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"Inferring Knowledge and Ignorance about Motion from the Limits of Vision and Physics"

by
Raymond Kozlowski

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Date . 2/19/90 . . . .
Thomas L. Dean

Approved by the Graduate Council

Date . . . .

Inferring Knowledge and Ignorance about Motion from the Limits of Vision and Physics

Master's Thesis

Raymond Kozlowski

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ABSTRACT

In [Davis88] a model allowing inferences about knowledge and ignorance from visual perception is presented. One limitation of this model is that in it objects are free to move about as long as they do not overlap. Consequently, a course of events normally dismissed by intelligent beings as physically impossible cannot be rejected. After all, a ball cannot stay suspended in mid-air, a person cannot circumnavigate the globe in one hour, and a motionless stone cannot start climbing a hill by itself. In this paper, we incorporate physical restrictions necessary to make inferences of this kind into Davis' model without defeating its primary purpose, the ability to infer ignorance.

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Chapter I

INTRODUCTION.

The motion of objects on the earth is limited by physical constraints such as gravity, conservation of energy, and mechanical limitations. Intelligent beings can predict a great deal about the motion of objects from perception and their knowledge of these constraints, and construct plans accordingly. On the other hand, the limits of vision or absence of applicable physical laws cause ignorance about certain facts simply because there is no way of finding out about them.

The model for inferring knowledge and ignorance from visual perception presented in [Davis88] is based on the assumption that whatever is physically possible and visually consistent with perceptions, is not known to be false. On the other hand, whatever is perceived or follows from perceptions *is* known. Our goal is to modify Davis' model by extending the concept of what is physically possible to include the effect of gravity on isolated objects (not in contact with any other object), the conservation of energy on inert objects, and maximum possible self-generated speeds of active objects. Any inference about the motion of an object will also have to involve knowing a bound on the size of the object lest the inference become meaningless.

We shall not consider actions and goals except in showing that physical laws may help an agent in determining the feasibility of a plan. In particular, we shall consider the role of physics in continuous coordination between perception and plan execution presented in [Davis89]. Allowing agents to reason about other agents' plans would greatly complicate the ability to infer ignorance and is beyond the scope of this work. The term "plan" is used loosely here - it merely means a course of events involving an agent.

We shall focus on the following problems:

Ia. Kim is standing 2 metres away from a waste-paper basket, holding a pit. Infer that Kim knows that if she throws the pit at a certain angle and with a certain speed then the pit will fall into the basket.

Ib. The situation is the same as the one described in the preceding example except now the distance is 10 metres. We consider two plans:

- Kim simply throws the pit into the basket
- Kim walks to within 2 m of the basket and then throws the pit into it.

We wish to distinguish between these two plans in that the first one is not feasible given Kim's accuracy while the second one is both physically and epistemically feasible.

II. Joe sees a picture with a wall in the background. Infer that he knows that the picture is hanging on the wall.

III. Isaac is lying on the ground underneath a tree. Suddenly, he sees an apple falling above his head. Infer that he knows that unless he quickly moves out of the way, the apple will hit his head.

IV. Judy sees Sharon standing facing a falling ball, and looking at the ball. Infer that Judy knows that Sharon knows that the ball is falling.

V. Fred and Max are in a room together. Max leaves the room. Fred knows that the nearest exit is over 55 metres away. Infer that after 15 sec Fred knows that Max is still in the building. Also infer that if Fred has not seen Max for an hour, then he does not know whether or not Max is still in the building.

VI. Jenny drops a stone into an empty well. Infer that Jenny knows that the stone is not going to jump out of the well.

VII. David sees a sled sliding downhill. Infer that David knows that the sled is not going to reverse its direction and start moving uphill.

These problems illustrate various aspects of inferring motion under physical constraints. (Ia) shows how we can infer that an agent can predict the motion of an isolated object given initial velocity with some accuracy. Kim knows that the pit will fall into the basket because she can see that the path to the basket is unblocked and she knows the effects of gravity. (Ib) shows how knowledge of physics helps to determine the feasibility of a plan. Kim "rejects" the first plan because she knows that given her accuracy she is too far from the basket; she also knows that the second plan will conditionally succeed. (II) shows the ability to infer that an object is connected to another from lack of motion. Joe knows that the picture is hanging on the wall because if it were not, it would have fallen to the floor. (III) shows how an agent can infer both the size limit of an object and its isolation directly from perception. Isaac could bound the size of the apple when it was hanging on the tree. Now that the apple is falling, he knows that it must be separated from the tree given its maximum size. (IV) shows how one agent can infer another agent's knowledge about motion in gravity. From where Judy is standing, she can see what Sharon sees and therefore must know that the ball has no support and is separated from the ground. (V) shows how an agent can infer how far a moving object may have gone, if he knows its maximum possible self-generated speed. Fred knows that Max is still in the building because if he were not, he would have had to move with a speed that Fred knows that he cannot generate. (VI) shows the limits in how inert objects move without contact with active objects. Jenny knows that the stone is not going to jump out of the well because that would mean an increase in the energy of the stone, and there is nothing in the well to supply the stone with extra energy. (VII) shows how an agent can rule out unnecessary discontinuities in the value of parameters such as velocity. David knows that the sled is not going to start climbing the hill because that would cause a discontinuity in its velocity while there is a perfectly possible course of events without such a discontinuity, namely one in which the sled continues sliding downhill.

In addition to these problems designed for inferring positive knowledge, we want to retain the ability to infer ignorance by solving Davis' benchmark:

O. Steve is in a closed room with no windows, and he crosses from one side of the room to the other. Claire is outside the room. Infer that Claire does not know now that Steve has crossed the room.

This problem shows how we can infer that an agent is ignorant of a fact from our knowledge of the physical limits of vision. Since Claire cannot see Steve inside the room, she does not know what is happening in the room.

Chapter II

PHYSICAL CONSTRAINTS.

We start by presenting a brief overview of the world model in [Davis88]: There is a fixed set of physical objects maintaining constant shape and moving continuously in time. The places occupied by two objects at a single time may not overlap. Some objects are agents. An object is assumed to be visible to an agent if it is not occluded from him by an object in between. The knowledge of agents is governed by the following axioms:

A.1 Knowledge of axioms: All general axioms - axioms of predicate calculus, geometry, time, physics, knowledge and perception - are known.

A.2 Consequential closure: Any logical implication of the agents knowledge is known.

A.3 Veridicality: All knowledge is true.

A.4 Positive introspection: If an agent knows a fact, he knows that he knows it.

A.5 Memory: If an agent knows a fact (with no time indexicals) at one time, he knows it at all later times.

A.6 Internal Clock: An agent always knows what time it is.

A.7 Anything that is perceived is known.

A.8 If a physical statement is physically possible, and it does not contradict any past or present perceptions, then it cannot be known to be false.

We modify Davis' model by adding more restrictions to what is physically possible. Otherwise, the model remains unaltered. First, we define the gravitational force. We assume that it always acts downwards and that the gravitational acceleration is constant, namely 9.81 m/sec^2. The effects of gravity on an object can only be inferred while it is isolated. An isolated set of objects behaves in such a way that its centre of mass moves with constant horizontal velocity and with gravitational acceleration directed downwards. The motion of the centre of mass of an isolated set of objects depends solely on its initial velocity. The only exception to the law of gravity is the earth, which is a unique object, always motionless, and visually distinguishable from any other object.

The knowledge of gravity is useful only when separation between objects can be determined. An agent can deduce that two objects are separated directly from perception if he can see the space between them. We call this *visual separation.*

Objects may be carried by other objects, called carriers. The earth is always the carrier; it is never *being* carried. In general, it is impossible to determine visually which one of two objects in contact with each other is the carrier (is a moving train being pulled by the locomotive or pushed by the caboose?). We assume that for every object there is a speed relative to a carrier that the object cannot exceed. These maximum possible self-generated speeds must be specified *a priori* by the system. The most obvious way is to specify a single value of speed for each type of object.

Some objects are inert, possessing only kinetic and gravitational potential energy. Their maximum self-generated speed is always zero. Non-inert objects are called active. All agents are active objects, but not vice-versa. Some objects, such as a train, are active but do not possess knowledge or perceptions. Since the classification of objects into inert and active ones is a part of our physics, every agent always knows whether or not a particular object is inert. Incidentally, this does not mean that an agent necessarily knows whether or not the object he is looking at is inert. Even though he knows whether that particular object in the world is inert, he may not recognize it, that is know that what he sees is, in fact, that object. While objects are not propelled, the sum of their kinetic and gravitational potential energy may not increase.

The physical world is characterized by a number of parameters, such as the velocity of objects and the *carrier* relation. A discontinuity in the value of a parameter may occur as a result of an action or when there is no other way to satisfy the axioms. Since we do not reason about plans other than determine their feasibility, all we can say is that an object capable of executing a plan, that is any active object, may perform an action upon an object in contact with it, causing, for instance, a discontinuity in its velocity. Spontaneous discontinuities occur because of the simplifications in our physics. Since all objects are assumed to be perfectly rigid at all times, a collision, for instance, necessitates a discontinuity in velocity in most circumstances. Otherwise, we disallow discontinuities in the values of parameters.

Within the modified model, we can pose the following analogues to problems I through VII:

Ia. Kim is in a closed room 2 metres away from a waste-paper basket, holding a pit. The space between Kim and the basket is free. The opening of the basket, located at the same height as the pit, is wholly visible and its diameter is 0.5 m. Kim knows that the diameter of the pit is less than 1 centimetre. Infer that Kim knows that if she throws the pit at an angle of $45^o \pm 5\%$ with an initial speed of 4.4 m/sec $\pm$ 5% and none of the other objects move for 0.7 sec, then the pit will fall into the basket (note that even though the range of initial velocities is designed to suggest Kim's throwing accuracy, in this example there is neither a plan nor knowledge on the part of Kim as to the pit being thrown in this or any other way; all Kim knows is that *if* the pit has an initial velocity in the given range, then it will behave in the specified manner).

Ib. The situation is the same as in example Ia except now the distance is 10 metres. Kim can throw objects at any angle with a 5% accuracy and with any initial speed with the same accuracy. We consider plan A:

A: Kim throws the pit at an angle $\theta_0 \pm 5\%$ and with an initial speed $V_0 \pm 5\%$.

Show that plan A is not feasible, i.e. Kim does not know whether or not the pit will fall into the basket. Now let us consider plan B:

B: Kim walks to within 2 m from the basket and throws the pit at an angle of $45^{\circ} \pm 5\%$ and with an initial speed of 4.4 m/sec $\pm$ 5%.

Show that plan B is both physically and epistemically feasible.

II. Joe sees a picture of height less than 1 metre, with a wall in the background. He does not see any point of contact between the picture and the wall. He knows that there is no hole in the wall. The picture is motionless for 1 sec. Therefore, at the end of the interval, he knows that the picture has not been separated from the wall during the entire interval.

III. Isaac is lying on the ground in some enclosure with a tree and an apple hanging from it 4 metres above his head. Now he sees the apple 3 m above. From what Isaac knows about the structure of the tree, he can infer that if he, the tree and the apple are the only objects within the enclosure, and he and the tree are motionless for the next 1 sec, then the apple will come in contact with him within the interval.

IV. Judy, Sharon, a ball, and the ground are the only objects within a convex enclosure over a certain interval. We assume that Sharon has a unique set of visible properties that allow her to be distinguished from anyone else. Due to the geometry of the situation, Judy can see Sharon, the ball, the space between them, and the space between the ball and the ground. She can also see that at least the visible part of Sharon has an unoccluded view of the space between the ball and the ground. Since Judy further knows that Sharon is a conscious agent, whose perceptions and knowledge are governed by the standard axioms, Judy can deduce that Sharon can see that the ball is separated from the ground and that Sharon knows that if there are no other objects in the enclosure, then the ball is falling.

V. At the beginning of the interval in question, Max is within 3 metres of Fred, and Fred has an unoccluded view of him. Fred knows that the horizontal distance from the nearest exit is over 55 m. Fred is motionless throughout the interval. He also knows that

Max has a diameter less than 2 m and that his maximum possible self-generated speed is less than 200 m/min. Infer that at the end of the interval, after 15 sec, Fred knows that if there were no other objects in the building, Max was not isolated, and he did not carry the building, then he is still inside the building. Furthermore, infer that if the length of the interval is 1 hour long, and Max is invisible to Fred throughout the interval, then Fred does not know at the end of the interval whether or not Max is still in the building.

VI. Jenny is holding a motionless stone completely inside a well. She knows that the stone has a diameter no greater than 0.1 m. She drops the stone and withdraws her hand. She can see that no point in the stone is less than 0.11 m below the opening of the well. After 0.5 sec, Jenny can see that there is a point in the stone less than 1.22 m below the opening of the well, and that the stone has not hit the walls or the bottom of the well at any time during that interval. Infer that Jenny now knows that unless something else enters the well, the stone will remain inside it.

VII. David has been watching a sled moving downhill during some interval. He can see that the space in front of the sled is free. Infer that David knows now that, unless there is an object behind the sled which might block the sled's path or actively change its direction, the sled will not reverse its velocity instantaneously.

In addition to these problems, we wish to be able to solve Davis' benchmark for inferring ignorance, slightly modified:

O. Claire is on one side of a wall, for some interval of time. On the other side of the wall, occluded from Claire, is an active object. The object lies within some larger region, which is entirely occluded from Claire and such that the object can move within it. It is in contact with the ground. Over a certain time interval, Claire stays motionless, the object stays within its envelope, it is in contact with the ground, and no other object ever overlaps the envelope. Then there is no way for Claire to know whether the object is motionless or whether it is moving around within its envelope, since either is equally compatible with the motions of objects that Claire does see.

Chapter III

FORMAL EXTENDED MODEL.

The model in [Davis88] is a variation of the possible world semantics for knowledge. Two levels of possible worlds are used: layouts, timeless physical descriptions of the world, and situations, states of the world including non-visible properties of objects and knowledge states of agents. A behaviour is a function of layouts over time while a chronicle is a function of situations over time. Layouts and behaviours may or may not be physically possible. An object is an atomic individual with a set of visual properties. An agent recognizes these properties if any part of the object is visible to him. Knowledge is represented by the accessibility relation over situations. The predicate *k(A, S1, S2)* holds if situation *S2* is knowledge-accessible to agent *A* in situation *S1*, i.e. as far as *A* knows in *S1*, *S2* may be the real situation.

We alter Davis' model by further constraining the physical possibility of layouts and behaviours. First, we introduce some new predicates. Two point sets are said to be separated if there is no path between them lying completely in space occupied by objects:

$$\text{separated}(X1, X2, L) \leftrightarrow$$

$$[\ \text{connected}(XP) \wedge \text{intersect}(XP, X1) \wedge \text{intersect}(XP, X2) \rightarrow$$

$$(\exists XP1)\ [\text{sub\_place}(XP1, XP) \wedge \text{free\_space}(XP1, L)]\ ]$$

An object is said to be *isolated* iff the figure occupied by it is separated from the figure occupied by every other object in the layout.

Two point sets are said to be *visually separated* relative to an agent if for any path between them there is a segment visible to the agent and in free space. Even if two objects are not visually separated but the agent knows an upper bound on the size of one of the objects, separation between them can sometimes be inferred. A *vd_envelope* of depth *D* enclosing an object *O* partially visible to an agent is defined as a connected set of points such that each point in its boundary is either in free space or is separated from a visible point in *O* by a distance greater than *D*. If the agent knows that the diameter of the object is less than *D*, then he knows that the object is separated from every object not completely inside the envelope.

