	98.8	95.8	3.1	98.8	96.3	2.5
9 h	99.6	81.5	22.3	99.6	99.6	0	99.6	98.9	0.8	99.6	99.6	0
10 h	100	88.6	12.8	100	100	0	100	100	0	100	100	0
11 h	100	94.3	6.0	100	100	0	100	100	0	100	100	0
Average	*99.6*	*84.4*	*19.1*	*99.6*	*99.3*	*0.3*	*99.6*	*98.7*	*1.0*	*99.6*	*99.0*	*0.6*
HN-2.5												
8 h	97.4	78.6	23.9	97.4	90.0	8.2	97.4	90.6	7.5	97.4	90.4	7.7
9 h	98.1	80.2	22.3	98.1	94.1	4.2	98.1	94.2	4.1	98.1	94.7	3.5
10 h	98.8	89.5	10.4	98.8	97.1	1.8	98.8	97.7	1.1	98.8	98.2	0.6
11 h	99.5	96.2	3.4	99.5	99.5	0	99.5	99.5	0	99.5	99.5	0
Average	*98.4*	*86.1*	*15.0*	*98.4*	*95.2*	*3.5*	*98.4*	*95.5*	*3.2*	*98.4*	*95.7*	*3.0*

Table 2.2: Results for the HN-1.7, 2.0, and 2.5 Instances on the Flooding Setting.

Chapter 3

The Convergent Evacuation Network Design Problem

The Convergent Evacuation Network Design Problem (CENDP) extends the Convergent Evacuation Planning Problem by allowing for the possibility of infrastructure upgrades. The problem was proposed and formulated by [15] in his thesis. There are two possible infrastructure upgrades: adding new lanes and elevating roads. Each edge has an existing number of lanes n_e as well as a maximum number of lanes that can be added n_e^+. We assume that capacity increases linearly with the number of lanes. Each road segment can also be elevated, extending its availability by a given amount of time. The costs of the upgrades are given by $c_l(e)$ for adding a single lane to arc e and $c_e(e)$ for elevating arc e to extend its availability by a single time step. These costs are given per unit length.

The *Convergent Evacuation Network Design Problem* can now be defined:

Definition 6. *The* Convergent Evacuation Network Design Problem (CENDP) *consists in finding a convergent evacuation plan that includes two kinds of infrastructure upgrades: lane additions and road elevations.*

3.1 The MIP Model

This section presents a MIP model for solving the CENDP, adapted from [15]. Here x_e is a binary variable that indicates whether arc e is selected, φ_{e_t} is a continuous variable representing the flow on arc $e_t \in \mathcal{A}^x$, z_e is an integer variable indicating the number of lanes added to arc e, and v_{e_t} is a binary variable indicating whether arc e is available at time t, corresponding to a road elevation. The objective (1) maximizes the total flow of evacuees, with $\delta^-(k)$ and $\delta^+(k)$ respectively denoting the set of incoming and outgoing edges of node k. Without loss of generality, we assume that all roads have the same cap on the number of additional lanes, n^+, and that the upgrade costs per unit distance are the same for all edges (c_l and c_e). The MIP model for the CENDP is given by

$$\max \sum_{k \in \mathcal{E}} \sum_{e_t \in \delta^+(k)} \varphi_{e_t} \tag{3.1}$$

s.t.

$$\sum_{e_t \in \delta^-(i)} \varphi_{e_t} - \sum_{e_t \in \delta^+(i)} \varphi_{e_t} = 0 \qquad \forall i \in \mathcal{T}^x \tag{3.2}$$

$$\sum_{e \in \delta^+(i)} x_e \leq 1 \qquad \forall i \in \mathcal{E} \cup \mathcal{T} \tag{3.3}$$

$$\varphi_{e_t} \leq x_e \left(1 + \frac{n^+}{n_e}\right) u_{e_t} \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \tag{3.4}$$

$$\varphi_{e_t} \leq v_{e_t} \left(1 + \frac{z_e}{n_e}\right) u_{e_t} \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \tag{3.5}$$

$$\sum_{e_t \in \delta^+(k)} \varphi_{e_t} \leq d_k \qquad \forall k \in \mathcal{E} \tag{3.6}$$

$$v_{e_t} \geq v_{e_{t+1}} \qquad \forall e_t, e_{t+1} \in \mathcal{A}^x \tag{3.7}$$

$$v_{e_t} = 1 \qquad \forall e \in A, \forall t \in [0, f_e) \tag{3.8}$$

$$\sum_{t=f_e}^{h} v_{e_t} = w_e \qquad \forall e \in \mathcal{A} \tag{3.9}$$

$$\sum_{e \in \mathcal{A}} l_e \left(c_l \cdot z_e + c_e \cdot w_e\right) \leq \mathcal{B} \tag{3.10}$$

$$z_e \leq n^+ \qquad \forall e \in \mathcal{A} \tag{3.11}$$

$$\varphi_{e_t} \geq 0 \qquad \forall e \in \mathcal{A} \tag{3.12}$$

$$x_e \in \{0,1\} \qquad \forall e \in \mathcal{A} \tag{3.13}$$

$$z_e, w_e \in \mathbb{Z}^+ \qquad \forall e \in \mathcal{A} \tag{3.14}$$

$$v_{e_t} \in \{0,1\} \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \tag{3.15}$$

Constraints (3.2) ensure flow conservation at each transit node, constraints (3.3) impose a convergent evacuation plan, and constraints (3.4) ensure that flow will only travel on selected edges. Constraints (3.5) limit the flows to the edge capacities, taking into account additional capacity due to road widening, and constraints (3.6) limit the total outflow of each evacuation node to its demand. Constraints (3.7) ensure that road blockages due to flooding will be permanent, and constraints (3.8) preserve road availability before the onset of the flood. Constraint (3.10) is the budget constraint, where l_e is the length of arc e and w_e is number of units of elevation upgrades on e.

Note that constraints (3.5) are nonlinear as they contain products of two decision variables. The next section, which presents the Benders decomposition, shows how to linearize these constraints.

