Applied Mathematics - Computer Science Capstone

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Title: Algorithm for solving a class of Hamilton-Jacobi equations in high dimensions.

Abstract: Classical methods to solve Hamilton-Jacobi (H-J) equations are not sufficiently robust to yield real-time solutions. Here we implement a convex minimization procedure adapted from the paper *Algorithms for overcoming the curse of dimensionality for certain Hamilton Jacobi equations arising in control theory and elsewhere* by Darbon and Osher to solve a class of high-dimensional H-J equations in real time. We use a numerical method to arrive at our solution. In particular, we use the Hopf-Lax equation to transform the H-J equation into an optimization problem where we can then use the Alternating Direction Method of Multipliers (Split-Bregman) algorithm to arrive at the solution. We analyze the performance of our method with various proximal maps and Hamiltonians. Additionally, we give a visual representation of our solutions in two dimensions.

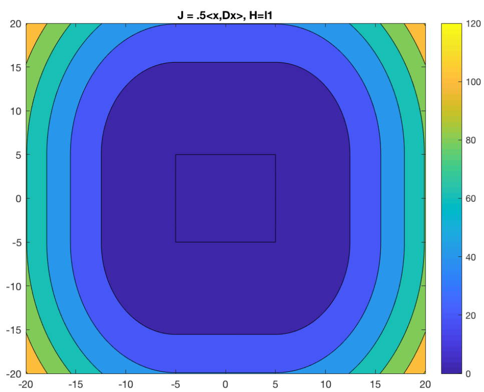
We consider H-J equations of the form (1) where $(x,t) \in \mathbb{R}^n \times (0, +\infty)$ and H denotes the Hamiltonian, $H \in \Gamma_0(\mathbb{R}^n)$ and $J \in \Gamma_0(\mathbb{R}^n)$ is the proximal map.

$$\begin{cases} \frac{\partial S}{\partial t}(x,t) + H(\nabla_x S(x,t)) = 0 \\ S(x,0) = J(x). \end{cases} \tag{1}$$

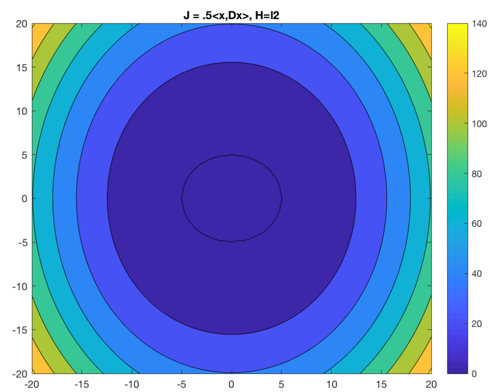
A function is said to be in $\Gamma_0(\mathbb{R}^n)$ if the following are satisfied:

- $f : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$
- f is convex
- f is lower semi-continuous
- f is proper.

Figures 1(a) and 1(b) give a visualization of the solution for (1) where J is the quadratic functional $J(x) = \langle x, Dx \rangle$ and H is the 1–norm and 2–norm, respectively.



(a) $H = ||\cdot||_1$



(b) $H = ||\cdot||_2$

Figure 1: $J = \frac{1}{2}\langle x, Dx \rangle$