We apply the cartesian coordinate system to the space; that is, we map each point to a set of coordinates (x(P), y(P), z(P)). Each object set in a layout has a unique point called the centre of mass. The centre of mass of a non-empty object set is a unique point lying in any convex enclosure containing it, and moving continuously in time, (that is, the function over time to points λ(T) *centre(Q, scene(B,T))* is continuous, for any object set *Q* and behaviour *B*). This continuity does not follow from the continuous motion of the objects themselves if the set is symmetric about some axis. A single object's centre of mass is a point fixed relative to the figure occupied by the object throughout time.

The velocity of an object set at a time instant *T* in each of the three dimensions, *xvel(Q, B, T)*, *yvel(Q, B, T)*, and *zvel(Q, B, T)* is the derivative of the position of the set's centre of mass at *T*. Velocity is assumed to be piecewise continuous. Zero velocity of an object is not equivalent to its being motionless. An object may by motionless but rotating in place with its centre of mass having non-zero velocity and it may also change its place by rotating around its motionless centre of mass.

Next, we define the *carrier_of* relation, which is a total ordering over any group of connected objects except that it is non-reflexive (by definition, an isolated object has no carriers). It is false for any pair of separated objects. Objects with carriers may be *propelled* by themselves or by their intersecting carriers. Each object in our world has a maximum possible self-generated speed relative to a carrier in contact with it, *vmax(O)*.

$$(\exists OC)\ \text{carrier\_of}(OC, O, \text{scene}(B, T)) \wedge \text{rvel}(O, OC, B, T) \leq \text{vmax}(O)$$

A layout is physically possible if no two objects overlap, the object *oground* occupies the place *xground*, and it has no carriers at any time.

Some objects are inert, possessing only kinetic and gravitational potential energy. We define *emu(O, B, T)* as the the sum of the kinetic and gravitational potential energy of a unit of mass of object *O* at time *T*:

$$\text{emu}(O, B, T) = g \cdot \text{ycoord}(\text{centre}(O, \text{scene}(B, T))) + \tfrac{1}{2}\text{vel}(O, B, T)^2$$

While an inert object is not in contact with a moving object, its *emu* may not increase. We say that it is in *inert motion*.

Gravity acts upon an isolated set of objects in such a way that its centre of mass moves with constant horizontal velocity and vertical gravitational acceleration:

$$(\forall T)\ [\text{time\_in}(T, I) \rightarrow \text{isolated}(Q, \text{scene}(B, T))] \rightarrow$$

$$\text{yvel}(Q, B, \text{end}(I)) = \text{yvel}(Q, B, \text{start}(I)) - 9.81 \cdot \text{metre/sec}^2 \cdot \text{time\_length}(I) \wedge$$

$$\text{xvel}(Q, B, \text{end}(I)) = \text{xvel}(Q, B, \text{start}(I)) \wedge$$

$$\text{zvel}(Q, B, \text{end}(I)) = \text{zvel}(Q, B, \text{start}(I))$$

A behaviour is physically possible if all its layouts are physically possible and each object always obeys gravity and at any time it is either in inert motion or being propelled by itself or another intersecting object. Based on this definition, the speed of a non-isolated object is always limited while it has a carrier. The motion of an isolated set of objects, on the other hand, depends on its initial velocity. This initial velocity must be restricted in some way. It is assumed that the velocity of an object at an instant such that it is not isolated during some interval before and isolated during some interval after may not increase. This restriction limits the initial speed of an isolated object except when it has been isolated at all times before some instant. This is not a serious problem, however, since once the object is observed over any interval, its speed can be bounded.

In order to limit the number of discontinuities in the values of parameters characterizing the physical world, such as velocity and the *carrier_of* relation, we use the filter preferential entailment theory presented in [Sande88], a variant of the chronological minimization of discontinuities (CMD). Since discontinuities resulting from actions must not be minimized, we must waive CMD where actions may occur. An action can only be performed by an active object, so discontinuities in the value of parameters for all active objects and those in contact with them are masked off. Other discontinuities are minimized by the $\lessdot$ relation defined for any pair of behaviours:

$$B1 \lessdot B2 \leftrightarrow$$
$$(\exists T0)\ [\ (\forall T,U)\ [T \leq T0 \rightarrow \text{value}(U, B1, T) = \text{value}(U, B2, T) \land$$
$$\text{breaks}(B1, T0) \subset \text{breaks}(B2, T0)\ ]$$

We obtain the filtering described in [Sande88] by minimizing behaviours in the *bv_compatible* relation before applying the *v_compatible* relation rather than only minimizing visually compatible behaviours.

Not all fluents are parameters. We want to minimize the discontinuities of only those fluents that directly characterize physics, not their consequences. For instance, it would be a mistake to minimize the discontinuities of an object's being visible to an agent, which is a consequence of motion and not a physical property. In this extension of physics, the only parameters are the positions of the centres of mass of all objects, all of their derivatives (velocity, acceleration, etc.), and the *carrier_of* relation. There may be many more parameters defined in the situations, as well as some other properties of physics applicable to CMD, such as angular velocity, but we are not concerned with them here.

An interesting case of discontinuities occurs in the *carrier_of* parameter. By definition, during a collision between two objects, one of them must be the carrier of the other. If the objects separate after the collision, neither of them can be the carrier of the other. Such a discontinuity is not a problem, however, since a collision causes a discontinuity anyway, and therefore a behaviour in which the objects stick together after the collision is not preferred over one in which they separate. Unfortunately, other cases cause problems.

Consider a ball rolling on a table. When the ball reaches the edge of the table, it may either stop or fall off the edge. The first case causes a discontinuity in velocity while the second one, in the *carrier_of* parameter. Neither of them would be preferred according to CMD. Therefore, we must mask discontinuities in the *carrier_of* relation of two objects when there is a discontinuity in their places intersecting.

We have used only very few parameters. In general, with a large number of parameters, it may be difficult to decide which ones to minimize and where to mask their discontinuities.

Chapter IV

PROOFS.

Each of our sample inferences (I) through (VII), as well as example (O), can be formalized as a proof from the axioms of perception and knowledge, together with suitable axioms of geometry and physics including the law of gravity, inertia, and the maximum possible self-generated speeds of objects. In this section we shall sketch the structure of these six proofs. The non-logical symbols introduced here are defined in appendix A or B. Free variables are assumed to be quantified over the entire formula. The complete proof of example (O) may be found in appendix C and that of example (VI), in appendix D.

I. The statement to be proved, that Kim knows that if the pit is thrown with initial speed of 4.4 m/sec ± 5% at an angle 45° ± 5% and no other object in the room moves during the interval, then the pit will fall into the basket, can be formalized as follows:

k(akim, s0, S1) $\wedge$

B1 = behaviour(chronicle(S1)) $\wedge$

L1A = scene(B1, start(i0)) $\wedge$ L1Z = scene(B1, end(i0)) $\wedge$

$0.95 \cdot 4.4$ metre/sec $< V_0 < 1.05 \cdot 4.4$ metre/sec $\wedge$

$0.95 \cdot 45^\circ < \theta_0 < 1.05 \cdot 45^\circ \wedge$

xvel(opit, B1, start(i0)) = $V_0 \cdot \cos\theta \wedge$

yvel(opit, B1, start(i0)) = $V_0 \cdot \sin\theta \wedge$

free_space(behind(place(opit, L1A), place(akim, L1A)) $\cap$ inside(oroom), L1A) $\wedge$

$(\forall$T,O) [time_in(T, i0) $\wedge$ O $\neq$ opit $\wedge$

sub_place(place(O, scene(B1, T)), inside(oroom)) $\rightarrow$ motionless(O, B1, I)] $\rightarrow$

sub_place(place(opit, L1Z), place(inside(obasket), L1Z))

We assume that Kim knows that 1 centimetre is an upper bound on the size of the pit, and thus she can estimate the initial position of its centre of mass to within a centimetre. In example II and in chapter V we discuss how an agent can determine an upper bound on the size of objects. Because the pit is very small, it is even reasonable to allow Kim to determine the bound based on the theory of vision adopted here, i.e. by constructing a *vd_envelope* whose boundary is wholly visible to Kim and entirely in free space, and which contains the pit. If the diameter of the envelope is less than 1 cm, then so is that of the pit.

After the throw, the pit is isolated since, by Kim's assumption, no other object in the room moves and the space between Kim and the basket is free. Therefore, Kim can apply gravity to predict the motion of the pit in the interval. Given the bounds on the initial position of the centre of mass and the initial speed and angle at which it is thrown, in all physically possible behaviours, the place of the pit will be inside the basket. The actual derivation is purely geometrical and need not concern us here. The set of possible trajectories of the pit defines a point set that Kim must know to be in free space except for the pit throughout the interval. This is satisfied because Kim can see the entire space except behind the pit and knows that it is in free space. As to the space behind the pit, she assumes that it is in free space. As we can see, the assumption about the other objects in the room being motionless could be relaxed. All that is required is that the set of all possible trajectories not be intersected by any object other than the pit.

Ib.A. The plan fails because it violates a knowledge precondition; Kim does not know whether the pit will fall into the basket given the assumptions. In order to show that, we have to find a possible behaviour satisfying the assumptions such that the pit is not inside the basket at the end of the interval for any value of initial velocity. The horizontal distance covered by the pit is the following:

$$d = V_0{}^2 \cdot \sin(2\,\theta)/g$$

Since the accuracy of V_0 is 5%, the accuracy of d is worse than 10%, or at least 1 m. Since the diameter of the opening of the basket is 0.5 m, there is a behaviour in which the pit misses the opening and hits the side of the basket or the floor. At that point the law of gravity does not apply; the pit may even become motionless. Thus, we have found a physically possible behaviour satisfying the assumptions, compatible with Kim's knowledge in *s0*, and one in which the pit does not fall into the basket regardless of its initial velocity. Therefore, Kim does not know that the pit will fall into the basket under plan A.

Ib.B. The first part of the plan can be represented as follows:

monitor(distance(akim, obasket) < 2 • metre, go_towards(obasket))

The action *go_towards(obasket)* is physically feasible because the space between Kim and the basket (and, in particular, between Kim and a place 2 m in front of the basket) is free and will remain so because the other objects are assumed to remain motionless. Furthermore, there is a possible carrier throughout the distance, namely the floor. Finally, Kim is not an inert object so she can generate the motion herself. The action is epistemically feasible for the same reason: Kim can see the basket and therefore she knows the direction towards it. Once Kim comes within 2 m from the basket, she can throw the pit into it. This action is obviously feasible because it is the same as the one in example Ia, and we have shown that it is physically possible and that Kim has all the required conditional knowledge. It ought to be stressed that Kim still does not know what will really happen. We have used the term "conditional knowledge" deliberately - the goal will be reached predicated upon the conditions being met.

II. The statement to be proved, that Joe knows at the end of the interval that the picture was not separated from the wall during the entire interval, can be formalized as follows:

k(ajoe, s0, S1) →

(∃T) [time_in(T, i0) ∧

¬separated(opicture, owall, scene(behaviour(chronicle(S1)), T))]

Joe has an unoccluded view of the picture and he can see the wall in the background all around the picture. Let us suppose that that the picture were separated from the wall throughout the interval. Then, by the geometry of the situation and from Joe's perceptions and his knowledge of the shape of the wall, there is an envelope *xenvelope* totally enclosing the picture and other objects that may be "hiding" behind it, whose boundary is in free space and whose vertical span is 1 metre. In turn, there is a convex enclosure enclosing *xenvelope*, whose vertical span is 1 m. Since the envelope is convex, the centre of mass of the picture and the objects behind it must lie within it. Even assuming that the initial velocity were directed upwards, the centre must have moved by a distance of more than 1 m in just half the interval. Thus, the centre is outside a convex enclosure containing the object set. That contradicts the perceptions. Therefore, *opicture* must have been in contact with *owall* at some point in the interval *i0*.

Note: If Joe assumed that there were no objects behind the picture, he could infer that the picture had been in contact with it during the *entire* interval.

III. The statement to be proved, that Isaac knows that if he, the ground, the tree, and the apple are the only objects within the enclosure, and he and the tree are motionless during the interval *i0*, which is 1 sec long, then the apple will come in contact with him within the interval, can be formalized as follows:

k(aisaac, s0, S1) ∧

B1 = behaviour(chronicle(S1)) ∧

(∀O,T) [time_in(T, i0) ∧

sub_place(place(O, scene(B1, T)), xenvelope) ↔

O = aisaac ∨ O = otree ∨ O = oapple ∨ O = oground] ∧

motionless(aisaac, B1, i0) ∧ motionless(otree, B1, i0) →

(∃T) [time_in(T, i0) ∧

intersect(place(oapple, scene(B1, T)), place(aisaac, scene(B1, T)))]

Isaac can bound the size of the apple in much the same way Joe was able to bound the size of the picture in example II, since he knows the shape of the tree and has an unoccluded view of the apple and the tree in the background all around the apple. Let us assume that Isaac knows that 10 centimetres is an upper bound on the size of the apple. Now Isaac can see some point of the apple only 2 metres above his head. The tree is still 4 m above, so, by the triangle inequality, any point of the apple is below the tree and thus is separated from it. Since there are no other objects in the enclosure, the apple must be isolated and obey the law of gravity. There is no doubt that the apple will fall to the ground. The only concern left is that the apple may be moving horizontally as well. Because of the upper bound on the size of the apple, the position of its centre of mass can be estimated and the horizontal speed bound from above.

IV. The statement to be proved is that Judy knows that Sharon knows that there is a ball in front of her (this is not strictly true and we shall explain the problem shortly) and that if there are no other objects in the enclosure, then the ball is falling and that Sharon can approximately predict its motion. Since motion in gravity is well known and must occur in any physically possible behaviour for isolated objects, we need only prove that Judy knows that Sharon knows that there is a ball in front of her and that if there are no other objects in the enclosure then the ball is isolated. Formally, let *s0* be the real situation, let *S1* be any situation accessible from *s0* through Judy's knowledge, and let *S2* be any situation accessible from *S1* through Sharon's knowledge. Then, in *S2*, there is a ball in front of Sharon, and if there are no other objects in the envelope in *S2*, then the ball is isolated.

k(ajudy, s0, S1) ∧ k(asharon, S1, S2) →

(∃OBALL) [true_in(S2, ball(OBALL)) ∧

in_front(place(OBALL, layout(S2)), place(asharon, layout(S2))) $\wedge$

$(\forall O)$ [sub_place(place(O, layout(S2)), xenvelope) $\leftrightarrow$

O = asharon $\vee$ O = ajudy $\vee$ O = OBALL $\vee$ O = oground] $\rightarrow$

isolated(OBALL, layout(S2))]

The proof that Sharon knows in *S2* that there is a ball in front of her is virtually the same as the proof of example V in [Davis88]. There is, however, an error in that proof. It assumes that Judy recognizes busses. Therefore, in *S1*, the object in question must be a bus. Since Sharon's view of the bus in the layout of *S1* is unblocked, she must see it. Davis claims that "in any layout visually compatible with *S1* to Sharon, there must be a bus in its place." This is not true. Sharon may not recognize busses. There must indeed be an object in the bus' place in *S2*, but it need not be a bus. We must remember that *S2* need not be knowledge-accessible to Judy, who recognizes busses and would be able to rule out situations in which the object is not a bus. Sharon may not have that ability. Therefore, all Judy knows is that if Sharon recognizes busses then Sharon knows that there is a bus in front of her.

Considering the above problem, we must modify the formula to be proved in our example:

k(ajudy, s0, S1) $\wedge$ k(asharon, S1, S2) $\rightarrow$

$(\exists OBALL)$ [in_front(place(OBALL, layout(S2)), place(asharon, layout(S2))) $\wedge$

$(\forall O)$ [sub_place(place(O, layout(S2)), xenvelope) $\leftrightarrow$

O = asharon $\vee$ O = ajudy $\vee$ O = OBALL $\vee$ O = oground] $\rightarrow$

isolated(OBALL, layout(S2))]

Since Judy can see the space between the ball and the ground and the space between that space and the visible part of Sharon, she knows that Sharon has an unoccluded view of that space. Thus Judy knows that the ball is visually separated from the ground relative to Sharon. Therefore, Sharon must know that the ball is separated from the ground.

