Title: Algorithm for solving a class of Hamilton-Jacobi equations in high dimensions.

Abstract: Classical methods to solve Hamilton-Jacobi (H-J) equations are not sufficiently robust to yield real-time solutions. Here we implement a convex minimization procedure adapted from the paper Algorithms for overcoming the curse of dimensionality for certain Hamilton Jacobi equations arising in control theory and elsewhere by Darbon and Osher to solve a class of high-dimensional H-J equations in real time. We use a numerical method to arrive at our solution. In particular, we use the Hopf-Lax equation to transform the H-J equation into an optimization problem where we can then use the Alternating Direction Method of Multipliers (Split-Bregman) algorithm to arrive at the solution. We analyze the performance of our method with various proximal maps and Hamiltonians. Additionally, we give a visual representation of our solutions in two dimensions.

We consider H-J equations of the form (1) where \((x, t) \in \mathbb{R}^n \times (0, +\infty)\) and \(H\) denotes the Hamiltonian, \(H \in \Gamma_0(\mathbb{R}^n)\) and \(J \in \Gamma_0(\mathbb{R}^n)\) is the proximal map.

\[
\begin{align*}
\frac{\partial S}{\partial t}(x, t) + H(\nabla_x S(x, t)) &= 0 \\
S(x, 0) &= J(x).
\end{align*}
\]

(1)

A function is said to be in \(\Gamma_0(\mathbb{R}^n)\) if the following are satisfied:

- \(f : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{+\infty\}\)
- \(f\) is convex
- \(f\) is lower semi-continuous
- \(f\) is proper.

Figures 1(a) and 1(b) give a visualization of the solution for (1) where \(J\) is the quadratic functional \(J(x) = \langle x, Dx \rangle\) and \(H\) is the 1–norm and 2–norm, respectively.
Figure 1: \[ J = \frac{1}{2} \langle x, Dx \rangle \]