3.2 Benders Decomposition

The MIP formulation is computationally intractable for real-sized instances because it attempts to simultaneously choose paths, add road upgrades, and schedule the departure times of evacuees. In order to rectify this, the decisions are split into two stages. The first stage is called the Tree Design Problem with Upgrades (TDP-U), and it selects a set of convergent paths with accompanying infrastructure upgrades. The second stage is called the Flow Scheduling Problem (FSP), and it schedules the departure times of evacuees. Together, the two stages are referred to as the TDFS-U. This section details a Benders decomposition approach (BD-U) for improving the two-stage method, where the first stage becomes the master problem and the second stage becomes the subproblem. The Magnanti-Wong method is applied in order to generate Pareto-optimal cuts.

3.2.1 Tree Design Problem with Upgrades (Master Problem)

The first stage in the algorithm, on which the restricted master problem will be based, is the Tree Design Problem with Upgrades (TDP-U), adapted from [15]. The TDP-U is formulated as a relaxation of the CENDP, where the flows and capacities have been aggregated over time. The objective of the TDP-U is to maximize the flow from evacuation nodes to safe nodes within the time horizon, given the infrastructure upgrade budget. The TDP-U is a MIP model with a binary arc selection variable x_e, continuous flow variables ψ_e for each arc $e \in \mathcal{A}$, integer lane addition variables z_e, and binary road elevation variables v_{e_t}:

$$\max \sum_{k \in \mathcal{E}} \sum_{e \in \delta^+(k)} \psi_e \tag{3.16}$$

s.t.

$$\sum_{e \in \delta^-(i)} \psi_e - \sum_{e \in \delta^+(i)} \psi_e = 0 \qquad \forall i \in \mathcal{T} \tag{3.17}$$

$$\sum_{e \in \delta^+(i)} x_e \leq 1 \qquad \forall i \in \mathcal{E} \cup \mathcal{T} \tag{3.18}$$

$$\psi_e \leq x_e \left(1 + \frac{n^+}{n_e}\right) \sum_{t \in \mathcal{H}} u_{e_t} \qquad \forall e \in \mathcal{A} \tag{3.19}$$

$$\psi_e \leq \left(1 + \frac{z_e}{n_e}\right) \sum_{t \in \mathcal{H}} v_{e_t} \cdot u_{e_t} \qquad \forall e \in \mathcal{A} \tag{3.20}$$

$$\sum_{e \in \delta^+(k)} \psi_e \leq d_k \qquad \forall k \in \mathcal{E} \tag{3.21}$$

$$v_{e_t} \geq v_{e_{t+1}} \qquad \forall e_t, e_{t+1} \in \mathcal{A}^x \tag{3.22}$$

$$v_{e_t} = 1 \qquad \forall e \in A, \forall t \in [0, f_e) \tag{3.23}$$

$$\sum_{t=f_e}^{h} v_{e_t} \leq w_e \qquad \forall e \in \mathcal{A} \tag{3.24}$$

$$\sum_{e \in \mathcal{A}} l_e \left(c_l \cdot z_e + c_e \cdot w_e\right) \leq \mathcal{B} \tag{3.25}$$

$$z_e \leq n^+ \qquad \forall e \in \mathcal{A} \tag{3.26}$$

$$\psi_e \geq 0 \qquad \forall e \in \mathcal{A} \tag{3.27}$$

$$x_e \in \{0,1\} \qquad \forall e \in \mathcal{A} \tag{3.28}$$

$$z_e, w_e \in \mathbb{Z}^+ \qquad \forall e \in \mathcal{A} \tag{3.29}$$

$$v_{e_t} \in \{0,1\} \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \tag{3.30}$$

Constraints (3.17) impose the aggregate flow conservation at each transit node, constraints (3.18) enforce a tree structure, and constraints (3.19) ensure that flow will only be sent on selected arcs. Constraints (3.20) and (3.21) are the capacity and demand constraints, and constraint (3.25) is the budget constraint. The objective (3.16) maximizes the aggregate flow. Constraints (3.20) are nonlinear as they contain products of variables $z_e \cdot v_{e_t}$. These constraints can be linearized by replacing each product with a new variable p_{e_t} to represent such a product and adding the following constraints:

$$p_{e_t} \le z_e \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \tag{3.31}$$

$$p_{e_t} \le v_{e_t} \cdot n^+ \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \tag{3.32}$$

$$p_{e_t} \in \mathbb{Z}^+ \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \tag{3.33}$$

Theorem 3.2.1. *The optimal solution of the RMP is an upper bound to the CENDP.*

Proof. The proof relies on showing that any optimal solution to the CENDP is also a feasible solution to the TDP-U with the same objective value, and is an extension of the result by [6] for the CEPP. Let $\Phi = (\{\varphi_{e_t}\}, \{x_e\}, \{z_e\}, \{v_{e_t}\})$ be an optimal solution to the CENDP, with an objective value of $z(\Phi)$. Clearly, constraints (3.18) and (3.22) through (3.30) in the TDP-U will be satisfied. Let

$$\psi_e = \sum_{t \in \mathcal{H}} \varphi_{e_t}$$

for each arc $e \in \mathcal{A}$. The objective value of the TDP-U will be the same as the CENDP because

$$\begin{aligned}
z(\Phi) &= \sum_{k\in\mathcal{E}} \sum_{e_t\in\delta^+(k)} \varphi_{e_t} \\
&\equiv \sum_{k\in\mathcal{E}} \sum_{e\in\delta^+(k)} \sum_{t\in\mathcal{H}} \varphi_{e_t} \\
&= \sum_{k\in\mathcal{E}} \sum_{e\in\delta^+(k)} \psi_e
\end{aligned}$$

Since Φ is a solution to the CENDP, we have

$$\begin{aligned}
&\sum_{e_t\in\delta^-(i)} \varphi_{e_t} - \sum_{e_t\in\delta^+(i)} \varphi_{e_t} = 0 && \forall i \in \mathcal{T}^x \\
&\Rightarrow \sum_{e\in\delta^-(i)} \sum_{t\in\mathcal{H}} \varphi_{e_t} - \sum_{e\in\delta^+(i)} \sum_{t\in\mathcal{H}} \varphi_{e_t} = 0 && \forall i \in \mathcal{T} \\
&\Rightarrow \sum_{e\in\delta^-(i)} \psi_e - \sum_{e\in\delta^+(i)} \psi_e = 0 && \forall i \in \mathcal{T}
\end{aligned}$$