In general it is possible for the ball to be visually separated relative to Judy but not relative to Sharon. Judy must see that there is no object on the other side of the ball from Sharon that might make the ball not visually separated from the ground relative to Sharon. Conversely, there might be an object behind the ball making the ball not visually separated from the ground relative to Judy but it might still be visually separated relative to Sharon. In that case Judy would be ignorant of Sharon's knowledge about the ball's isolation.

V. The statement to be proved, that Fred knows at the end of the interval that if there are no other objects in the building during the interval, Max is not isolated, and he is not the carrier of the building, then Max is still in the building, can be formalized as follows:

k(afred, s0, situation(C1, end(i0))) ∧
B1 = behaviour(C1) ∧ L1 = scene(B1, end(i0)) ∧
(∀T,O) [time_in(T, i0) ∧
    sub_place(place(O, scene(B1, T)), place(inside(obuilding), scene(B1, T))) →
        O = afred ∨ O = amax] ∧
(∀T) [time_in(T, i0) →
    ¬isolated(amax, scene(B1, T)) ∧
    ¬carrier_of(amax, obuilding, scene(B1, T))] →
        sub_place(place(amax, L1), place(inside(obuilding), L1))

Fred perceives at the beginning of the interval *i0* that some point of Max is within 3 metres of him, so any knowledge-accessible situation must have some point of Max within 3 m of Fred. Moreover, Fred knows that Max has a diameter of less than 2 m, so that in any knowledge-accessible situation any point of Max is less than 2 m from any other point in Max. By the triangle inequality, then, in any knowledge-accessible situation at time *ta* any point of Max is less than 5 m from Fred.

We are given that Fred knows the size and shape of the building, specifically that the distance to the nearest exit is greater than 55 m throughout the interval. (The room is a part of the building - its location relative to the rest of the building is fixed. Fred must have seen enough of the building, in particular all its outer walls, and know that he has done so. The problem is similar to walking around a possibly rotating object in order to determine its shape, except now Fred is inside an object that may be rotating.)

Now Max leaves the room. Since the assumption is that Max was not isolated during the interval, and he was not the carrier of the building, and Fred knows that he did not intersect Max and so could not be his carrier, the building must have been the carrier of Max. Hence, Fred knows that the speed of Max relative to the building did not exceed 200 m/min, and so the maximum distance covered by Max in 15 sec is 50 m. Thus, every point of Max must lie within 55 m from him after 15 sec. Therefore, Fred must still be in the building.

Now we assume that the length of the interval is 1 hour, instead, and Max is invisible to Fred throughout the interval:

time_length(i1) = 1 • hour ∧

• (∀T) [time_in(T, i1) →

wholly_invisible(place(amax, scene(b0, T)), afred, scene(b0, T))]

The statement to be proved, that Fred does not know at the end of the interval whether or not Max is in the building, can be formalized as follows:

(∃C1,B1,L1) [k(afred, s0, situation(C1, end(i1))) ∧

B1 = behaviour(C1) ∧ L1 = scene(B1, end(i1)) ∧

(∀T,O) [time_in(T, i1) ∧

sub_place(place(O, scene(B1, T)), place(inside(obuilding), scene(B1, T))) →

O = afred ∨ O = amax] ∧

(∀T) [time_in(T, i1) →

¬isolated(amax, scene(B1, T)) ∧

$\neg$carrier_of(amax, obuilding, scene(B1, T))] $\wedge$

sub_place(place(amax, L1), place(inside(obuilding), L1))] $\wedge$

($\exists$C2,B2,L2) [k(afred, s0, situation(C2, end(i1))) $\wedge$

B2 = behaviour(C2) $\wedge$ L2 = scene(B2, end(i1)) $\wedge$

($\forall$T,O) [time_in(T, i1) $\wedge$

sub_place(place(O, scene(B2, T)), place(inside(obuilding), scene(B2, T))) $\rightarrow$

O = afred $\vee$ O = amax] $\wedge$

($\forall$T) [time_in(T, i1) $\rightarrow$

$\neg$isolated(amax, scene(B2, T)) $\wedge$

$\neg$carrier_of(amax, obuilding, scene(B2, T))] $\wedge$

$\neg$sub_place(place(amax, L2), place(inside(obuilding), L2))]

To prove this, we construct two particular behaviours. In the first one, every object moves just as it does in the real world except that Max stays inside the buiding. In the second one, every object moves just as it does in the real world except that Max is outside the building at the end of the interval. The first one is obviously physically possible since nothing prevents Max from staying inside the building; he may even be motionless. It is compatible with Fred's perceptions if the space inside the building invisible to him is large enough to hide Max and if he does not see Max outside the building. The second one is physically possible because Max has enough time now to leave the building given his maximum possible self-generated speed relative to the building. Max can propel himself because the building is its carrier. Finally, the exit is large enough for Max to pass through it. The behaviour is compatible with Fred's perceptions since Max is invisible to him throughout the interval.

VI. The statement to be proved, that Jenny knows at the end of the interval *i0* that if no object other than the stone and the ground intersects the well during *i0* then, while no other object intersects the well, the stone will remain inside the well, can be formalized as follows: In the chronicles of all situations accessible to Jenny in situation *s0z* (the situation at the end of *i0*), if no object other than the stone and the ground intersects the well between the beginning of *i0* and the end of an interval *I*, then the place of the stone is inside the well at the end of *I*.

k(ajenny, s0z, situation(C1, end(i0))) ∧ start(I) = end(i0) ∧

(∀O) [start(i0) ≤ T ∧ T ≤ end(I) ∧

intersect(place(O, scene(behaviour(C1), T)), xwell) →

O = ostone ∨ O = oground] →

sub_place(place(ostone, scene(behaviour(C1), end(I))), xwell)

Based on Jenny's perceptions during the interval and her knowledge of a bound on the size of the stone, she can estimate the y coordinate of its centre of mass. She also knows that if there are no other objects in the well then the stone has been isolated during the interval *i0*, because she has seen the space around it throughout *i0* and she knows the maximum size of the stone and the depth of the well. Given this knowledge, Jenny can bound the initial velocity and thus the energy of a unit of mass of the stone at the beginning of *i0*. (It is worthwhile to point out that Jenny cannot infer that the initial speed of the stone is zero since she dropped it without applying force, because an agent in our model cannot reason about actions, including those performed by himself.) If the stone is outside the well at the end of an interval *I* immediately following interval *i0*, then at some time in *I* it must have been inside the well just under the opening. At that point its potential energy alone would have been greater than at the end of *i0*. If there are no other objects in the well, however, then the stone has been separated from all moving and active objects. Being an inert object itself, by the conservation of energy, the stone cannot increase its energy by itself, and, therefore, must remain inside the well.

VII. The statement to be proved, that David knows that if no object other than the ground intersects the sled then the sled will not immediately reverse its velocity, can be formalized as follows:

$$\text{k(adavid, s0, S1)} \wedge$$
$$(\forall \text{O}) \text{ [intersect(place(O, layout(S1)), place(osled, layout(S1)))} \rightarrow$$
$$\text{O = osled} \vee \text{O = oground]} \rightarrow$$
$$\text{yvel}_{+}\text{(osled, behaviour(chronicle(S1)), time(S1))} < 0$$

Since David has observed the sled sliding downhill up to the time *t0*, he knows that the velocity of the sled at *t0* in the y direction is negative (rotation is ruled out due to the shape of the sled). He also knows that there is a consistent, physically possible behaviour in which the sled continues sliding without a discontinuity in its velocity, since the space in front of the sled is free in all situations accessible to David and the assumption is that there are no objects behind the sled in contact with it. Such a behaviour obeys gravity because the ground is the sled's carrier and it is consistent with the sled being in inert motion since its energy does not increase. Since no active object is assumed to be in contact with the sled and the sled itself is inert, a discontinuity in the sled's velocity would constitute a breakpoint, increasing the set of breakpoints at time instant *t0*. Thus, the behaviour without a discontinuity in the sled's velocity is preferred according to CMD. Therefore, in all situations accessible to David, the sled does not reverse its velocity at time *t0*.

O. The proof of this example is a modified version of the proof of example I in [Davis88]. The statement to be proved, that Claire does not know at the end of the time interval *i0* whether or not the object *omystery* is moving within the envelope during *i0*, can be formalized as follows: At the end of *i0*, there is one knowledge-accessible situation that follows on a chronicle in which the object is motionless; there is another knowledge-accessible situation that follows on a chronicle in which the object is in motion.

$(\exists C1)$ [k(aclaire, s0z, situation(C1, end(i0))) $\wedge$

motionless(omystery, behaviour(C1), i0)]

$(\exists C2)$ [k(aclaire, s0z, situation(C2, end(i0))) $\wedge$

motionless(omystery, behaviour(C2), i0)]

To prove this, we construct two particular behaviours. In the first one, every object moves just as it does in the real world except that *omystery* stays motionless throughout *i0*. In the second one, every object moves just as it does in the real world except that *omystery* moves continuously within *xenvelope* throughout *i0*. We show that both of these are physically possible, since no other object comes within the envelope, by hypothesis, and so no other object interacts with *omystery*. The size and shape of *xenvelope* is assumed to be such that *omystery* is free to move within it. Since *omystery* is in contact with the ground, the law of gravity does not constrain its motion or lack thereof. Since *omystery* is not inert, it can propel itself. Both behaviours are compatible with Claire's perceptions, since the identical objects are visible to Claire in the identical places. Hence, by axiom A.8, Claire does not know of either of them that it did not occur.

Chapter V

DISCUSSION.

The knowledge, perception, and physics both in the original model and in the extension are highly simplified. The assumption that a point unblocked from any point in the agent is visible to the agent is obviously unrealistic. Moreover, the requirement of the presence of a light source is not even considered, not to mention reflection and refraction of light. Knowledge is simplified by assuming perfect memory and full deductive power of agents. The relationship between visual perception and knowledge ignores the possibility of knowledge gained via other senses.

Even with the extension, our physics is still very unrealistic. The assumption that an object has constant shape at all times is very important for deducing its shape and particularly its size limit, but it is obviously not true in the real world. Treating objects as units with no way to distinguish their subparts is particularly unsatisfying in problems where mechanical actions, such as throwing a ball, are involved.

Our gravitational system, including using the earth as the reference, is entirely reasonable for the type of problems we are interested in, except for the absence of air resistance. We have assumed that the gravitational force always acts downwards. This, of course, is unrealistic on a macroscopic scale, but for our purposes it is more than adequate (we are not concerned with satellites, for instance). The restriction that gravitational acceleration is constant is even less significant since precise predictions are hardly our goal here; indeed, they would be unrealistic. The absence of air resistance, however, has serious consequences. We could allow horizontal deceleration (dampened trajectory) and constant descent (like a parachute). Constant descent should not matter that much since accuracy in predicting the fall of an object is not usually critical, but allowing any horizontal deceleration is unreasonable. We could remedy this problem by allowing only a fraction of the kinetic energy to be lost due to air resistance. That would not, however, solve the main problem, which is the absence of flying objects.

One possible solution would seem to be to allow the ability to fly as a situation-dependent property. The problem with this approach is that now in order to infer that an agent can predict the motion of an object we would also have to deduce that he knows that the object is incapable of flying. It appears that the best solution would be to make the ability to fly a part of our physics, so that every agent would always know whether or not a particular object is capable of flying. In our model, any physical restriction is common knowledge. Davis argues that, for instance, disallowing layouts with unicorns (or, more precisely, objects shaped like unicorns) has the undesired consequence that every agent would have to know that there are no unicorns. This problem stems from the weakness of the relationship between physics and knowledge. Either everybody knows a physical restriction or nobody can possibly know it. We could introduce the concept of "knowable", that is possibly but not necessarily known, but then we would be forced to define exactly what *is* knowable.

Our maximum possible self-generated speeds represent true mechanical limitations of real-world objects. It may be argued that these speeds do not exist, or at least that they are unknown. This is not much of a problem, however, if the desirable inferences can be made. If a particular value is too high, some impossible behaviours may not be ruled out. If it is too low, an invalid inference may be made. The more reasonable the specified values are, the better the system will function. In reality, the maximum speed that an object can reach depends on other factors such as the weight of its load and the characteristics of its carrier: slope, coefficient of friction, etc. Throwing objects is approximated in our model by increasing the velocity of the carrier, even if only for an instant of time, thereby allowing the object to increase its kinetic energy. In the real world, there is little correspondence between the force with which an active object can propel itself and that with which it can throw things.

Refining our physics is not always a good approach. In order to allow airplanes to fly, we would have to account for Bernoulli's equation, which is practically impossible in this model. And even that would not solve the entire problem. The only way to deal with air pressure realistically seems to be to treat air as a fluid object. This would, however, not only greatly complicate the model by having to allow the presence of non-rigid objects, but it would invalidate the kind of inferences that have been our goal here.

Even if we could make our physics realistic, the ability to reason about motion would still be limited due to the fact that a lot of relevant information would not be available. After all, the shape of an airplane alone does not guarantee the ability to fly. Most physical limitations of objects result from their invisible properties.

It would appear reasonable to use the conservation of energy and momentum in our model, at least for inert objects. We have avoided this approach for two reasons. First, the mass of an object cannot be determined visually, and thus all or at least a wide range of positive values would have to be considered. Interestingly, perception might provide an agent with *some* information about the mass of an object. For instance, the behaviour of a beach ball and a bowling ball after a collision would betray the greater mass of the latter. The second reason to avoid the conservation of energy is that it allows undesirable behaviours. If two balls hit the ground at the same time, it is consistent with the conservation of energy that the energy of one of the balls is transferred to the other and therefore our example VI could not be proved.

The inferences about knowledge and ignorance that we have been able to make in our model depend on our strong axioms of physics and knowledge. We have avoided the frame problem by not relying on the persistence of facts unless our general assumptions such as the constant shape and continuous motion of objects, perfect memory of agents, etc., require it. The key is axiom A.8, which disallows drawing undesirable causal relationships between events, not supported by our physics or perceptions. This is why we could not reason about actions and plans. Once we allow agents to rule out physically possible and consistent behaviours because they are in conflict with a plan, inferring ignorance becomes much more difficult.

The fact is that many physically possible behaviours are not plausible. In our examples, we have faced the problem of not being able to infer the isolation of an object. Consequently, most problem formulations had to be of the form: "Agent knows that if there are no hidden objects then ...". An agent must know two things in order to make inferences of this kind. First, he must know that the object is separated from visible objects. One method is shown in [Davis88] in solving the problem that Fred knows that since Max is with him, he is not five miles away. It requires the agent to know the maximum size of objects. We have also shown the ability to infer the separation of objects directly from perception.

The second requirement for meaningful inferences from physical limitations is that the size of the object be bounded. Otherwise, the agent can infer very little about the location and velocity of the object's centre of mass. As shown in [Davis88], one way to bound the size of an object is to walk around it and know that one has seen all sides of it. Depending on the shape of the object, the agent may often be able to determine that the object is not rotating. If an object with a round side facing an agent rotates in place as the agent walks around it, however, then, as far as the agent is concerned, the object may look the same from all sides or its hidden back may extend indefinitely.

Another way to bound the size of an object is with the aid of another object behind it, as we have done in the case of the picture on the wall and the apple on the tree. There is still another way, resulting from the definition of vision in this model, i.e. if there is a line segment between point P and any point in agent A, then P is assumed to be visible to A.

Thus, if the agent is larger than the object, then the lines of vision converge and the size can be bounded. This solution, however, results from a highly unrealistic assumption and, in my opinion, ought to be avoided in most circumstances. We have used this method in example Ia.

Unfortunately, the two requirements for inferring motion under physical constraints are frequently in conflict. The size is difficult to bound if the object is visually separated from other objects. Conversely, if there is an object behind the one in question, then it is difficult to deduce that they are separated.

Maximum possible self-generated speeds do not constrain the motion of an object enough to be truly useful. It would be more reasonable to use average speeds to infer the plausibility of a behaviour, but there is no such mechanism in this model. The problem is exacerbated by the perpetual possibility that the object in question has fast carriers. Unless we are able to dismiss a behaviour in which a person jumps on an airplane once he is out of sight, we are forced to consider the possibility that he will be out of town in a matter of minutes. Moreover, free-falling objects gain speed very quickly, and it is unclear how to limit the transfer of the kinetic energy gained in this manner into horizontal motion.

Knowledge of inertia does allow interesting inferences about the motion of inert objects. Nevertheless, ignoring the dissipation of energy of real-world objects causes many unrealistic behaviours to be considered possible. After all, a ball endlessly bouncing off the ground, even though consistent with the conservation of energy, is absurd. It would be desirable to allow an agent to infer that if there are no active objects in a room, then, after some time (very soon, in fact), all of the inert objects in the room will definitely be motionless.

We were able to infer that a sliding object cannot reverse its velocity instantaneously, using the chronological minimization of discontinuities. Even though a behaviour in which the object continuously decelerates to zero and starts moving uphill cannot be ruled out by CMD, it will be ruled out because it does not satisfy the condition for inert motion. Nevertheless, there are many behaviours consistent with CMD and inert motion, in which the object gradually reverses its direction and starts moving uphill. We would wish to prefer the direction to an object to remain constant if that is consistent with the axioms and observations. In order to achieve this goal, we could define some kind of a predicate denoting direction, whose discontinuities would be minimized by CMD.

As we have seen, the extended physics contains important simplifications and there are inherent limitations in its usefulness for inferring knowledge about motion - much of such knowledge possessed by intelligent beings comes from sources other than the limits of physics.

Chapter VI

CONCLUSION

As we have shown, incorporating physical constraints into the model allows agents to infer some basic but interesting facts about the motion of objects. It is a step towards a more realistic model of knowledge and ignorance about the course of events derived from perception.

From the standpoint of investigating the relationship between perception and knowledge, the most important feature of our extension is the introduction of the concept of visual separability. The underlying assumption that objects separated from each other do not interact and that objects that are not separated interact in limited ways allows interesting inferences and is fairly realistic in most circumstances.

The main limitation of our approach is the same as the one described in [Davis88] - because of the very strong axiom A.8 which does not allow an agent to rule out a physically possible situation that is compatible with his perceptions, reasoning about actions and probable behaviours is not allowed.

Appendix A

ORIGINAL LOGIC

For completeness, we repeat here the original first-order sorted logic with equality contained in [Davis88], without proofs.

The sorts of variables and constants in our logic will be indicated by their first letter, using the key in the following table:

First Letter	Sort
A	Agent
B	Behaviour
C	Chronicle
I	Closed Interval of Time
L	Layout
O	Object
P	Geometric Point
S	Situation
T	Instant of Time
X	Point Set

We will indicate variables by symbols beginning with upper-case letters; symbols beginning with lower-case letters will be non-logical (predicate, function, or constant) symbols. Free variables are taken to be universally quantified with a scope of the entire formula.

Definitions are sometimes followed by logical formulas. Sometimes these are equivalent in meaning; sometimes, they capture only part of the meaning. Whatever is in a definition that is not expressed in a following formula is given only as an informal explanation.

A.1 Theory of Geometry

Definition 1.1: The predicate *point_in(P, X)* holds if *P* is a point in *X*.

Axiom 1.2: A point set is determined by the points in it.

$X1 = X2 \leftrightarrow (\forall P)\ \text{point\_in}(P, X1) = \text{point\_in}(P, X2)$

Definition 1.3: The predicate *sub_place(X1, X2)* holds if *X1* is a subset of *X2.*

$\text{sub\_place}(X1, X2) \leftrightarrow (\forall P)\ \text{point\_in}(P, X1) \rightarrow \text{point\_in}(P, X2)$

Lemma 1.4: The *sub_place* relation is a partial ordering.

$\text{sub\_place}(X1, X1)$

$\text{sub\_place}(X1, X2) \wedge \text{sub\_place}(X2, X1) \rightarrow X1 = X2$

$\text{sub\_place}(X1, X2) \wedge \text{sub\_place}(X2, X3) \rightarrow \text{sub\_place}(X1, X3)$

Definition 1.5: The predicate *intersect(X1, X2)* holds if the two point sets intersect.

$\text{intersect}(X1, X2) \leftrightarrow (\exists P)\ \text{point\_in}(P, X1) \wedge \text{point\_in}(P, X2)$

Definition 1.6: The function *interior(X)* maps a point set *X* to its interior. (Interior here is used in the topological sense; the interior of *X* is *X* minus its boundary).

Axiom 1.7: If *X1* is a subset of *X2* then the interior of *X1* is a subset of the interior of *X2.*

$\text{sub\_place}(X1, X2) \rightarrow \text{sub\_place}(\text{interior}(X1), \text{interior}(X2))$

Definition 1.8: The predicate *overlap(X1, X2)* holds if the interiors of *X1* and *X2* intersect.

$\text{overlap}(X1, X2) \leftrightarrow \text{intersect}(\text{interior}(X1), \text{interior}(X2))$

Lemma 1.9: If *X1* overlaps *XA* and *XA* is a subset of *XB*, then *X1* overlaps *XB.*

$\text{overlap}(X1, XA) \wedge \text{sub\_place}(XA, XB) \rightarrow \text{overlap}(X1, XB)$

Definition 1.10: The predicate *connected(X)* holds if *X* is a connected set of points.

Definition 1.11: The predicate *strictly_inside(X1, X2)* means that *X1* is a subset of *X2* and, moreover, their boundaries are disjoint.

strictly_inside(X1, X2) → sub_place(X1, X2)

Definition 1.12: The function *line_seg(P1, P2)* maps two points *P1* and *P2* onto the line segment connecting them (a point set).

Axiom 1.13: *Line_seg(P1, P2)* is a symmetric function.

line_seg(P1, P2) = line_seg(P2, P1)

Axiom 1.14: *P1* lies on the line segment from *P1* to *P2*.

point_in(P1, line_seg(P1, P2))

Definition 1.15: The predicate *blocked(PA, X, PB)* means that point set *X* blocks the view of point *PA* from point *PB* (The order corresponds to the physical layout: *X* comes between *PA* and *PB*.). That is, the line from *PA* to *PB* intersects the interior of *X*.

blocked(PA, X, PB) ↔ intersect(interior(X), line_seg(PA, PB))

We will overload the predicate *blocked* to take point-set arguments either in its second and third arguments or in all three arguments.

Definition 1.16: The predicate *blocked(P, XB, XC)* means that *P* is blocked by *XB* from every point in *XC*. The predicate *blocked(XA, XB, XC)* means that each point of *XA* is blocked by *XB* from any point in *XC*.

blocked(P, XB, XC) ↔ (∀PC) [point_in(PC, XC) → blocked(P, XB, PC)]

blocked(XA, XB, XC) ↔ (∀PA) [point_in(PA, XA) → blocked(PA, XB, XC)]

Lemma 1.17: If *XA* is blocked by *XB* from *XC*, then any subplace of *XA* is blocked from any subplace of *XC*.

blocked(XA, XB, XC) ∧ sub_place(XA1, XA) ∧ sub_place(XC1, XC) →

blocked(XA1, XB, XC1)

Definition 1.18: The predicate *unblocked(XA, XB, XC)* holds if no point in *XA* is blocked by *XB* from *XC*.

$$\text{unblocked}(XA, XB, XC) \leftrightarrow$$

$$(\forall PA)\ [\text{point\_in}(PA, XA) \rightarrow \neg\text{blocked}(PA, XB, XC)]$$

A.2 Theory of Time and Motion

Definition 2.1: The predicate $T1 \leq T2$ (written infix) holds if time instant *T1* is earlier or equal to *T2*.

Axiom 2.2: Time instants are totally ordered.

$$T1 \leq T2 \lor T2 \leq T1$$

$$T1 \leq T2 \land T2 \leq T1 \rightarrow T1 = T2$$

$$T1 \leq T2 \land T2 \leq T3 \rightarrow T1 \leq T3$$

Definition 2.3: The function *start(I)* maps a time interval *I* to its starting time instant.

Definition 2.4: The function *end(I)* maps a time interval *I* to its ending time instant.

Definition 2.5: The predicate *time_in(T, I)* holds if the time instant *T* is part of the time interval *I*.

$$\text{time\_in}(T, I) \leftrightarrow \text{start}(I) \leq T \leq \text{end}(I)$$

Axiom 2.6: Any two ordered unequal time points determine an interval.

$$T1 \leq \land\ T1 \neq T2 \rightarrow (\exists I)\ [T1 = \text{start}(I) \land T2 = \text{end}(I)]$$

Definition 2.7: The predicate *sub_interval(I1, I2)* holds if interval *I1* is a subset of interval *I2*.

$$sub\_interval(I1, I2) \leftrightarrow [(\forall T)\ time\_in(T, I1) \rightarrow time\_in(T, I2)]$$

Definition 2.8: The function *scene(B, T)* maps a behaviour *B* and a time *T* onto the layout of *B* at time *T*.

Axiom 2.9: Two layouts are equal just if they have the same objects and they assign them to the same places. (We do not consider rotations in place to make a difference).

$$L1 = L2 \leftrightarrow (\forall O)\ [object\_of(O, L1) \leftrightarrow object\_of(O, L2)] \wedge$$
$$[object\_of(O, L1) \rightarrow place(O, L1) = place(O, L2)]$$

Axiom 2.10: Two behaviours are equal just if corresponding layouts are equal.

$$B1 = B2 \leftrightarrow (\forall T)\ layout(B1, T) = layout(B2, T)$$

Definition 2.11: The function *chronicle(S)* maps a situation *S* onto the chronicle containing it.

Definition 2.12: The function *time(S)* maps a situation *S* onto the time when it occurs.

Definition 2.13: The function *situation(C, T)* maps a chronicle *C* and a time instant *T* into the situation of *C* at time *T*.

$$situation(chronicle(S), time(S)) = S$$

Definition 2.14: Situation *S1* precedes *S2* if they belong to the same chronicle and *S1* occurs earlier.

$$precedes(S1, S2) \leftrightarrow chronicle(S1) = chronicle(S2) \wedge time(S1) \leq time(S2)$$

Definition 2.15: The function *layout(S)* maps a situation *S* into the physical layout present in *S*.

Definition 2.16: The function *behaviour(C)* maps a chronicle *C* onto its behaviour.

Axiom 2.17: A chronicle at a given time has the same layout whether you go through the behaviour or through the situation.

scene(behaviour(C), T) = layout(situation(C, T))

Axiom 2.18: Two behaviours *B1* and *B2* which have equal layouts at some time *TS* may be spliced together across time into a new behaviour that agrees with *B1* up to *TS* and with *B2* after *TS*.

layout(B1, TS) = layout(B2, TS) →

(∃B3) [(∀T≤TS) layout(B3, T) = layout(B1, T) ∧

(∀T≥TS) layout(B3, T) = layout(B2, T)]

Axiom 2.19: Any two behaviours *B1* and *B2* can be spliced together across objects to form a new behaviour *B3* which agrees with *B1* on the objects of *B1* and agrees with *B2* on the objects that are in *B2* but not in *B1*. Note that *B3* may not be physically possible.

(∃B3) (∀O,T)

[object_of(O, B1) →

object_of(O, B3) ∧ place(O, layout(B3, T)) = place(O, layout(B1, T))] ∧

[object_of(O, B2) ∧ ¬object_of(O, B1) →

object_of(O, B3) ∧ place(O, layout(B3, T)) = place(O, layout(B1, T))]

Axiom 2.20: Given a layout *L* and an object *O* in *L*, there exists a layout *LO* which has *O* in the same place as in *L*, and which has no other objects.

object_of(O, L) →

(∃LO) [object_of(O1, LO) ↔ O1 = O] ∧ place(O, LO) = place(O, L)

Definition 2.21: The predicate *motionless(O, B, I)* holds in the place occupied by *O* in *B* does not change during *I*. (By this definition, we are not counting rotation in place as motion.)

$$\text{motionless}(O, B, I) \leftrightarrow (\forall T1,T2)\ [\ \text{time\_in}(T1, I) \wedge \text{time\_in}(T2, I) \rightarrow \text{place}(O, \text{scene}(B, T1)) = \text{place}(O, \text{scene}(B, T2))\ ]$$

Definition 2.22: The predicate *continual_motion(O, B, I)* holds if *O* is never motionless in *B* throughout *I*.

$$\text{continual\_motion}(O, B, I) \leftrightarrow [(\forall IS)\ \text{sub\_interval}(IS, I) \rightarrow \neg\text{motionless}(O, B, IS)]$$

Axiom 2.23: Given any layout *L*, there is a behaviour including *L* in which everything is motionless.

$$(\exists B)\ L = \text{layout}(B, TF) \wedge (\forall I,O)\ \text{object\_of}(O, B) \rightarrow \text{motionless}(O, B, I)$$

Axiom 2.24: If an object *O* is strictly inside a region *XE* in a layout *L*, then there exists a behaviour *B* which includes *L*, in which *O* is in continual motion but stays inside *XE*.

$$\text{object\_of}(O, L) \wedge \text{strictly\_inside}(\text{place}(O, L), XE) \rightarrow (\exists B)\ [\ L = \text{scene}(B, TF) \wedge (\forall T)\ \text{strictly\_inside}(\text{place}(O, \text{scene}(B, T)), XE) \wedge (\forall I)\ \text{continual\_motion}(X, B, I)\ ]$$

A.3 Theory of Physics

Definition 3.1: The predicate *object_of(O, L)* holds if object *O* is within layout *L*. The predicate *object_of(O, B)* holds if object *O* is within behaviour *B*.

Axiom 3.2: A behaviour has the same objects as each of of its layouts. The predicate *object_of(O, B)* holds if object *O* is within behaviour *B*.

$$\text{object\_of}(O, B) \leftrightarrow \text{object\_of}(O, \text{scene}(B, T))$$

Definition 3.3: The function *place(O, L)* maps an object *O* and a layout *L* into the point set occupied by *O* during *L*.

Definition 3.4: The predicate *same_vprops(O1, O2)* holds if *O1* and *O2* have the same visual properties.

Axiom 3.5: The relation *same_vprops(O1, O2)* is an equivalence relation.

$$\text{same\_vprops}(O, O)$$

$$\text{same\_vprops}(O1, O2) \rightarrow \text{same\_vprops}(O2, O1)$$

$$\text{same\_vprops}(O1, O2) \wedge \text{same\_vprops}(O2, O3) \rightarrow \text{same\_vprops}(O1, O3)$$

Definition 3.6: The predicate *free_space(X, L)* holds if point set *X* does not intersect the place of any bodies in layout *L*.

$$\text{free\_space}(X, L) \leftrightarrow [(\forall O)\ \text{object\_of}(O, L) \rightarrow \neg \text{intersect}(X, \text{place}(O, L))]$$

Definition 3.7: The predicate *visible(P, A, L)* holds if the point *P* is visible to agent *A* in layout *L*. That is, there is no object between *A* and *P*. (Note that each point in *A* itself is visible to *A* by definition.)

$$\text{visible}(P, A, L) \leftrightarrow [(\forall O)\ \text{object\_of}(O, L) \wedge O \neq A \rightarrow$$
$$(\exists PA)\ \text{point\_in}(PA, \text{place}(A, L)) \wedge \neg \text{blocked}(P, \text{place}(O, L), PA)]$$

Axiom 3.8: If a point *P* is invisible to agent *A* in *L* then, for any point *PA* in *A*, there is some visible point on an object that blocks *P* from *PA*.

$$\neg \text{visible}(P, A, L) \wedge \text{point\_in}(PA, \text{place}(A, L)) \rightarrow$$
$$(\exists O)\ [\ \text{blocked}(P, \text{place}(O, L), PA) \wedge$$
$$(\exists PO)\ [\ \text{point\_in}(PO, \text{place}(O, L)) \wedge \text{visible}(PO, A, L) \wedge$$

point_in(PO, line_seg(PA, P))]]

Definition 3.9: The predicate *wholly_visible(X, A, L)* holds if each point in *X* is visible to *A* in *L*.

wholly_visible(X, A, L) $\leftrightarrow$ [($\forall$P) point_in(P, X) $\rightarrow$ visible(P, A, L)]

Definition 3.10: The predicate *wholly_invisible(X, A, L)* holds if no point of *X* is visible to *A* in *L*.

wholly_invisible(X, A, L) $\leftrightarrow$ [($\forall$P) point_in(P, X) $\rightarrow$ $\neg$visible(P, A, L)]

Definition 3.11: The predicate *phys_poss(L)* holds if *L* is a physically possible layout.