so that constraints (3.17) are satisfied. Similarly,

$$\begin{aligned}
&\varphi_{e_t} \le x_e \left(1 + \frac{n^+}{n_e}\right) u_{e_t} && \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \\
&\Rightarrow \sum_{t\in\mathcal{H}} \varphi_{e_t} \le \sum_{t\in\mathcal{H}} x_e \left(1 + \frac{n^+}{n_e}\right) u_{e_t} && \forall e \in \mathcal{A} \\
&\Rightarrow \psi_e \le x_e \left(1 + \frac{n^+}{n_e}\right) \sum_{t\in\mathcal{H}} u_{e_t} && \forall e \in \mathcal{A}
\end{aligned}$$

satisfying constraints (3.19). Also,

$$\begin{aligned}
&\varphi_{e_t} \le v_{e_t} \left(1 + \frac{z_e}{n_e}\right) u_{e_t} && \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \\
&\Rightarrow \sum_{t\in\mathcal{H}} \varphi_{e_t} \le \sum_{t\in\mathcal{H}} v_{e_t} \left(1 + \frac{z_e}{n_e}\right) u_{e_t} && \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \\
&\Rightarrow \psi_e \le \left(1 + \frac{z_e}{n_e}\right) \sum_{t\in\mathcal{H}} v_{e_t} \cdot u_{e_t} && \forall e \in \mathcal{A}
\end{aligned}$$

Finally,

$$\begin{aligned}
&\sum_{e_t \in \delta^+(k)} \varphi_{e_t} \leq d_k && \forall k \in \mathcal{E} \\
&\equiv \sum_{e \in \delta^+(k)} \sum_{t \in \mathcal{H}} \varphi_{e_t} \leq d_k && \forall k \in \mathcal{E} \\
&\Rightarrow \sum_{e \in \delta^+(k)} \psi_e \leq d_k && \forall k \in \mathcal{E}
\end{aligned}$$

□

3.2.2 Flow Scheduling Problem (Subproblem)

The TDP-U produces a convergent upgraded evacuation graph G. Next, the Flow Scheduling Problem (FSP) determines the departure times of evacuees. The TDP produces a convergent evacuation graph $\mathcal{G}$ with infrastructure upgrades, specified by $\overline{x}_e$, $\overline{z}_e$, and $\overline{v}_{e_t}$. The Flow Scheduling Problem (FSP) uses these optimal values as inputs to determine the departure times of evacuees in the time-expanded graph so as to maximize the number of evacuees reaching safety. The FSP can be formulated as follows (adapted from [15]):

$$\max \sum_{k \in \mathcal{E}} \sum_{e_t \in \delta^+(k)} \varphi_{e_t} \tag{3.34}$$

s.t.

$$\sum_{e_t \in \delta^-(i)} \varphi_{e_t} - \sum_{e_t \in \delta^+(i)} \varphi_{e_t} = 0 \qquad \forall i \in \mathcal{T}^x \tag{3.35}$$

$$\varphi_{e_t} \leq \overline{x}_e \cdot u_{e_t} \left(1 + \frac{n^+}{n_e}\right) \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \tag{3.36}$$

$$\varphi_{e_t} \leq \overline{v}_{e_t} \cdot u_{e_t} \left(1 + \frac{\overline{z}_e}{n_e}\right) \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \tag{3.37}$$

$$\sum_{e_t \in \delta^+(k)} \varphi_{e_t} \leq d_k \qquad \forall k \in \mathcal{E} \tag{3.38}$$

$$\varphi_{e_t} \geq 0 \qquad \forall e_t \in \mathcal{A}^x \tag{3.39}$$

Constraints (3.35) are the flow conservation constraints. Constraints (3.36) ensure that flow will only be sent on edges in the network. Constraints (3.37) and (3.38) are the capacity and demand constraints. Note that the right-hand sides of constraints (3.36), (3.37), and (3.38) are constants. The objective (3.34) maximizes the flow.

3.2.3 The Benders Cuts

The Benders cuts are of the form:

$$
\begin{aligned}
z \leq & \sum_{e \in \mathcal{A}} x_e \left(1 + \frac{n^+}{n_e}\right) \sum_{t \in \mathcal{H}} u_{e_t} \cdot y_{e_t} \\
& + \sum_{e \in \mathcal{A}} \sum_{t \in \mathcal{H}} \left(v_{e_t} \cdot u_{e_t} + \frac{p_{e_t} \cdot u_{e_t}}{n_e}\right) y'_{e_t} \\
& + \sum_{k \in \mathcal{E}} d_k \cdot y_k
\end{aligned} \tag{3.40}
$$

where $\{y_{e_t}\}$, $\{y'_{e_t}\}$, and $\{y_k\}$ are the dual variables associated with SP constraints (3.36), (3.37), and (3.38) respectively. The Restricted Master Problem (RMP) is then an extension of the TDP-U with the set of Benders cuts $\mathcal{C}$.

3.3 Magnanti-Wong Method

The Magnanti-Wong Method is used again in order to strengthen the Benders cuts. The dual of the Magnanti-Wong Problem is solved:

$$
\max \sum_{k \in \mathcal{E}} \left(\sum_{e_t \in \delta^+(k)} \varphi_{e_t} + \xi \sum_{e_t \in \delta^+(k)} \bar{\varphi}_{e_t} \right) \tag{3.41}
$$

s.t.

$$
\sum_{e_t \in \delta^-(i)} \varphi_{e_t} - \sum_{e_t \in \delta^+(i)} \varphi_{e_t} = 0 \qquad \forall i \in \mathcal{T}^x \tag{3.42}
$$

$$
\varphi_{e_t} + \bar{x}_e \cdot u_{e_t} \left(1 + \frac{n^+}{n_e}\right) \cdot \xi \leq x_e^0 \cdot u_{e_t} \left(1 + \frac{n^+}{n_e}\right) \qquad \forall e \in \mathcal{A}, t \in \mathcal{H} \tag{3.43}
$$

$$
\varphi_{e_t} + \bar{v}_{e_t} \cdot u_{e_t} \left(1 + \frac{\bar{z}_e}{n_e}\right) \cdot \xi \leq v_{e_t}^0 \cdot u_{e_t} \left(1 + \frac{z_e^o}{n_e}\right) \qquad \forall e \in \mathcal{A}, t \in \mathcal{H} \tag{3.44}
$$

$$\sum_{e_t \in \delta^+(k)} \varphi_{e_t} + d_k \cdot \xi \leq d_k \qquad \forall k \in \mathcal{E} \tag{3.45}$$

$$\varphi_{e_t} \geq 0 \qquad \forall e_t \in \mathcal{A}^x \tag{3.46}$$

where $\{\bar{x}_e\}$, $\{\bar{z}_e\}$ and $\{\bar{v}_{e_t}\}$ are from the optimal solution to the RMP and $\{\bar{\varphi}_{e_t}\}$ are from the optimal solution to the SP. As a core point, we take