Axiom 3.12: modified version given in appendix B as axiom 3.28.

A.4 Theory of Perception and Knowledge

Definition 4.1: The predicate *v_compatible(A, L1, L2)* means that layout *L2* is visually compatible with layout *L1* relative to agent *A*. That is, as far as *A* can see in *L1*, the world might be in state *L2*.

Axiom 4.2: Layout *L2* is *v_compatible* with *L1* with respect to agent *A* iff the following two conditions hold: (i) *L1* and *L2* are both physically possible. (ii) Let *X* be a connected set of points, such that every point in *X* is visible to *A* in *L1*. Then each point of *X* is visible to *A* in *L2*. Moreover, if *X* lies entirely in *A* in *L1*, then *X* lies entirely in *A* in *L2*; if *X* lies entirely in some object *O1* in *L1* then, in *L2*, *X* lies entirely within some object *O2* with the same visible properties as *O1*; if *X* lies entirely in free space in *L1* then *X* lies entirely in free space in *L2*.

v_compatible(A, L1, L2) $\leftrightarrow$

object_of(A, L1) $\wedge$ object_of(A, L2) $\wedge$ phys_poss(L1) $\wedge$ phys_poss(L2) $\wedge$

($\forall$X,O) [connected(X) $\wedge$ wholly_visible(X, A, L1) $\rightarrow$

wholly_visible(X, A, L2) ∧

[sub_place(X, place(A, L1)) ↔ sub_place(X, place(A, L2))] ∧

[object_of(O, L1) ∧ sub_place(X, place(A, L2)) ↔

(∃O2) object_of(O2, L2) ∧ same_vprops(O, O2) ∧

sub_place(X, place(O2, L2))] ∧

[object_of(O, L2) ∧ sub_place(X, place(O, L2)) ↔

(∃O1) object_of(O1, L1) ∧ same_vprops(O, O1) ∧

sub_place(X, place(O1, L1))] ∧

[free_space(X, L1) ↔ free_space(X, L2)]]

Lemma 4.3: *V_compatibility* is an equivalence relation on layouts.

v_compatible(A, L, L)

v_compatible(A, L1, L2) → v_compatible(A, L2, L1)

v_compatible(A, L1, L2) ∧ v_compatible(A, L2, L3) →

v_compatible(A, L1, L3)

Lemma 4.4: Let *L1* and *L2* be physically possible layouts with the following properties: (i) The same objects are partly visible in the two layouts. (ii) Any object which is partly visible in the two is in the same position in both. Then the two layouts are visually compatible.

object_of(A, L1) ∧ object_of(A, L2) ∧ phys_poss(L1) ∧ phys_poss(L2) ∧

[object_of(O, L1) ∧ ¬wholly_invisible(O, A, L1) →

object_of(O, L2) ∧ place(O, L1) = place(O, L2)] ∧

[object_of(O, L2) ∧ ¬wholly_invisible(O, A, L2) →

object_of(O, L1) ∧ place(O, L1) = place(O, L2)] →

v_compatible(A, L1, L2)

Definition 4.5: The predicate *bv_compatible(A, B1, B2, T)* means that behaviour *B2* is visually compatible with behaviour *B1* up to time *T*. That is, *B2* is consistent with everything that *A* can see in *B1* up to (and including) time *T*.

Axiom 4.6: modified version given in appendix B.

Lemma 4.7: given in appendix B.

Definition 4.8: The predicate *k(A, S1, S2)* holds if *S2* is accessible in *S1* via *A*'s knowledge. That is, *S2* is consistent with everything that *A* knows in *S1*.

Axiom 4.9: Veridicality: All that is known is true. Formally, the knowledge accessibility relation is reflexive.

k(A, S, S)

Axiom 4.10: Positive introspection: If *A* knows ϕ then he knows that he knows ϕ. Formally, the knowledge accessibility relation is transitive.

$$k(A, S1, S2) \wedge k(A, S2, S3) \rightarrow k(A, S1, S3)$$

Axiom 4.11: Memory: An agent does not forget what he knows. Formally, let situation *S1B* be accessible from *S0B*, and let *S0A* precede *S0B* in the same chronicle. Then there is a situation *S1A* in the chronicle of *S1B* which is accessible from *S0A*.

$$k(A, S0B, S1B) \wedge precedes(S0A, S0B) \rightarrow$$
$$(\exists S1A)\ k(A, S0A, S1A) \wedge precedes(S1A, S1B)$$

Axiom 4.12: Internal clock: An agent always knows the time. Formally, if *S1* is knowledge accessible from *S0*, then the two situations occur at the same time.

$$k(A, S0, S1) \rightarrow time(S0) = time(S1)$$

Axiom 4.13: An agent knows what he perceives. Formally, if *S1* is knowledge accessible from *S0*, then the layout of *S1* is visually compatible with the layout of *S0*.

$$k(A, S0, S1) \rightarrow v\_compatible(A, layout(S0), layout(S1))$$

Axiom 4.14: Perception is the only source of knowledge of the source of events. Formally, let *C0* be the real chronicle. Let *B1* be a behaviour that is visually compatible with the behaviour of *C0* up to time *T* relative to agent *A*; thus, as far as *A* could have seen up to time *T*, *B1* could be the real behaviour. Then it is consistent with *A*'s knowledge that *B1* actually was the real behaviour; that is, there is a chronicle *C1* whose situation in *T* is knowledge accessible from the situation of *C0* in *T*.

$$bv\_compatible(A, behaviour(C0), B1, T) \rightarrow$$
$$(\exists C1)\ [k(A, situation(C0, T), situation(C1, T)) \wedge B1 = behaviour(C1)]$$

Theorem 4.15: Facts perceived over time are known. Formally, if *S1* is knowledge accessible from *S0*, then the behaviour of the chronicle of *S0* up *S0* is visually compatible with the behaviour of the chronicle of *S1* up to *S1*.

$$k(A, S0, S1) \rightarrow$$
$$bv\_compatible(A, behaviour(chronicle(S0)), behaviour(chronicle(S1)), time(S0))$$

Theorem 4.16: Negative introspection: If an agent does not know a fact about the course of events, then he knows that he doesn't know it. Formally, if a behaviour is compatible with an agent's perceptions, then, from any knowledge accessible situation, there is a knowledge accessible chronicle exhibiting that behaviour.

$$bv\_compatible(A, behaviour(C0), B1, T) \wedge k(A, situation(C0, T), SM) \rightarrow$$
$$(\exists C1)\ [k(A, SM, situation(C1, T)) \wedge bv\_compatible(A, behaviour(C1), B1, T)]$$

Appendix B

EXTENSION OF THE LOGIC

We present here an extension of the logic in appendix A. All of the conventions introduced there are followed here as well. Two new types of variables are introduced. The first one is a non-empty object set, indicated by Q as its first letter. The other one is a parameter, indicated by U. Each parameter is a piecewise continuous real-valued or propositional fluent, a function of time in any behaviour. The left and right limit values of parameters are denoted by U_- and U_+, respectively. As in [Sande89], our logic becomes non-monotonic, since continuity of parameters at breakpoints is inferred if it is consistent with all the axioms.

B.1 Theory of Geometry

Definition 1.19: The predicate *convex(X)* holds if X is a convex set of points.

convex(X) ↔ (∀P1,P2) [point_in(P1, X) ∧ point_in(P2, X) →

sub_place(line_seg(P1, P2), X)]

Definition 1.20: The function *behind(XB, XC)* maps point sets *XB* and *XC* to the largest point set blocked by *XB* from *XC* not intersecting *XB*.

sub_place(XA, behind(XB, XC)) ↔ blocked(XA, XB, XC) ∧ ¬intersect(XA, XB)

Definition 1.21: The function *boundary(X)* maps point set X to its boundary.

boundary(X) = X − interior(X)

Lemma 1.22: If a point set intersects another point set whose boundary is in free space, then the former is strictly inside the latter.

$$\text{connected}(X1) \wedge \text{intersect}(X1, X2) \wedge \text{free\_space}(\text{boundary}(X2)) \rightarrow \text{strictly\_inside}(X1, X2)$$

Proof: Let us suppose that *X1* were not strictly inside *X2*. Then there exists point *P* in *X1* not inside *X2*. Since *X*1 is connected, there exists a path completely in *X1* between a point in *X2* and *P*. This path must cross the boundary of *X2*. This contradicts the assumption that the boundary is in free space.

Definition 1.23: The functions *x(P)*, *y(P)*, and *z(P)* map point *P* to its cartesian coordinates.

Definition 1.24: The function *distance(P1, P2)* maps points *P1* and *P2* to the distance between them.

$$\text{distance}(P1, P2)^2 = [x(P2) - x(P1)]^2 + [y(P2) - y(P1)]^2 + [z(P2) - z(P1)]^2$$

Definition 1.25: The function *diameter(X)* maps point set *X* to its diameter.

$$(\forall P1,P2)\ [\text{point\_in}(P1, X) \wedge \text{point\_in}(P2, X) \rightarrow \text{distance}(P1, P2) \leq \text{diameter}(X)] \wedge$$
$$(\exists P1,P2)\ [\text{point\_in}(P1, X) \wedge \text{point\_in}(P2, X) \wedge \text{distance}(P1, P2) = \text{diameter}(X)]$$

B.2 Theory of Time and Motion

Lemma 2.25: If at the beginning of interval *I* object *O* intersects point set *X* whose boundary is entirely in free space throughout *I*, then *O* is strictly inside *X* at the end of *I*.

Let L0 = scene(B, start(I)). Then

$$\text{intersect}(X, \text{place}(O, L0), L0) \wedge [\text{time\_in}(T, I) \rightarrow \text{free\_space}(\text{boundary}(X), \text{scene}(B, T))] \rightarrow \text{strictly\_inside}(\text{place}(O, \text{scene}(B, \text{end}(I))), X)$$

Proof: Let us suppose that O were not inside X at *end(I)*. Then, since O intersects X at *start(I)* and objects move continuously in time, there must be a time T in I when O intersects the boundary of X, which contradicts the assumption that the boundary lies in free space throughout I.

Definition 2.26: The function *time_length(I)* maps interval I to its length.

B.2.1 Preferential Entailment

Definition 2.27.1: The function *value(U, B, T)* maps a parameter U to its value in behaviour B at time T.

Definition 2.27.2: The predicate $mask_{-}(U, B, T)$ holds if parameter U is masked on the left in behaviour B at time T, i.e. it is "allowed" to be discontinuous on the left side of T.

The predicate $mask_{+}(U, B, T)$ holds if parameter U is masked on the right in behaviour B at time T, i.e. it is "allowed" to be discontinuous on the right side of T.

Definition 2.27.3: The predicate *ec(U, B, T)* holds if parameter U is essentially continuous in behaviour B at time T, i.e. it is either continuous or masked at T.

$$\begin{aligned} &ec(U, B, T) \leftrightarrow \\ &\quad [mask_{-}(U, B, T) \lor value_{-}(U, B, T) = value(U, B, T)] \land \\ &\quad [mask_{+}(U, B, T) \lor value_{+}(U, B, T) = value(U, B, T)] \end{aligned}$$

Definition 2.27.4: The function *breaks(B, T)* maps a behaviour B and time T into the set of parameters discontinuous in B at time T.

$$breaks(B, T) = \{U \mid \neg ec(U, B, T)\}$$

Definition 2.27.5: The predicate $B1 \lessdot B2$ (written infix) holds if behaviour $B1$ is preferred over behaviour $B2$ according to the chronological minimization of "spontaneous" discontinuities (CMD).

$B1 \lessdot B2 \leftrightarrow$

$(\exists T0)\ [\ (\forall T,U)\ [T \leq T0 \rightarrow \text{value}(U, B1, T) = \text{value}(U, B2, T)] \land$

$\text{breaks}(B1, T0) \subset \text{breaks}(B2, T0)\ ]$

B.3 Theory of Physics

Definition 3.13: The predicate *oset_of(Q, L)* holds if object set *Q* is within layout *L*.

$\text{oset\_of}(Q, L) \leftrightarrow$

$Q \neq \phi \land Q \subseteq \{O \mid \text{object\_of}(O, L)\}$

Definition 3.14: The predicate *separated(X1, X2, L)* holds if for any path (or any connected set of points) connecting sets *X1* and *X2* there is a segment (subset) entirely in free space in layout *L*.

$\text{separated}(X1, X2, L) \leftrightarrow$

$[\ \text{connected}(XP) \land \text{intersect}(XP, X1) \land \text{intersect}(XP, X2) \rightarrow$

$(\exists XP1)\ [\text{sub\_place}(XP1, XP) \land \text{free\_space}(XP1, L)]\ ]$

We allow the predicate *separated* to take an object in its first and/or second argument.

$\text{separated}(O1, O2, L) \leftrightarrow \text{separated}(\text{place}(O1, L), \text{place}(O2, L), L)$

Lemma 3.15: The relation *separated(X1, X2, L)* is symmetric.

$\text{separated}(X1, X2, L) \rightarrow \text{separated}(X2, X1, L)$

Proof: Immediate from definition.

Lemma 3.16: The relation *¬separated(X1, X2, L)* is an equivalence relation over objects. It is symmetric on any pair of point sets.

$\neg\text{separated}(O, O, L)$

$\neg\text{separated}(X1, X2, L) \rightarrow \neg\text{separated}(X1, X2, L)$

$\neg\text{separated}(O1, O2, L) \land \neg\text{separated}(O2, O3, L) \rightarrow \neg\text{separated}(O1, O3, L)$

Proof: Reflexivity: Any subset of an object intersects it and lies entirely in occupied space. Symmetry: Follows immediately from definition. Transitivity: Let *X12* be a set intersecting *place(O1, L)* and *place(O2, L)* and lying entirely in occupied space in *L*. Let *X23* be a set intersecting *place(O2, L)* and *place(O3, L)* and lying entirely in occupied space in *L*. Then the union $X12 \cup X23 \cup place(O2, L)$ is connected, lies entirely in occupied space, and intersects *place(O1, L)* and *place(O3, L)* in *L*.

Lemma 3.17: If point set *X1* is separated from set *X2*, then any of its subsets *X3* is also separated from *X2*.

separated(X1, X2, L) ∧ sub_place(X3, X1) → separated(X3, X2, L)

Proof: Immediate since the set of all paths connecting *X3* and *X2* is a subset of all paths connecting *X1* and *X2*.