$x_e^o = \frac{1}{|\delta^+(i)|+1}$, $\forall i \in \mathcal{N}$, $\forall e \in \delta^+(i)$
$z_e^o = \min\left(\frac{\mathcal{B}}{2 \cdot l_e \cdot c_l \cdot |\mathcal{A}|}, \frac{n^+}{2}\right)$
$w_e^o = \frac{\mathcal{B}-1}{2 \cdot l_e \cdot c_e \cdot |\mathcal{A}|}$

$$v_{e_t}^o = \begin{cases} 1 & \text{if } t < f_e + w_e^o - 1 \\ 0 & \text{if } t \geq f_e + w_e^o - 1 \end{cases}$$

Note that this is not a true core point, since the unfixed $\{v_{e_t}^o\}$ should strictly decrease with time due to constraints (3.22). However, this would cause issues with numerical precision. The Benders cuts use dual variables from the Magnanti-Wong Problem: which means $\{y_{e_t}\}$, $\{y'_{e_t}\}$, and $\{y_k\}$ are the dual variables associated with constraints (3.43), (3.44), and (3.45) respectively.

3.4 Results

This section compares the results of using the simple two-stage approach versus the Benders decomposition on the Hawkesbury-Nepean case study. Each experiment was run with a time horizon of 600 min, with flooding arriving 5, 6, or 7 hours into the evacuation period. The time horizon was discretized into 5 minute intervals. The upgrade costs were taken to be 5 units per kilometer of additional lanes built and 0.01 units per kilometer for elevating a road to extend its availability by one time step. The budget is 100 units. The algorithms were compared on three high population instances, HN-1.7, HN-2.0, and HN-2.5, representing a 70%, 100%, and 150% increase in population, respectively, in order to reflect future population growth in the Hawkesbury- Nepean region. Each instance was run ten hours, unless it converged

	2-Stage			Benders			Stationary M-W			Moving M-W		
Instance	**TDP** (%)	**FSP** (%)	**Gap** (%)	**LR** (%)	**MPBD** (%)	**Gap** (%)	**LR** (%)	**MPBD** (%)	**Gap** (%)	**LR** (%)	**MPBD** (%)	**Gap** (%)
HN-1.7												
5 h	100	91.1	9.7	100	99.4	0.6	100	99.0	1.0	100	98.2	1.8
6 h	100	90.1	11.0	100	100	0	100	100	0	100	100	0
7 h	100	98.5	1.5	100	100	0	100	100	0	100	100	0
HN-2.0												
5 h	100	92.2	8.4	100	96.1	4.1	100	97.8	2.3	100	96.4	3.8
6 h	100	92.7	7.9	100	100	0	100	99.5	0.5	100	99.9	0.1
7 h	100	95.8	4.4	100	100	0	100	100	0	100	100	0
HN-2.5												
5 h	100	88.9	12.5	100	93.4	7.0	100	92.0	8.6	100	93.1	7.5
6 h	100	91.5	9.3	100	96.5	3.6	100	97.2	2.8	100	97.0	3.1
7 h	100	94.3	6.1	100	99.0	1.0	100	98.6	1.4	100	98.6	1.5

Table 3.1: Results for the HN-1.7, 2.0, and 2.5 Instances on the Flooding Setting.

earier. The algorithms were implemented with JAVA 8 and GUROBI 6.0. The FSP algorithm from [6] was reimplemented. The results were obtained on a 64 bit machine with a 1.4 GHz Intel Core i5 processor and 4 GB of RAM.

Table 3.1 compares the results of the two-stage approach to the Benders method, the Magnanti-Wong method with a stationary core point and the Magnanti-Wong method with a moving core point. The two-stage method yields duality gaps ranging from 1.5% to 12.5%. The Benders approaches are able to close three of the instances, and reduce the duality gap substantially in other instances.

Chapter 4

The Cell Transmission Model

So far, the progress of the evacuation has been modeled macroscopically. Vehicles have been modeled as a uniform flow traveling from one road segment to the next, sent only if road capacity allowed. Due to its simplicity, this kind of macroscopic modeling misses key traffic flow phenomena, such as congestion propagation when traffic density is near capacity, which is commonly seen during evacuations. This chapter presents the implementation of an alternate, mesoscopic, traffic simulation method called the Cell Transmission Model.

4.1 Background

The Cell Transmission Model (CTM), introduced by [4], is a model of vehicle flow in networks, where roads are divided into segments called cells. The CTM is a discrete approximation to the hydrodynamic traffic model. Through a series of difference equations, the CTM expresses the following density-flow relationship:

$$q = \min\left(vk, q_{max}, v(k_j - k)\right)$$

where q is the flow, v is the free-flow speed, k is the density, q_{max} is the maximum flow, and k_j is the jam density. The term $v(k_j - k)$ represents a backwards-moving shock-wave, limiting q when the density is sufficiently close to the jam density. Substituting into the differential flow conservation equation,

$$\frac{\partial q(x,t)}{\partial x} = -\frac{\partial k(x,t)}{\partial t}$$

one obtains

$$\frac{\partial \min\left(vk, q_{max}, v(k_j - k)\right)}{\partial x} = -\frac{\partial k(x,t)}{\partial t}$$

This coincides with the differential hydrodynamic traffic model [4].

A time step h is fixed, and vehicle counts in each cell are maintained incrementally at each clock tick, with several difference equations relating flows to densities. The length of a cell is set to be the distance traveled by a vehicle moving at free-flow speed during time h through that road. The road network has a source node, linked to all evacuation nodes by single time step length cells. Vehicles leaving evacuation nodes go through a series of transit nodes to eventually arrive at one of the safety nodes, with S collectively denoting the set of safety nodes.

4.1.1 CTM Simulation

The CTM simulation maintains the amounts of vehicles in each cell at each time step through update equations, for a given evacuation graph described by binary values $\{y_c\}$ and evacuee departure schedule given by $\{r_c(t)\}$.