Definition 3.18: The predicate *isolated(Q, L)* holds if the objects in set *Q* are separated from every other object in layout *L*.

isolated(Q, L) ↔ oset_of(Q, L) ∧

(∀O1,O2) [object_of(O1, L) ∧ O1 ∉ Q ∧ O2 ∈ Q →

¬intersect(place(O1, L), place(O2, L))]

We allow the predicate *isolated* to take an object as its first argument.

isolated(O, L) ↔ isolated({O}, L)

Definition 3.19: The predicate *carrier_of(OC, O, L)* holds if object *OC* is carrying object *O* in layout *L*. Written with two arguments, *carrier_of(OC, O)* denotes a truth-valued parameter.

carrier_of(OC, O, L) →

object_of(OC, scene(B, T)) ∧ object_of(O, scene(B, T))

value(carrier_of(OC, O), B, T) ≡ carrier_of(OC, O, scene(B, T))

Axiom 3.20: The relation *carrier_of* is a total ordering over any group of connected objects except that it is non-reflexive. It is false for separated objects.

¬carrier_of(O, O, L)

¬separated(O1, O2, L) → carrier_of(O1, O2, L) ∨ carrier_of(O2, O1, L)

carrier_of(O1, O2, L) → ¬carrier_of(O2, O1, L)

carrier_of(O1, O2, L) ∧ carrier_of(O2, O3, L) → carrier_of(O1, O3, L)

separated(O1, O2, L) → ¬carrier_of(O1, O2, L)

Axiom 3.21: Discontinuities of the *carrier_of* relation are *masked*, i.e. the *carrier_of(OC, O)* parameter is considered to be *essentially continuous*, if either object is active or there is a discontinuity in their places intersecting.

[mask-(carrier_of(OC, O), B, T) ↔

¬inert(OC) ∨ ¬inert(O) ∨

[¬intersect(place(OC, scene(B, T-)), place(O, scene(B, T-))) ∧

intersect(place(OC, scene(B, T)), place(O, scene(B, T)))]] ∧

[$mask_{+}$(carrier_of(OC, O), B, T) ↔

¬inert(OC) ∨ ¬inert(O) ∨

[intersect(place(OC, scene(B, T)), place(O, scene(B, T))) ∧

¬intersect(place(OC, scene(B, T_{+})), place(O, scene(B, T_{+})))]]

Definition 3.22: The function *centre(Q, L)* maps object set *Q* to its centre of mass in layout *L*.

Axiom 3.23: The centre of mass of an object set *Q* lies in any convex enclosure containing it.

convex(X) ∧ (∀O) [O ε Q → sub_place(place(O, L), X)] →

point_in(centre(Q, L), X)

Lemma 3.24: The distance between the centre of mass of an object O and any point in it is at most equal to its diameter.

object_of(O, L) $\wedge$ point_in(P, place(O, L)) $\rightarrow$

distance(P, centre(O, L)) $\leq$ diameter(place(O, L))

Proof: Immediate from axiom 3.23, since the set of all points separated from any point in an object by a distance not greater than its diameter is convex.

Axiom 3.25: The centre of mass of object set Q in layout L is a unique point.

P1 = centre(Q, L) $\wedge$ P2 = centre(Q, L) $\rightarrow$ P1 = P2

Definition 3.26: The constant *oground* denotes the ground.

Axiom 3.27: *Oground* is inert and visually distinquishable from any other object.

inert(oground)

same_vprops(O, oground) $\rightarrow$ O = oground

Axiom 3.28 (modified axiom 3.12 from [Davis88]): A *layout* L *is physically possible if no two objects overlap and oground* is an object in L occupying the space *xground* and not carried by any object.

phys_poss(L) $\leftrightarrow$

($\forall$O1,O2) [object_of(O1, L) $\wedge$ object_of(O2, L) $\rightarrow$

$\neg$overlap(place(O1, L), place(O2, L))] $\wedge$

object_of(oground, L) $\wedge$ place(oground, L) = xground $\wedge$

($\forall$OC) $\neg$carrier_of(OC, oground, L)

Definition 3.29: The functions *xvel(Q, B, T)*, *yvel(Q, B, T)*, and *zvel(Q, B, T)* map object set Q to its velocity in behaviour B at time T, i.e. the derivative of the position of the centre of mass of Q. Written with one argument, *xvel(Q)*, *yvel(Q)*, and *zvel(Q)* denote real-valued parameters.

xvel(Q, B, T) $\equiv$ value(xvel(Q), B, T) = dx(λ(T) centre(Q, scene(B, T)))

$$\text{yvel}(Q, B, T) \equiv \text{value}(\text{yvel}(Q), B, T) = dy(\lambda(T)\ \text{centre}(Q, \text{scene}(B, T)))$$

$$\text{zvel}(Q, B, T) \equiv \text{value}(\text{zvel}(Q), B, T) = dz(\lambda(T)\ \text{centre}(Q, \text{scene}(B, T)))$$

The functions *xvel(O, B, T)*, *yvel(O, B, T)*, and *zvel(O, B, T)* map object *O* to its velocity in behaviour *B* at time *T*.

Axiom 3.30: Discontinuities of the velocity of an object *O* are *masked*, i.e. the velocity is considered to be *essentially continuous*, if the object is active or has an active carrier.

$$U = \text{xvel}(O) \lor U = \text{yvel}(O) \lor U = \text{zvel}(O) \rightarrow$$

$$[\ \text{mask}_-(U, B, T) \leftrightarrow$$

$$\neg\text{inert}(O) \lor (\exists OC)\ [\text{carrier\_of}(OC, O, \text{scene}(B, T)) \land \neg\text{inert}(OC)]\ ] \land$$

$$[\ \text{mask}_+(U, B, T) \leftrightarrow$$

$$\neg\text{inert}(O) \lor (\exists OC)\ [\text{carrier\_of}(OC, O, \text{scene}(B, T)) \land \neg\text{inert}(OC)]\ ]$$

Definition 3.31: The functions *rxvel(O1, O2, B, T)*, *ryvel(O1, O2, B, T)*, and *rzvel(O1, O2, B, T)* map object *O1* to its velocity relative to object *O2* at time *T* in behaviour *B*.

$$\text{rxvel}(O1, O2, B, T) = \text{xvel}(O1, B, T) - \text{xvel}(O2, B, T)$$

$$\text{ryvel}(O1, O2, B, T) = \text{yvel}(O1, B, T) - \text{yvel}(O2, B, T)$$

$$\text{rzvel}(O1, O2, B, T) = \text{zvel}(O1, B, T) - \text{zvel}(O2, B, T)$$

Definition 3.32: The function *vel(O, B, T)* maps object *O* to the magnitude of its velocity vector at time *T* in behaviour *B*.

$$\text{vel}(O, B, T)^2 = \text{xvel}(O, B, T)^2 + \text{yvel}(O, B, T)^2 + \text{zvel}(O, B, T)^2$$

Definition 3.33: The function *rvel(O1, O2, B, T)* maps object *O1* to the magnitude of its velocity relative to object *O2* at time *T* in behaviour *B*.

$$\text{rvel}(O1, O2, B, T)^2 = \text{rxvel}(O1, O2, B, T)^2 + \text{ryvel}(O1, O2, B, T)^2 + \text{rzvel}(O1, O2, B, T)^2$$

Definition 3.34: For convenience, we shall use the constant g to denote the gravitational acceleration.

$$g = 9.81 \cdot \text{metre/sec}^2$$

Definition 3.35: The function *vmax(O)* maps object *O* to its maximum possible self-generated speed relative to a carrier.

Definition 3.36: The predicate *inert(O)* holds if object *O* is inert, that is unable to propel itself.

$$\text{inert(O)} \leftrightarrow \text{vmax(O)} = 0$$

Axiom 3.37: The predicate *propelled(O, B, T)* holds if the speed of object *O* relative to an intersecting carrier *OC* in behaviour *B* at time *T* is not greater than the maximum possible self-generated speed of *O*.

$$\begin{aligned}
&\text{propelled(O, B, T)} \leftrightarrow \\
&\quad (\exists \text{OC}) \; [\; \text{carrier_of(OC, O, scene(B, T))} \wedge \\
&\qquad \text{intersect(place(OC, scene(B, T)), place(O, scene(B, T)), scene(B, T))} \wedge \\
&\qquad \text{rvel(O, OC, B, T)} \leq \text{vmax(O)} \wedge \\
&\qquad (\exists \text{I}) \; (\forall \text{OC'}) \; [\text{time_in(T, I)} \rightarrow \neg \text{carrier_of(OC', O, scene(B, T))}] \rightarrow \\
&\qquad\quad \text{rvel}_{+}\text{(O, OC, B, T)} \leq \text{vmax(O)} \;]
\end{aligned}$$

Definition 3.38: The function *emu(O, B, T)* maps object *O* to the sum of the kinetic and gravitational potential energy of a unit of mass of *O* in behaviour *B* at time *T*.

$$\text{emu(O, B, T)} = g \cdot \text{y(centre(O, scene(B, T)))} + \tfrac{1}{2}\text{vel(O, B, T)}^2$$

Axiom 3.39: The predicate *inert_motion(O, B, T)* holds if the sum of the kinetic and gravitational potential energy of object *O* does not increase in behaviour *B* at time *T*.

$$\begin{aligned}
&\text{inert_motion(O, B, T)} \leftrightarrow \\
&\quad \text{emu}_{-}\text{(O, B, T)} \geq \text{emu(O, B, T)} \geq \text{emu}_{+}\text{(O, B, T)}
\end{aligned}$$

Axiom 3.40: Gravity: Behaviour *B* is gravitationally possible if, for all object sets *Q* not including *oground* and all intervals *I*, the centre of mass of *Q* moves in the negative y direction with gravitational acceleration and with constant velocity in x and z directions throughout *I*.

$$\text{bgrav\_poss}(B) \leftrightarrow$$

$$(\forall Q,I)\ [\ \text{oground} \notin Q \wedge (\forall T)\ [\text{time\_in}(T, I) \rightarrow \text{isolated}(Q, \text{scene}(B, T))] \rightarrow$$

$$\text{yvel}(Q, B, \text{end}(I)) = \text{yvel}(Q, B, \text{start}(I)) - g \cdot \text{time\_length}(I) \wedge$$

$$\text{xvel}(Q, B, \text{end}(I)) = \text{xvel}(Q, B, \text{start}(I)) \wedge$$

$$\text{zvel}(Q, B, \text{end}(I)) = \text{zvel}(Q, B, \text{start}(I))\ ]$$

Lemma 3.41: The distance travelled by object *O* while being isolated during interval *I* in gravitationally possible behaviour *B* is the following:

$$\text{bgrav\_poss}(B) \wedge$$

$$(\forall T)\ [\ \text{object\_of}(O, \text{scene}(B, T)) \wedge \text{time\_in}(T, I) \rightarrow \text{isolated}(O, \text{scene}(B, T))\ ] \rightarrow$$

$$y(\text{centre}(O, \text{scene}(B, \text{end}(I)))) = y(\text{centre}(O, \text{scene}(B, \text{start}(I)))) +$$

$$\text{yvel}(O, B, \text{start}(I)) \cdot \text{time\_length}(I) -$$

$$\tfrac{1}{2}g \cdot \text{time\_length}(I)^2 \wedge$$

$$x(\text{centre}(O, \text{scene}(B, \text{end}(I)))) = x(\text{centre}(O, \text{scene}(B, \text{start}(I)))) +$$

$$\text{xvel}(O, B, \text{start}(I)) \cdot \text{time\_length}(I) \wedge$$

$$z(\text{centre}(O, \text{scene}(B, \text{end}(I)))) = z(\text{centre}(O, \text{scene}(B, \text{start}(I)))) +$$

$$\text{zvel}(O, B, \text{start}(I)) \cdot \text{time\_length}(I)\ ]$$

Proof: Follows from axiom 3.40.

Axiom 3.42: Behaviour *B* is physically possible if all its layouts are physically possible, it obeys gravity, and each object at any time in *B* is either being propelled or in inert motion.

$$\text{bphys\_poss}(B) \leftrightarrow$$

$$(\forall T)\ \text{phys\_poss}(\text{scene}(B, T)) \wedge$$

$bgrav\_poss(B) \land$

$(\forall O,T)$ [inert_motion(O, B, T) $\lor$ propelled(O, B, T)]

Lemma 3.43: While an inert object *O* does not intersect any moving object, the sum of its kinetic and gravitational potential energy does not increase in a physically possible behaviour *B*.

bphys_poss(B) $\land$

$(\forall$O', T) [O' $\neq$ O $\land$ time_in(T, I) $\land$

intersect(place(O, scene(B, T)), place(O', scene(B, T))) $\rightarrow$

vel(O', B, T) = 0] $\rightarrow$

emu(O, B, start(I)) $\geq$ emu(O, B, end(I))

Proof: If the *emu* of *O* had increased during *I* then, by axiom 3.42, it would have had to be propelled at some time during *I*. Since *O* is inert, its *vmax* is zero. Since all objects that *O* is in contact with during *I*, if any, have zero speed, the speed of *O* cannot be propelled above zero, by axiom 3.37. If an object's speed is zero then, by definition 3.38, neither its kinetic nor gravitational potential energy may increase.

B.4 Theory of Perception and Knowledge

Axiom 4.6 (modified from [Davis88]): Two behaviours are visually compatible up to time *TS* if both behaviours are physically possible and minimal according to CMD, and their corresponding layouts up to time *TS* are compatible.

bv_compatible(A, B1, B2, TS) $\leftrightarrow$

bphys_poss(B1) $\land$ bphys_poss(B2) $\land$

$(\forall$B') [bphys_poss(B') $\rightarrow$ $\neg$ [B' $\lessdot$ B1] $\land$ $\neg$ [B' $\lessdot$ B2]] $\land$

$(\forall T \leq TS)$ v_compatible(A, scene(B1, T), scene(B2, T))

Lemma 4.7 (originally in [Davis88]): *Bv_compatibility* is an equivalence relation over behaviours.

bv_compatible(A, B, B, T)

bv_compatible(A, B1, B2, T) → bv_compatible(A, B2, B1, T)

bv_compatible(A, B1, B2, T) ∧ bv_compatible(A, B2, B3, T) →

bv_compatible(A, B1, B3, T)

Proof: By lemma 4.3, the relation *v_compatible* is an equivalence relation. Reflexivity holds since the real behaviour must be physically possible and minimal according to CMD. Symmetry and transitivity hold since both behaviours in the *bv_compatible* relation must be physically possible and minimal according to CMD.

Lemma 4.17: An agent can always measure time (not just know what time it is).

k(A, S0A, S1A) ∧ k(A, S0B, S1B) ∧

start(I0) = time(S0A) ∧ end(I0) = time(S0B) ∧

start(I1) = time(S1A) ∧ end(I0) = time(S1B) →

time_length(I0) = time_length(I1)

Proof: From axiom 4.12, *time(S0A) = time(S1A)* and *time(S0B) = time(S1B).* Thus, by axiom 2.6, *I0 = I1.* Therefore, *time_length(I0) = time_length(I1).*

Definition 4.18: The predicate *v_separated(A, X1, X2, L)* holds if point sets *X1* and *X2* are not wholly invisible to agent *A* and for any path connecting them there is a segment wholly visible to *A* and lying entirely in free space in layout *L.*

v_separated(A, X1, X2, L) ↔

¬wholly_invisible(X1, A, L) ∧ ¬wholly_invisible(X2, A, L) ∧

(∀XP) [connected(XP) ∧ intersect(XP, X1) ∧ intersect(XP, X2) →

(∃X1P) [sub_place(X1P, XP) ∧ free_space(X1P, L) ∧

wholly_visible(X1P, A, L)]]

We allow the predicate *v_separated* to take an object as its second and/or third argument.

v_separated(A, O1, O2, L) $\leftrightarrow$ v_separated(A, place(O1, L), place(O2, L), L)

Lemma 4.19: The relation *v_separated* is non-reflexive, symmetric, and not transitive.

$\neg$v_separated(A, X, X, L)

v_separated(A, X1, X2, L) $\leftrightarrow$ v_separated(A, X2, X1, L)

Proof: Immediate from definition.

Lemma 4.20: If point sets *X1* and *X2* are visually separated in layout *L*, then they are separated in *L*.

v_separated(A, X1, X2, L) $\rightarrow$ separated(X1, X2, L)

Proof: Immediate since the conditions for being separated are a subset of the conditions for being visually separated.

Lemma 4.21: If point set *X1* is not visually separated from set *X2*, then any superset *X3* of *X1* is not visually separated from *X2*.

$\neg$v_separated(A, X1, X2, L) $\wedge$ wholly_visible(X1, A, L) $\wedge$

wholly_visible(X2, A, L) $\wedge$ sub_place(X1, X3) $\rightarrow$

$\neg$v_separated(X3, X2, L)

Proof: Since *X1* and *X2* are not visually separated but are wholly visible to *A* in *L*, by definition there must exist a path *XP* connecting *X1* and *X2* and such that none of its subpaths is wholly visible to *A* and entirely in free space in *L*. Since *X1* is a subset of *X3*, *XP* connects *X3* and *X1*. Therefore, *X3* and *X2* are not visually separated.

Lemma 4.22: If two point sets *X1* and *X2* are visually separated relative to agent *A*, then he knows that they are visually separated.

k(A, S0, S1) $\wedge$ v_separated(A, X1, X2, layout(S0)) $\rightarrow$

v_separated(A, X1, X2, layout(S1))

Proof: Let *XP* be any path connecting *X1* and *X2* in the layout of *S0*. Let *X1P* be a subset of *XP* wholly visible to *A* and lying entirely in free space. From axiom 4.2, *X1P* must also be wholly visible to *A* and lie entirely in free space in the layout of *S1* for it to be visually compatible with the layout of *S0*. Thus, *X1* and *X2* are visually separated in *S1*, i.e. *A* knows in *S0* that they are visually separated.

Lemma 4.23: If the spaces occupied by objects *O1* and *O2* are visually separated relative to agent *A*, then he knows that the spaces occupied by the objects partially visible to him are separated.

k(A, S0, S1) $\wedge$ L0 = layout(S0) $\wedge$ L1 = layout(S1) $\wedge$

object_of(O1, L0) $\wedge$ object_of(O2, L0) $\wedge$ v_separated(A, O1, O2, L0) $\rightarrow$

($\exists$O1', O2') [object_of(O1', L1) $\wedge$ object_of(O2', L1) $\wedge$

same_vprops(O1', O1) $\wedge$ same_vprops(O2', O2) $\wedge$

($\forall$X1, X2) [wholly_visible(X1, A, L0) $\wedge$ sub_place(X1, place(O1, L0)) $\wedge$

wholly_visible(X2, A, L0) $\wedge$ sub_place(X2, place(O2, L0)) $\rightarrow$

wholly_visible(X1, A, L1) $\wedge$ sub_place(X1, place(O1', L1)) $\wedge$

wholly_visible(X2, A, L1) $\wedge$ sub_place(X2, place(O2', L1))] $\wedge$

separated(O1', O2', L1)]

Note: The difficulty here is that objects *O1* and *O2* need not occupy the same space in *S1* as in *S0* for the behaviours to be visually compatible.

Proof: The existence of objects *O1'* and *O2'* with the stated properties in layout *L1* except that they are separated is guaranteed by axiom 4.2. Let us suppose that there is a knowledge-accessible situation *S1* in which *O1'* and *O2'* are not separated. Then there exists a path *X* connecting *O1'* and *O2'* and lying entirely in occupied space in *L1*. Let *XU* = *X* $\cup$ *place(O1', L1)* $\cup$ *place(O2', L1)*. *XU* is connected, intersects *X1* and *X2*, and lies entirely in occupied space in *L1*. Thus, by definition 3.14, *X1* and *X2* are not separated in

L1. Then, by lemma 4.20, they are not visually separated in *L1*. Hence, by lemma 4.22, they are not visually separated in *L0*. Thus, by lemma 4.21, since *X1* and *X2* are wholly visible, *O1* and *O2* are not visually separated in *L0*, which contradicts the antecedent. Therefore, in all knowledge-accessible situations *S1*, *O1'* and *O2'* are separated, i.e. *A* knows in *S0* that they are separated.

Lemma 4.24: If the spaces occupied by objects *O1* and *O2* are visually separated relative to agent *A* and he recognizes them, then he knows that they are separated.

$$
\begin{aligned}
&k(A, S0, S1) \land L0 = layout(S0) \land L1 = layout(S1) \land \\
&object\_of(O1, L0) \land object\_of(O2, L0) \land v\_separated(A, O1, O2, L0) \land \\
&[same\_vprops(O1', O1) \rightarrow true\_in(O1'=O1, S1)] \land \\
&[same\_vprops(O2', O2) \rightarrow true\_in(O2'=O2, S1)] \rightarrow \\
&\quad separated(O1, O2, L1)
\end{aligned}
$$

Proof: Immediate from lemma 4.23 since now *O1'* = *O1* and *O2'* = *O2*.

Definition 4.25: The predicate *vd_envelope(XS, D, O, A, L)* holds if there is a point in object *O* visible to agent *A* in layout *L* and inside convex point set *XS* such that any point in *XS* is either (i) in free space and visible to *A* in *L*, or (ii) separated by a distance more than *D* from some point in *O* visible to *A* in *L*.

$$
\begin{aligned}
&vd\_envelope(XS, D, O, A, L) \leftrightarrow \\
&\quad convex(XS) \land (\exists PO)\ [point\_in(PO, place(O, L)) \land visible(PO, A, L)] \land \\
&\quad (\forall P)\ [\ point\_in(P, boundary(XS)) \rightarrow \\
&\qquad [visible(P, A, L) \land free\_space(\{P\}, L)] \lor \\
&\qquad (\exists PS)\ [point\_in(PS, place(O, L)) \land visible(PS, A, L) \land \\
&\qquad\quad distance(PS, P) > D]\ ]
\end{aligned}
$$

Note: We refer to the distance *D* as the depth of the *vd_envelope*.

Lemma 4.26: If point set *XS* is a *vd_envelope* of depth *D* of an object *O* whose diameter is less than *D*, then *O* is strictly inside *XS*.

vd_envelope(XS, D, O, A, L) ∧ diameter(place(O, L)) < D →

strictly_inside(place(O, L), XS)

Proof: Let us suppose that *O* were not strictly inside *XS*. Then, there is a point *P* in *O* outside the interior of *XS*. Since *XS* is a *vd_envelope* of *O*, there must be a point *PO* in *O* inside *XS*. Since *place(O, L)* must be connected, there must be a path between *P* and *PS* completely within *O*. This path must cross the boundary of *XS*. It cannot cross it where it is in free space because it is completely within *O*. All other points in *boundary(XS)*, however, are separated from *PO* by a distance more than *D*, and, by definition 1.25, no point in *O* may be separated from another point in *O* by more than *D* since the diameter of *O* is less than *D*. We have thus arrived at a contradiction. Therefore, *O* must be strictly inside *XS*.

Appendix C

EXAMPLE O: INFERRING IGNORANCE

The following proof is a modified version of the proof of example I in [Davis88]. We shall omit those proofs of lemmas that apply here without modifications. The main problem with the proof in [Davis88] in the extended model is that the object is assumed to be strictly inside the envelope and no other object overlaps the envelope throughout the interval. Consequently, the object is isolated throughout the interval and therefore is not motionless in gravitational field. Fortunately, the object need not be strictly inside the envelope for the proof to go through if we make the assumption that the object can move within the envelope (hypothesis 5.5a). In order to satisfy the other restrictions, the object must not be inert (hypothesis 5.5b).

The basic constants of our example are the following:

Constants:

aclaire	-	Claire
c0	-	actually occuring chronicle
i0	-	time interval in question
omystery	-	the mystery object
owall	-	the wall
xclaire	-	place occupied by Claire
xenvelope	-	the spatial envelope containing the object
xwall	-	place occupied by the wall

We define a few additional constants as convenient abbreviations:

Definition 5.1 (in [Davis88]):

b0	=	behaviour(c0)	-	the real behaviour
ta	=	start(i0)	-	starting time
tz	=	end(i0)	-	ending time
la	=	scene(b0,ta)	-	starting layout
s0z	=	situation(c0,tz)	-	ending situation

Hypothesis 5.2 (in [Davis88]): Claire, the wall, and the mystery object are distinct objects in the chronicle.

object_of(aclaire, b0) ∧ object_of(owall, b0) ∧ object_of(omystery, b0) ∧

aclaire ≠ owall ∧ owall ≠ omystery ∧ aclaire ≠ omystery

Hypothesis 5.3 (in [Davis88]): Claire occupies the fixed place *xclaire* throughout the interval *i0.*

time_in(T, i0) → place(aclaire, scene(b0, T)) = xclaire

Hypothesis 5.4 (in [Davis88]): The wall occupies the fixed place *xwall* throughout the interval *i0.*

time_in(T, i0) → place(owall, scene(b0, T)) = xwall

Hypothesis 5.5 (modified from [Davis88]): The mystery object remains inside the envelope throughout the interval *i0.*

time_in(T, i0) → sub_place(place(omystery, scene(b0, T)), xenvelope)

Hypothesis 5.5a (new): The mystery object can move inside the envelope no matter where in the envelope the object is found at some time.

(∀L,TL) sub_place(place(omystery, L), XE) →

(∃B) [scene(B, T) = L ∧

(∀T) sub_place(place(omystery, scene(B, T)), XE) ∧

(∀I) continual_motion(omystery, B, I)]

Note: This restriction is relatively benign - for most regions larger than an object, there is some way the latter can move within the former. One way to violate it would be to make *xenvelope* equal to *place(omystery, L).*

Hypothesis 5.5b (new): The mystery object is not inert.

¬inert(omystery)

Hypothesis 5.6 (in [Davis88]): No other object comes inside the envelope within the interval *i0*.

$$O \neq \text{omystery} \wedge \text{time\_in}(T, i0) \rightarrow \neg\text{overlap}(\text{place}(O, \text{scene}(b0, T)), \text{xenvelope})$$

Hypothesis 5.7 (in [Davis88]): The envelope is blocked by the wall from Claire.

blocked(xenvelope, xwall, xclaire)

Hypothesis 5.8 (modified from [Davis88]): *b0* is a physically possible behaviour.

bphys_poss(b0)

To prove (in [Davis88]): Claire does not know whether or not *omystery* was motionless during *i0*. We express this as follows: There is a situation accessible to Claire in situation *s0z* (the ending situation) that follows on a chronicle in which *omystery* was motionless throughout *i0*. There is also an accessible situation that follows on a chronicle in which it was not motionless.

$$(\exists C1)\ k(\text{aclaire}, s0z, \text{situation}(C1, tz)) \wedge \text{motionless}(\text{omystery}, \text{behaviour}(C1), i0)$$

$$(\exists C2)\ k(\text{aclaire}, s0z, \text{situation}(C2, tz)) \wedge \neg\text{motionless}(\text{omystery}, \text{behaviour}(C2), i0)$$

Lemma 5.9 (in [Davis88]): There is a behaviour, which we henceforth call *b1*, containing the same objects as *b0*, such that (i) all layouts of *b1* are the same as those of *b0*, up to and including time *ta*; (ii) every object except *omystery* has the same behaviour in *b1* as in *b0*; (iii) *omystery* is motionless after time *ta*.

$$\begin{aligned}
&(\exists B1\ \forall O)\ [\text{object_of}(O, B1) \leftrightarrow \text{object_of}(O, b0)] \wedge \\
&\quad (\forall T \leq ta)\ \text{layout}(B1, T) = \text{layout}(B0, T) \wedge \\
&\quad [\ \text{object_of}(O, B1) \wedge O \neq \text{omystery} \rightarrow \\
&\qquad (\forall T)\ \text{place}(O, \text{scene}(B1, T)) = \text{place}(O, \text{scene}(b0, T))\ ] \wedge \\
&\quad (\forall T \leq ta)\ \text{sub_place}(\text{place}(\text{omystery}, \text{scene}(B1, T), \text{xenvelope}) \wedge \\
&\quad (\forall I)\ [ta \leq \text{start}(I) \rightarrow \text{motionless}(\text{omystery}, B1, I)\ ]
\end{aligned}$$

Lemma 5.10 (modified from [Davis88]): There is a behaviour, which we henceforth call *b2*, containing the same objects as *b0*, such that (i) all layouts of *b2* are the same as those of *b0*, up to and including time *ta*; (ii) every object except *omystery* has the same behaviour in *b1* as in *b0*; (iii) *omystery* stays inside *xenvelope* in a state of continual motion.

$(\exists B2)\ (\forall O)\ [\text{object\_of}(O, B2) \leftrightarrow \text{object\_of}(O, b0)] \wedge$

$(\forall T \leq ta)\ \text{layout}(B2, T) = \text{layout}(B0, T) \wedge$

$[\ \text{object\_of}(O, B2) \wedge O \neq \text{omystery} \rightarrow$

$(\forall T)\ \text{place}(O, \text{scene}(B2, T)) = \text{place}(O, \text{scene}(b0, T))\ ] \wedge$

$(\forall T \leq ta)\ \text{sub\_place}(\text{place}(\text{omystery}, \text{scene}(B2, T), \text{xenvelope}) \wedge$

$(\forall I)\ [ta \leq \text{start}(I) \rightarrow \text{continual\_motion}(\text{omystery}, B2, I)\ ]$

Proof: By axiom 2.20, there is a layout *LAP* whose only object is *omystery*, placed in the same place as in the layout *la*. By hypothesis 5.5a, there is a behaviour *BX* containing *LAP* at time *ta* with *omystery* in continual motion throughout *BX*. By axiom 2.19, there is a behaviour *BY* combining the behaviour of *omystery* from *BX* and the behaviour of the other objects from *b0*. From the construction of *BY*, and from axiom 2.9, it follows that the layout of *BY* at time *ta* is equal to the layout *la*. Therefore, by axiom 2.18, we can define *b2* to be the behaviour which agrees with *b0* up to time *ta*, and with *BY* after *ta*. The above properties follow directly from the construction.

Lemma 5.11 (modified from [Davis88]): In behaviour *b1* at all times after *ta*, *omystery* is inside *xenvelope*.

$(\forall T \leq ta)\ \text{sub\_place}(\text{place}(\text{omystery}, \text{scene}(b1, T)), \text{xenvelope})$

Proof: Even though this lemma is slightly modified, the proof in [Davis88] applies.

Lemma 5.12 (modified from [Davis88]): Behaviours *b1* and *b2* are physically possible.

$\text{bphys\_poss}(b1) \wedge \text{bphys\_poss}(b2)$

Proof: The proof that all the layouts are physically possible is given in [Davis88]. The only difference is that now *omystery* is not strictly inside *xenvelope*, but this is not required. Another object may intersect but not overlap *omystery*. As to *oground*, nothing prevents it from occupying *xground*. Therefore, all the layouts are physically possible.

The law of gravity holds in *b1* and *b2* if *omystery* is not isolated. It it were, it obviously could not be motionless. It could be in continual motion, but it might or might not be possible for it to stay within *xenvelope*. Anyway, we assume that *omystery* is not isolated throughout *I* but intersects the ground. This is possible through the boundary of *xenvelope*. That way no other object overlaps *xenvelope* and yet the place occupied by *omystery* is a subset of *xenvelope*. The object may obviously be in inert motion in *b1*. It can propel itself in *b2* since, by hypothesis 5.5b, it is not inert. It has not been shown that a breakpoint in velocity at time *ta* can be avoided. This is not a problem, however, since the object is not inert and so its parameters are masked at all times. We have now shown that both *b1* and *b2* are physically possible.