Inputs:

y_c: A binary value that equals 1 if a cell is included into the evacuation network, and 0 otherwise

$r_c(t)$: The number of vehicles entering a cell c that is going out of the source at time t

Values Maintained:

$x_c(t)$: The number of vehicles in cell c (not going out of the source) at time t

$f_c^{in}(t)$: The number of vehicles flowing into cell c at time t

$f_c^{out}(t)$: The number of vehicles flowing out of cell c at time t

Parameters:

$u_c(t)$: The capacity of cell c at time t, measured in number of vehicles/ time step in c

w_c: Congestion wave speed, in lengths of c per time step

x_c^j: The jam density of cell c, in vehicles

The number of people successfully evacuated is maintained by *count*.

Initialization

Initially, the road network is empty, i.e. $x_c(0)$ is set to 0 for all cells c.

Propagation

For each cell c, the next edge along the evacuation route, c', is identified through examining the edge binary values (as long as c doesn't enter into a safe node). For a time $t \geq 0$, the following updates are performed. The notation $\sum_{c' \to c}$ means "sum over all cells c' immediately upstream from c."

1. *For edges leaving the source:*

$$w_c(t) = w_{c'}(x_{c'}^j - x_{c'}(t)) - f_{c'}^{in}(t) + f_c^{out}(t-1)$$

$$f_c^{out}(t) = \min(x_c(t), w_c(t))$$

$$x_c(t+1) = x_c(t) + r_c(t) - f_c^{out}(t)$$

   The outflow is the minimum of what is currently in the cell and the shock wave quantity $w_c(t)$. A shock wave is a traffic phenomenon where high density traffic causes delays to occur upstream, and is a distinctive feature of the CTM. The value of $w_c(t)$ models the shock wave coming from the downstream cell. If $x_c(t)$ is very close to the jam density x_c^j, the flow into cell c will be limited. The outflow from c into c' is also limited by how much flow enters c' at time t, except that which arrives from c itself.

2. *For edges between transit nodes:*

$$w_c(t) = w_{c'}(x_{c'}^j - x_{c'}(t)) - f_{c'}^{in}(t) + f_c^{out}(t-1)$$

$$f_c^{out}(t) = \min(u_c(t), x_c(t), w_c(t))$$
$$x_c(t+1) = x_c(t) + f_c^{in}(t) - f_c^{out}(t)$$

The min for these edges additionally limits flow to the capacity of the cell.

3. *For edges coming into safe nodes:*

$$f_c^{out}(t) = \min(u_c(t), x_c(t))$$
$$x_c(t+1) = x_c(t) + f_c^{in}(t) - f_c^{out}(t)$$
$$count = count + f_c^{out}(t)$$

4. In order to move the exiting flow along, set $f_c^{in}(t+1) = \sum_{c' \to c} f_{c'}^{out}(t)$ for all cells c with incoming edges, and add this inflow on to $x_c(t+1)$.

Finally, at the end of the time horizon, the number of people successfully evacuated is reported to be *count*.

4.1.2 CTM Optimization

Optimization over the CTM is largely similar to the CTM simulation. The parameters remain the same, and the values $\{y_c\}$, $\{x_c(t)\}$, $\{r_c(t)\}$, $\{f_c^{out}(t)\}$, and $\{f_c^{in}(t)\}$ become variables.

4.1.3 The CTM Equations

The implementation of the CTM mixed-integer linear programming model is based on the work of [13].

The number of vehicles in a cell c at time $t+1$ is equal to the number of vehicles in c at time t plus the net inflow:

$$x_c(t+1) = x_c(t) + r_c(t) - f_c^{out}(t) \qquad \forall e \in \delta^+(s), \forall (t, t+1) \in \mathcal{H} \times \mathcal{H}$$
$$x_c(t+1) = x_c(t) + f_c^{in}(t) - f_c^{out}(t) \quad \forall e : e \notin \delta^+(s), e \notin \delta^-(\mathcal{S}), \forall (t, t+1) \in \mathcal{H} \times \mathcal{H}$$

$$x_c(t+1) = f_c^{in}(t) \qquad \forall e \in \delta^-(\mathcal{S}), \forall (t, t+1) \in \mathcal{H} \times \mathcal{H}$$

Flow conservation is ensured by

$$\sum_{c \in \delta^-(i)} f_c^{out}(t) - \sum_{c \in \delta^+(i)} f_c^{in}(t+1) = 0 \qquad \forall i \in \mathcal{E} \cup \mathcal{T}, \forall (t, t+1) \in \mathcal{H} \times \mathcal{H}$$

The demand constraints are expressed as:

$$\sum_{t \in \mathcal{H}} r_c(t) \leq d_{c.head} \qquad \forall e \in \delta^+(s)$$

Outflows are governed by three main equations:

$$f_c^{out}(t) \leq r_c(t) \qquad \forall e \in \delta^+(s) \tag{4.1}$$

$$f_c^{out}(t) \leq x_c(t) \qquad \forall e : e \notin \delta^+(s), e \notin \delta^-(S) \tag{4.2}$$

$$f_c^{out}(t) \leq u_c(t) \qquad \forall e : e \notin \delta^+(s), e \notin \delta^-(S) \tag{4.3}$$

Equation (4.1) limits outflow from cells leaving the source to the current inflow, equation (4.2) limits outflow from intermediary cells to be less than what is currently inside the cell, and (4.3) is the capacity constraint.

Inflows are limited according to the shock wave quantity:

$$f_c^{in}(t) \leq w_c \left(x_c^j - x_c(t) \right) \qquad \forall e : e \notin \delta^+(s), e \notin \delta^-(S)$$

Taken together, the inequalities governing f^{out} and f^{in} mimic the flow-density relationship in the CTM simulation. However, they do not capture the min part; as min is nonlinear, we must settle for a series of inequalities instead.

Cell densities are related to the availability of cells by the following equations:

$$x_c(t) \leq M y_c \qquad \forall e \in \mathcal{A}, t \in \mathcal{H}$$

$$\sum_{c' \in \delta^+(e)} y_{c'} \leq 1 \qquad \forall e \in \mathcal{A}$$

$$y_c \in \{0,1\} \qquad \forall e \in \mathcal{A}$$

Additionally, there are several boundary conditions on flows and densities. Taken together, they ensure a clear network at the beginning and end of the simulation.

4.2 Evacuation Planning over the Cell Transmission Model

This section proposes a Mixed Integer Programming model for maximizing the number of evacuees reaching safety using the Cell Transmission Model to simulate vehicle flow during evacuation. The objective function maximizes the number of evacuees entering the network (who all reach safety), based on the equations governing vehicle flows in the CTM.