Lemma 5.13 (in [Davis88]): In each of the behaviours *b0*, *b1*, and *b2*, at all times during the interval *i0*, the object *omystery* is invisible to Claire.

$$[B = b0 \vee B = b1 \vee B = b2] \wedge L = scene(B, T) \rightarrow$$

$$wholly\_invisible(omystery, aclaire, L)$$

Lemma 5.14 (in [Davis88]): The behaviours *b1* and *b2* are each visually compatible with Claire's perceptions in *b0* up to the end of *i0*.

bv_compatible(aclaire, behaviour(C), b1, tz)

bv_compatible(aclaire, behaviour(C), b2, tz)

Lemma 5.15 (in [Davis88]): The behaviour *b1* is visually compatible with Claire's perceptions up to the end of *i0*, and *omystery* is motionless in *b1* during *i0*.

bv_compatible(aclaire, behaviour(C), b1, tz) $\wedge$ motionless(omystery, b1, i0)

Lemma 5.16 (in [Davis88]): The behaviour *b2* is visually compatible with Claire's perceptions up to the end of *i0*, and *omystery* is motionless in *b2* during *i0*.

bv_compatible(aclaire, behaviour(C), b2, tz) ∧ ¬motionless(omystery, b1, i0)

Theorem 5.17 (in [Davis88]): Claire does not know, at the end of *i0*, whether or not *omystery* has been motionless during *i0*.

(∃C1) [k(aclaire, s0z, situation(C1, tz)) ∧ motionless(omystery, behaviour(C1), i0)]

(∃C2) [k(aclaire, s0z, situation(C2, tz)) ∧ ¬motionless(omystery, behaviour(C2), i0)]

Appendix D

EXAMPLE VI: INERTIA

The basic constants of our example are the following:

Constants:

ajenny	-	Jenny
b0	-	the real behaviour
i0	-	initial time interval
ostone	-	the stone
xjenny	-	place occupied by Jenny
xwell	-	the well, a pseudo-object
y0	-	the y coordinate of the edge of the well

We define a few additional constants as convenient abbreviations:

Definition 6.1:

ta	=	start(i0)	-	starting time
tz	=	end(i0)	-	ending time
la	=	scene(b0,ta)	-	starting layout
lz	=	scene(b0,tz)	-	ending layout
s0z	=	situation(c0,tz)	-	ending situation

Hypothesis 6.2: Jenny, the stone, and the ground are distinct objects in the chronicle.

$$\text{object\_of}(\text{ajenny}, \text{b0}) \land \text{object\_of}(\text{ostone}, \text{b0}) \land \text{object\_of}(\text{oground}, \text{b0}) \land$$

$$\text{ajenny} \neq \text{ostone} \land \text{ostone} \neq \text{oground} \land \text{ajenny} \neq \text{oground}$$

Hypothesis 6.3: Jenny occupies the fixed place *xjenny* throughout the interval *i0*.

$$\text{time\_in}(T, \text{i0}) \rightarrow \text{place}(\text{ajenny}, \text{scene}(\text{b0}, T)) = \text{xjenny}$$

Hypothesis 6.4: The point set *xwell* is a pseudo-object not overlaping the ground such that every point in its boundary below *y0* is in the ground.

$$\text{connected}(\text{xwell}) \land \neg\text{overlap}(\text{xwell}, \text{xground}) \land$$

$(\forall P)$ [point_in(P, boundary(xwell)) $\wedge$ y(P) $<$ y0 $\rightarrow$ point_in(P, xground)]

Note: This hypothesis does not involve time - it is always true because the ground is motionless at all times.

Hypothesis 6.5: The interval *i0* is 0.5 sec long.

time_length(i0) = 0.5 $\cdot$ sec

Hypothesis 6.6: The stone is an inert object.

inert(ostone)

Hypothesis 6.7: Jenny recognizes the stone.

k(ajenny, S0, S1) $\wedge$ same_vprops(O, ostone) $\rightarrow$ true_in(O=ostone, S1)

Hypothesis 6.8: Jenny knows that 0.1 m is an upper bound on the size of the stone.

k(ajenny, s0z, S1) $\rightarrow$ diameter(place(ostone, layout(S1))) $<$ 0.1 $\cdot$ metre

Note: We do not specify here how Jenny has acquired this knowledge. The possibility of the acqusition of such knowledge has been shown elsewhere in this paper. It is also worthwhile to point out that it is not absolutely necessary for Jenny to recognize the stone (hypothesis 6.7). If she could bound the size of the object in the well visible to her, which need not be *ostone* is *S1*, then the final inference would involve that object. She would still have to know that the object is inert, though. This would be true if she knew that the object is a stone, since all stones are inert.

Hypothesis 6.9: Jenny has seen every point in the boundary of *xwell* below *y0* before *tz*.

$(\forall P)$ [point_in(P, boundary(xwell)) $\wedge$ y(P) $<$ y0 $\rightarrow$

$(\exists T)$ [T $\leq$ tz $\wedge$ visible(P, ajenny, scene(b0, T))]]

Hypothesis 6.10: At any time *T* in *i0*, there is a *vd_envelope* of depth 0.1 m strictly inside the well and such that every point in it is more than 0.11 m below *y0*.

$$\text{time\_in}(T, i0) \rightarrow$$
$$(\exists XE)\ [\ \text{vd\_envelope}(XE, 0.1 \cdot \text{metre}, \text{ostone}, \text{ajenny}, \text{scene}(b0, T)) \wedge$$
$$\text{strictly\_inside}(XE, \text{xwell}) \wedge$$
$$(\forall P)\ [\text{point\_in}(P, XE) \rightarrow y(P) < y0 - 0.11 \cdot \text{metre}]\ ]$$

Hypothesis 6.11: At time *ta*, there is a point in the stone visible to Jenny and less than 0.12 m below *y0*.

$$(\exists P)\ [\ \text{point\_in}(P, \text{place}(\text{ostone}, la)) \wedge \text{visible}(P, \text{ajenny}, la) \wedge$$
$$y(P) > y0 - 0.12 \cdot \text{metre}\ ]$$

Hypothesis 6.12: At time *tz*, there is a point in the stone visible to Jenny and less than 1.22 m below *y0*.

$$(\exists P)\ [\ \text{point\_in}(P, \text{place}(\text{ostone}, lz)) \wedge \text{visible}(P, \text{ajenny}, lz) \wedge$$
$$y(P) > y0 - 1.22 \cdot \text{metre}\ ]$$

To prove: Jenny knows at the end of the interval *i0* that if no object other than the stone and the ground intersects the well during *i0*, then, while no other object intersects the well, the stone will remain inside the well. We express this as follows: In the chronicles of all situations accessible to Jenny in situation *s0z* (the ending situation), if no object other than the stone and the ground intersects the well between the start of the interval *i0* and the end of an interval *I*, then the place of the stone is inside the well at the end of *I*.

$$k(\text{ajenny}, s0z, \text{situation}(C1, tz)) \wedge \text{start}(I) = \text{end}(i0) \wedge$$
$$(\forall T,O)\ [\ ta \leq T \wedge T \leq \text{end}(I) \wedge$$
$$\text{intersect}(\text{place}(O, \text{scene}(\text{behaviour}(C1), T)), \text{xwell}) \rightarrow$$
$$O = \text{ostone} \vee O = \text{oground}\ ] \rightarrow$$
$$\text{sub\_place}(\text{place}(\text{ostone}, \text{scene}(\text{behaviour}(C1), \text{end}(I))), \text{xwell})$$

Lemma 6.13: Jenny knows that every point in the boundary of *xwell* below *y0* intersects the ground at all times.

k(ajenny, s0z, situation(C1, tz)) ∧ point_in(P, boundary(xwell)) ∧ y(P) < y0 →

point_in(P, xground)

Proof: By hypothesis 6.9, Jenny has seen every point in the boundary of *xwell* below *y0*. Jenny also knows that, by axiom 3.28 (possible layout), the ground remains in the same place at all times. Thus, by axiom 4.2, in all situations accessible to Jenny, the boundary of *xwell* below *y0* must intersect the ground.

Lemma 6.14: Jenny knows that at any time during the interval *i0* the stone is strictly inside a *vd_envelope* of depth 0.1 m that is strictly inside the well and entirely at least 0.11 m below *y0*.

Proof: Immediate from hypotheses 6.8 and 6.10 and lemma 4.26.

Lemma 6.15: Jenny knows that the centre of mass of *ostone* is more than 0.11 m below *y0* throughout the interval *i0*.

k(ajenny, s0z, situation(C1, tz)) ∧ time_in(T, i0) →

y(centre(ostone, scene(behaviour(C1), T))) < y0 − 0.11 • metre

Proof: Immediate from lemma 6.14 and axiom 3.23.

Lemma 6.16: Jenny knows that the centre of mass of *ostone* is less than 0.22 m below *y0* at the beginning of the interval *i0*.

k(ajenny, s0z, situation(C1, tz)) ∧ time_in(T, i0) →

y(centre(ostone, scene(behaviour(C1), ta))) > y0 − 0.22 • metre

Proof: By hypothesis 6.11, there is a point in the stone less than 0.12 m below y0. Thus, by hypothesis 6.8, lemma 3.24, and the triangle inequality, the centre of mass of the stone is less than 0.22 m below y0.

Lemma 6.17: Jenny knows that the centre of mass of *ostone* is less than 1.32 m below *y0* at the end of the interval *i0.*

k(ajenny, s0z, situation(C1, tz)) ∧ time_in(T, i0) →

y(centre(ostone, scene(behaviour(C1), tz))) > y0 − 1.32 • metre

Proof: Same as that of lemma 6.16 except now we use hypothesis 6.12 rather than hypothesis 6.11.

Lemma 6.18: Jenny knows that if there are no objects inside the well other than the stone throughout the interval *i0*, then the stone is isolated throughout *i0.*

k(ajenny, s0z, situation(C1, tz)) ∧ time_in(T, i0) ∧

(∀O) [overlap(place(O, scene(behaviour(C1), T)), xwell) → O = ostone] →

isolated(ostone, scene(behaviour(C1), T))

Proof: By lemma 6.14, Jenny knows that *ostone* is strictly inside *xwell* throughout the interval *i0*, since *strictly_inside* is transitive. The ground never overlaps *xwell.* Thus, if no object other than *ostone* overlaps *xwell*, then *ostone* must be isolated.

Lemma 6.19: Jenny knows that if there are no objects inside the well other than the stone throughout the interval *i0*, then the speed of the stone at time *ta* is less than 0.2 m/sec.

k(ajenny, s0z, situation(C1, tz)) ∧

(∀O) [overlap(place(O, scene(behaviour(C1), T)), xwell) → O = ostone] →

vel(ostone, behaviour(C1), ta) < 0.2 • metre/sec

Proof: Follows from axiom 3.40 (gravity) and lemmas 6.15 - 6.18.

Lemma 6.20: Jenny knows that if there are no objects inside the well other than the stone throughout the interval *i0*, then the energy of a unit of mass of the stone is less than (y0 − 0.1 m/sec) • g.

k(ajenny, s0z, situation(C1, tz)) ∧

(∀O) [overlap(place(O, scene(behaviour(C1), T)), xwell) → O = ostone] →

$$emu(ostone, behaviour(C1), ta) < (y0 - 0.1 \cdot metre/sec) \cdot g.$$

Proof: Follows from lemmas 6.15 and 6.19.

Lemma 6.21: Jenny knows that the stone will not get out the well while it stays below the coordinate *y0*.

$$k(ajenny, s0z, situation(C1, tz)) \wedge start(I) = tz \wedge$$
$$(\forall T,P)\ [\ time\_in(T, I) \wedge point\_in(P, place(ostone, scene(behaviour(C1), T))) \rightarrow$$
$$y(P) < y0\ ] \rightarrow$$
$$sub\_place(place(ostone, scene(behaviour(C1), end(I))), xwell)$$

Proof: By hypothesis 6.13, axiom 4.2 (visual compatibility), and axiom 4.11 (memory), Jenny knows that below *y0* there is ground all around the well. Thus, if the stone stays below *y0* and at least partially outside the well at the end of *I*, then there must be a time in *I* when the stone overlaps the ground since all objects move continuously in time and the stone is completely inside the well at the beginning of the interval *I*. This is physically impossible by axiom 3.28 (possible layout). Therefore, while the stone stays below *y0*, it stays inside the well.

Lemma 6.22: Jenny knows that if the stone is at least partially outside the well at the end of an interval *I* immediately following the interval *i0*, then there must have been a time in *I* when the stone was in the well with some point in it at coordinate *y0*.

$$k(ajenny, s0z, situation(C1, tz)) \wedge start(I) = tz \wedge$$
$$\neg sub\_place(place(ostone, scene(behaviour(C1), end(I))), xwell) \rightarrow$$
$$(\exists T)\ [\ time\_in(T, I) \wedge$$
$$sub\_place(place(ostone, scene(behaviour(C1), T)), xwell) \wedge$$
$$(\exists P)\ [point\_in(P, place(ostone, scene(behaviour(C1), T))) \wedge y(P) = y0]\ ]$$

Proof: Immediate since all objects move continuously in time and the stone is completely inside the well at the beginning of the interval *I*.

Lemma 6.23: Jenny knows that if the stone is at least partially outside the well at the end of an interval *I* immediately following the interval *i0*, then there must have been a time in *I* when the stone was in the well and the energy of a unit of mass of the stone was greater than (y0 − 0.1 m)• g.

k(ajenny, s0z, situation(C1, tz)) ∧ start(I) = tz ∧

¬ sub_place(place(ostone, scene(behaviour(C1), end(I))), xwell) →

(∃T) [time_in(T, I) ∧

sub_place(place(ostone, scene(behaviour(C1), T)), xwell) ∧

emu(ostone, behaviour(C1), T) > (y0 − 0.1• metre)• g]

Proof: In the layout of lemma 6.22, the centre of mass of the stone must lie less than 0.1 m below *y0* by lemma 3.24 and the triangle inequality. Thus, the potential energy of a unit of mass of the stone alone is greater than (y0 − 0.1 m)• g. Therefore, since the kinetic energy is always non-negative, the total energy of a unit of mass of the stone must be greater than (y0 − 0.1 m)• g.

Lemma 6.24: Jenny knows that if no object other than the stone and the ground intersects the well during the interval *i0* and some interval *I* immediately following *i0*, and if the stone is at least partially outside the well at the end of *I*, then there must have been a subinterval of *I* starting at time *tz*, during which the stone was in the well and the energy of a unit of mass of the stone was greater than at the beginning of *I*.

k(ajenny, s0z, situation(C1, tz)) ∧ start(I) = tz ∧

(∀T,O) [ta ≤ T ∧ T ≤ end(I) ∧

intersect(place(O, scene(behaviour(C1), T)), xwell) →

O = ostone ∨ O = oground] ∧

¬ sub_place(place(ostone, scene(behaviour(C1), end(I))), xwell) →

(∃I1) [sub_interval(I1, I) ∧ start(I1) = tz ∧

(∀T) [time_in(T, I1) →

sub_place(place(ostone, scene(behaviour(C1), T)), xwell)] ∧

emu(ostone, behaviour(C1), end(I1)) >

emu(ostone, behaviour(C1), start(I1))]

Proof: Immediate from lemmas 6.20 and 6.23.

Theorem 6.25: Jenny knows at the end of the interval *i0* that if no object other than the stone and the ground intersects the well during *i0*, then while no other object enters the well, the stone will remain inside the well.

k(jenny, s0z, situation(C1, tz)) ∧ start(I) = end(i0) ∧

(∀T,O) [ta ≤ T ∧ T ≤ end(I) ∧

intersect(place(O, scene(behaviour(C1), T)), xwell) →

O = ostone ∨ O = oground] →

sub_place(place(ostone, scene(behaviour(C1), end(I))), xwell)

Proof: Immediate from hypothesis 6.6, lemma 6.24, and lemma 3.43 (motion of an inert object).

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