4.2.1 MIP Model

$$\max \sum_{t\in\mathcal{H}} \sum_{c\in\delta^+(s)} r_c(t) \tag{4.4}$$

s.t.

$$x_c(t+1) = x_c(t) + r_c(t) - f_c^{out}(t) \qquad \forall e \in \delta^+(s), \forall (t,t+1) \in \mathcal{H}\times\mathcal{H} \tag{4.5}$$

$$x_c(t+1) = x_c(t) + f_c^{in}(t) - f_c^{out}(t) \qquad \forall e : e \notin \delta^+(s), e \notin \delta^-(\mathcal{S}), \forall (t,t+1) \in \mathcal{H}\times\mathcal{H} \tag{4.6}$$

$$x_c(t+1) = f_c^{in}(t) \qquad \forall e \in \delta^-(\mathcal{S}), \forall (t,t+1) \in \mathcal{H}\times\mathcal{H} \tag{4.7}$$

$$\sum_{c\in\delta^-(i)} f_c^{out}(t) - \sum_{c\in\delta^+(i)} f_c^{in}(t+1) = 0 \qquad \forall i \in \mathcal{E}\cup\mathcal{T}, \forall (t,t+1) \in \mathcal{H}\times\mathcal{H} \tag{4.8}$$

$$\sum_{t\in\mathcal{H}} r_c(t) \leq d_{c.head} \qquad \forall e \in \delta^+(s) \tag{4.9}$$

$$f_c^{out}(t) \leq r_c(t) \qquad \forall e \in \delta^+(s) \tag{4.10}$$

$$f_c^{out}(t) \leq x_c(t) \qquad \forall e : e \notin \delta^+(s), e \notin \delta^-(S) \tag{4.11}$$

$$f_c^{out} \leq u_c(t) \qquad \forall e : e \notin \delta^+(s), e \notin \delta^-(S) \tag{4.12}$$

$$f_c^{in}(t) \leq w_c \left(x_c^j - x_c(t)\right) \qquad \forall e : e \notin \delta^+(s), e \notin \delta^-(S) \tag{4.13}$$

$$x_c(t) \leq M y_c \qquad \forall e \in \mathcal{A}, t \in \mathcal{H} \tag{4.14}$$

$$\sum_{c'\in\delta^+(e)} y_{c'} \leq 1 \qquad \forall e \in \mathcal{A} \tag{4.15}$$

$$y_c \in \{0,1\} \qquad \forall e \in \mathcal{A} \tag{4.16}$$

$$f_c^{in}(0) = 0 \qquad \forall e \in \mathcal{A} \tag{4.17}$$

$$f_c^{out}(T) = 0 \qquad \forall e \in \mathcal{A} \tag{4.18}$$

$$f_c^{in}(t) = 0 \qquad \forall e \in \delta^+(s) \tag{4.19}$$

$$f_c^{out}(t) = 0 \qquad \forall e \in \delta^-(S) \tag{4.20}$$

$$r_c(T) = 0 \qquad \forall e \in \delta^+(s) \tag{4.21}$$

$$r_c(t) \geq 0 \qquad \forall e \in \delta^+(s), \forall t \in \mathcal{H} \tag{4.22}$$

$$x_c(0) = 0 \qquad \forall e \in \mathcal{A} \tag{4.23}$$

$$x_c(T) = 0 \qquad \forall e \notin \delta^-(\mathcal{S}) \tag{4.24}$$

$$x_c(t) \geq 0 \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \tag{4.25}$$

4.3 Benders Decomposition

The CTM MIP model is computationally intractable for real-sized problem instances, so the Benders decomposition is applied again. The master problem will be the TDP-CTM, which is a relaxation of the MIP. The subproblem, called the FSP-CTM, schedules the departure times of evacuees on the paths generated by the TDP-CTM.

4.3.1 TDP-CTM (Master Problem)

The Master Problem is the Tree Design Problem on the Cell Transmission Model (TDP-CTM), and is a relaxation of the MIP. R_c, X_c, F_c^{in}, and F_c^{out} represent aggregated inflow, occupancy, and flow variables respectively. All constraints have been summed over time.

$$\max \sum_{c \in \delta^+(s)} R_c \tag{4.26}$$

s.t.

$$R_c - F_c^{out} = 0 \qquad \forall e \in \delta^+(s) \tag{4.27}$$

$$F_c^{in} - F_c^{out} = 0 \qquad \forall e : e \notin \delta^+(s), e \notin \delta^-(\mathcal{S}) \tag{4.28}$$

$$X_c - F_c^{in} = 0 \qquad \forall e \in \delta^-(\mathcal{S}) \tag{4.29}$$

$$\sum_{c \in \delta^-(i)} F_c^{out} - \sum_{c \in \delta^+(i)} F_c^{in} = 0 \qquad \forall i \in \mathcal{E} \cup \mathcal{T} \tag{4.30}$$

$$R_c \leq d_{c.head} \qquad \forall e \in \delta^+(s) \tag{4.31}$$

$$F_c^{out} \leq R_c \qquad \forall e \in \delta^+(s) \tag{4.32}$$

$$F_c^{out} \leq X_c \qquad \forall e : e \notin \delta^+(s), e \notin \delta^-(S) \tag{4.33}$$

$$F_c^{out} \leq \sum_{t \in \mathcal{H}} u_c(t) \qquad \forall e : e \notin \delta^+(s), e \notin \delta^-(S) \tag{4.34}$$

$$F_c^{in} \leq w_c \left(|\mathcal{H}| x_c^j - X_c \right) \qquad \forall e : e \notin \delta^+(s), e \notin \delta^-(S) \tag{4.35}$$

$$X_c \leq M y_c \qquad \forall e \in \mathcal{A}, t \in \mathcal{H} \tag{4.36}$$

$$\sum_{c' \in \delta^+(e)} y_{c'} \leq 1 \qquad \forall e \in \mathcal{A} \tag{4.37}$$

$$y_c \in \{0, 1\} \qquad \forall e \in \mathcal{A} \tag{4.38}$$

$$F_c^{in} = 0 \qquad \forall e \in \delta^+(s) \tag{4.39}$$

$$F_c^{out} = 0 \qquad \forall e \in \delta^-(S) \tag{4.40}$$

$$R_c \geq 0 \qquad \forall e \in \delta^+(s) \tag{4.41}$$

$$X_c \geq 0 \qquad \forall e \in \mathcal{A} \tag{4.42}$$

4.3.2 FSP-CTM (Subproblem)

The Flow Scheduling Problem on the Cell Transmission Model (FSP-CTM) takes the edges selected by the TDP-CTM and schedules the departure times of evacuees, propagating flows according to the CTM.

$$\max \sum_{t \in \mathcal{H}} \sum_{c \in \delta^+(s)} r_c(t) \tag{4.43}$$

s.t.

$$x_c(t+1) = x_c(t) + r_c(t) - f_c^{out}(t) \qquad \forall e \in \delta^+(s), \forall (t, t+1) \in \mathcal{H} \times \mathcal{H} \tag{4.44}$$

$$x_c(t+1) = x_c(t) + f_c^{in}(t) - f_c^{out}(t) \qquad \forall e : e \notin \delta^+(s), e \notin \delta^-(\mathcal{S}), \forall (t, t+1) \in \mathcal{H} \times \mathcal{H} \tag{4.45}$$

$$x_c(t+1) = f_c^{in}(t) \qquad \forall e \in \delta^-(\mathcal{S}), \forall (t, t+1) \in \mathcal{H} \times \mathcal{H} \tag{4.46}$$

$$\sum_{c\in\delta^-(i)} f_c^{out}(t) - \sum_{c\in\delta^+(i)} f_c^{in}(t+1) = 0 \qquad \forall i \in \mathcal{E}\cup\mathcal{T}, \forall (t,t+1) \in \mathcal{H}\times\mathcal{H} \tag{4.47}$$

$$\sum_{t\in\mathcal{H}} r_c(t) \le d_{c.head} \qquad \forall e \in \delta^+(s) \tag{4.48}$$

$$f_c^{out}(t) \le r_c(t) \qquad \forall e \in \delta^+(s) \tag{4.49}$$

$$f_c^{out}(t) \le x_c(t) \qquad \forall e : e \notin \delta^+(s), e \notin \delta^-(S) \tag{4.50}$$

$$f_c^{out} \le u_c(t) \qquad \forall e : e \notin \delta^+(s), e \notin \delta^-(S) \tag{4.51}$$

$$f_c^{in}(t) \le w_c \left(x_c^j - x_c(t)\right) \qquad \forall e : e \notin \delta^+(s), e \notin \delta^-(S) \tag{4.52}$$

$$x_c(t) \le M\bar{y}_c \qquad \forall e \in \mathcal{A}, t \in \mathcal{H} \tag{4.53}$$

$$f_c^{in}(0) = 0 \qquad \forall e \in \mathcal{A} \tag{4.54}$$

$$f_c^{out}(T) = 0 \qquad \forall e \in \mathcal{A} \tag{4.55}$$

$$f_c^{in}(t) = 0 \qquad \forall e \in \delta^+(s) \tag{4.56}$$

$$f_c^{out}(t) = 0 \qquad \forall e \in \delta^-(S) \tag{4.57}$$

$$r_c(T) = 0 \qquad \forall e \in \delta^+(s) \tag{4.58}$$

$$r_c(t) \ge 0 \qquad \forall e \in \delta^+(s), \forall t \in \mathcal{H} \tag{4.59}$$

$$x_c(0) = 0 \qquad \forall e \in \mathcal{A} \tag{4.60}$$

$$x_c(T) = 0 \qquad \forall e \notin \delta^-(\mathcal{S}) \tag{4.61}$$

$$x_c(t) \geq 0 \qquad \forall e \in \mathcal{A}, \forall t \in \mathcal{H} \tag{4.62}$$

4.3.3 Benders Cuts

The Benders cuts are of the form

$$z \leq \sum_{k \in \mathcal{E}} d_k v_k + \sum_{t \in \mathcal{H}} \sum_{c \in \mathcal{C}} u_e(t) v_c(t) + \sum_{t \in \mathcal{H}} \sum_{e \notin \delta^+(s), e \notin \delta^-(S)} w_c x_c^j v_c(t)' + M \sum_{c \in \mathcal{A}} y_c v_c$$

where $\{v_k\}$ are the dual values associated with the demand constraints (4.48), $\{v_c(t)\}$ are the dual values associated with the capacity constraints (4.51), $\{v_c(t)'\}$ are the dual values associated with the shock wave flow constraints (4.52), and $\{v_c\}$ are the dual values associated with the "flow on paths" constraints (4.53). The cuts are added one by one to the Restricted Master Problem, which is based on the TDP-CTM. In practice, the M constant is dropped because constraints (4.53) could have been represented as

$$\frac{1}{M} x_c(t) \leq \bar{y}_c \qquad \forall e \in \mathcal{A}, t \in \mathcal{H}$$

4.4 Results

The algorithms were evaluated on two of the higher population instances, HN-1.7 and HN-2.0. The time horizon was 600 min, with flooding arriving 5 or 8 hours into the horizon. The jam density was set to 56 vehicles per kilometer lane and the congestion wave speed was set to 20 kilometers per hour (as suggested by [23]). Each instance was run for one hour, unless the algorithm converged earlier. The algorithms were implemented using JAVA 8 and GUROBI 6.0 and the results were obtained on a 64 bit machine with a 1.4 GHz Intel Core i5 processor and 4 GB of RAM. The Cell Transmission Model (CTM) was compared to the macroscopic model from Chapter 1 (Macro) on the same evacuation graph. When constructing evacuation graphs from the original road network, two settings were used. "A" denotes the standard setting, and "B" denotes the setting where additional cells are included at intersections. Through a transformation of the evacuation graph, the "B" setting allows for additional patterns of flow at intersections that would have ordinarily been classified as divergent, but are actually easy for authorities to separate.

Overall, the results show that the macroscopic model evacuates more people than the CTM. This suggests that either the macroscopic model overestimates the number of people that can be evacuated, or the CTM is much slower to converge. The next section evaluates the plans proposed by the CTM and the macroscopic model by running them on CTM and macroscopic simulators.

The "B" setting led to drastic improvements in the number of people evacuated compared to the "A" setting. This was not entirely obvious, since though the presence of extra edges around intersections in the "B" setting greatly increases the number of paths that can be chosen from, adding edges comes at a price of making the model more complicated and slower to run.

	2-Stage			Benders			
Instance	**TDP** (%)	**FSP** (%)	**Gap** (%)	**CPU** (s)	**RMP** (%)	**BD** (%)	**Gap** (%)
HN-1.7, 5h							
Macro-A	97.8	84.5	15.7	2370.3	97.8	93.0	5.1
CTM-A	97.7	87.3	11.9	3358.4	97.7	91.6	6.6
Macro-B	99.3	84.0	18.2	2635.5	99.3	93.9	5.7
CTM-B	99.2	85.9	15.5	3322.5	99.2	87.5	13.3
HN-1.7, 8h							
Macro-A	100	95.9	4.3	1050.5	100	99.7	0.3
CTM-A	100	89.5	11.8	730.2	100	94.8	5.4
Macro-B	100	94.3	6.0	3131.2	100	99.8	0.2
CTM-B	100	87.4	14.4	2898.5	100	94.3	6.0
HN-2.0, 5h							
Macro-A	87.3	77.6	12.5	927.9	87.3	84.9	2.9
CTM-A	92.8	79.1	17.3	1403.8	92.8	80.9	14.8
Macro-B	98.5	84.2	16.9	857.0	98.5	94.6	4.1
CTM-B	98.4	80.9	21.6	3306.4	98.4	87.9	13.1
HN-2.0, 8h							
Macro-A	100	95.3	5.0	19.2	100	97.0	3.1
CTM-A	99.4	85.8	15.8	2619.6	99.4	87.9	13.1
Macro-B	100	93.8	6.6	259.0	100	99.6	0.4
CTM-B	100	86.7	15.4	479.9	100	92.7	7.8

Table 4.1: Results for the HN-1.7 and 2.0 Instances

4.5 Model Evaluation

The macroscopic and CTM models were verified by running their proposed solutions on a macroscopic simulator and a CTM simulator, for the HN-1.7 instance. The macroscopic simulator propagates flows just as in the optimization models that use the macroscopic approach. Specifically, flow moves through the network without waiting as long as there is capacity. The CTM simulator, however, propagates flows differently than the optimization models that use the CTM approach. Namely, the CTM simulator makes one of the flow equations tight through the use of a min. With the min replaced by inequalities, the CTM optimization model allows for lagged flow, which expands the feasible solution set greatly. Results show that the macroscopic plans perform well when they are tested on the CTM simulator, validating the macroscopic model. By contrast, the CTM plans perform poorly on both the macroscopic

Instance	Obj. (%)	Macro (%)	CTM (%)
HN-1.7, 5h			
Macro-A	93.0	93.0	90.7
CTM-A	91.6	48.8	42.8
Macro-B	93.9	93.9	90.9
CTM-B	87.5	47.2	48.8
HN-1.7, 8h			
Macro-A	99.7	99.7	97.5
CTM-A	94.8	50.3	55.8
Macro-B	99.8	99.8	95.9
CTM-B	94.3	53.7	56.8

Table 4.2: Model Evaluation Results

simulator and the CTM simulator. The likely reason for the difference in performance between the CTM plans and the macroscopic plans is that the CTM solution fully permits waiting, whereas the macroscopic simulation forbids it and the CTM simulation controls it. Inspection of the flows over time in the CTM plans shows that many times there is a sizeable gap between the outflow and the minimum of the quantities limiting the outflow.

It is possible to represent the CTM exactly through a MIP model. For example, to represent a constraint of the form $z = \min(x, y)$, one can introduce a binary variable b and write

$$
\begin{aligned}
&z \leq x \\
&z \leq y \\
&z \geq x - bM \\
&z \geq y - (1 - b)M
\end{aligned}
$$

where M is a constant and $M \geq \max(x, y)$. The case $b = 0$ means $z = x$ and $b = 1$ means $z = y$. However, with this approach one would not be able to apply the Benders decomposition since the subproblem would be a MIP. The linear relaxation of the MIP could be solved instead of course, but it is not clear that such an approach would perform better than the present one.

Chapter 5

Conclusion

This thesis applied the Benders decomposition to the Convergent Evacuation Planning Problem (CEPP) and the Convergent Evacuation Network Design Problem (CENDP). For both problems, the master problem in the decomposition selected a convergent road network while the subproblem scheduled the departure times of evacuees. The Magnanti-Wong method was used to generate stronger, Pareto-optimal, Benders cuts. Compared to the two-stage approaches, the Benders decomposition methods yielded substantial improvements, often on the order of 10%, which is significant in the context of evacuation planning. The improvements were especially pronounced for severe flood scenarios, which shows that the Benders decomposition approach can be used to extend the two-stage method to flood scenarios, as well as to other evacuation scenarios involving road blockages such as bushfires, earthquakes, and volcanic eruptions. One limitation of the Benders decomposition approach is that it does not always converge to an optimal solution, which is due to the high degree of solution degeneracy.

The work on the CENDP has several limitations. Lacking data on infrastructure upgrade costs, we assumed that costs per kilometer did not differ among roads. In reality, some roads may be more expensive to upgrade than others. The costs are unitless, but we do assume a certain ratio between the cost to add a lane and the cost to elevate a road, which may not be accurate. It was also assumed that all roads had the same maximal number of additional lanes that could be added. It may not be the case that capacity increases linearly with the number of lanes. Once more data becomes available, this model will be better equipped to make recommendations regarding infrastructure upgrades.

Chapter 4 proposed using the Cell Transmission Model for evacuation planning. Results show that the Benders approach is effective when using the CTM as well. Unfortunately, the CTM optimization model does not accurately translate the non-linearities of the CTM, and permits lagging flow. When plans generated from the CTM model were tested on the CTM simulation, the number of people was substantially lower than reported by the CTM model. The CTM simulation can be used to evaluate the accuracy of plans proposed from macroscopic models. Preliminary experiments show that plans made from macroscopic models perform very well on the CTM, evacuating nearly the same number of people as reported by the macroscopic models. This is encouraging, because the macroscopic models are easier to implement and faster to run than the CTM models. Unfortunately, there is no way to re-express the nonlinearities of the CTM simulation in a linear program, which is what we are restricted to in the Benders decomposition framework.

A major underlying limitation of this work is that everything in the model is deterministic, yet there is a high degree of uncertainty in any evacuation scenario, particularly with regards to road capacities, which can change drastically in the event of a traffic accident. Future work will address this limitation by building a probabilistic model of edge capacities that depend on the flow, and will consider minimizing some notion of risk in the evacuation.

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