Skeptical Inheritance: Computing the Intersection of Credulous Extensions[1]

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Skeptical Inheritance:
Computing the Intersection of Credulous Extensions

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Abstract

Ideally skeptical inheritance supports exactly those inferences true in every credulous extension of an inheritance hierarchy. We provide a formal definition of ideally skeptical inheritance. We then give a linear-time algorithm which computes a plausible approximation of ideally skeptical inheritance. However, we show that there are inheritance hierarchies for which this algorithm computes only a subset of the inferences sanctioned by ideally skeptical inheritance. Reasoning by cases is necessary to compute the complete set of inferences. Finally, we give an algorithm which computes ideally skeptical inheritance using a limited form of Boolean satisfiability. It is not known whether this problem is $\mathcal{NP}$-complete.

1 Introduction

Inheritance, like other forms of defeasible reasoning, sanctions uncertain conclusions. In unambiguous contexts, these conclusions are only as reliable as the general rules from which they are derived: concluding that a particular bird can fly is reasonable only if birds generally fly. Ambiguity introduces another kind of uncertainty: if we have evidence that Charlie is a bird, and other evidence that he is not, our reasoning is still more uncertain because our assumptions are questionable.

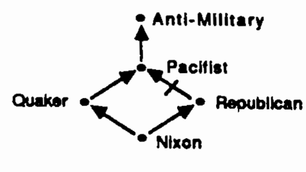

Figure 1: A modified "Nixon Diamond."

Credulous reasoning involves this second kind of uncertainty. A credulous reasoner sanctions any internally consistent state of the world. For example, in figure

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1 (a modificiation of Reiter and Criscuolo's "Nixon diamond" [?]), credulous inheritance allows either the conclusion that Nixon is a pacifist, or that Nixon is not a pacifist. In either case, the choice is arbitrary. Once we accept one of these conclusions, however, we can reason further: *if* we assume that Nixon is a pacifist, we can conclude that, like a typical pacifist, he is anti-military. Credulous inheritance permits us to believe that Nixon *is not* a pacifist, or *is* a pacifist, and therefore *is* anti-military, but *either way our beliefs must be internally consistent.*

While credulous inheritance offers many alternatives, skeptical inheritance yields a single, unambiguous set of conclusions for any inheritance hierarchy. *Ideally skeptical inheritance* supports exactly those conclusions true in every credulous extension. In the next section, we provide formal definitions of ambiguity, credulous extensions, and ideally skeptical inheritance. We then present Horty *et. al.*'s approach to skeptical inheritance [2] and discuss its over-credulity.[1] In section 4, we describe a linear-time algorithm for a plausible approximation of ideally skeptical inheritance. The failure of this algorithm to draw all the conclusions supported by ideally skeptical inheritance indicates the importance of reasoning by cases. In section 5, we give an algorithm for ideally skeptical inheritance based on a limited form of Boolean satisfiability.

2 Ambiguity and credulous extensions

An *inheritance hierarchy* $\Gamma = \langle V_\Gamma, E_\Gamma \rangle$ is a directed acyclic graph with edges labeled + or − and intended to denote "is-a" and "is-not-a" respectively. There is a ***positive path*** from a to x in Γ, written $a \rightarrow x$, if either $a = x$ or there is a node $b \in V_\Gamma$ with a positive path $a \rightarrow b$ in Γ and a positive edge $((b,x),+) \in E_\Gamma$. $a \not\rightarrow x$ if there is a node $b \in V_\Gamma$ with a positive path $a \rightarrow b$ in Γ and a negative edge $((b,x),-) \in E_\Gamma$. We use $\rightsquigarrow$ as a variable ranging over the set $\{\rightarrow, \not\rightarrow\}$, and say that $a \rightsquigarrow x$—there is a (positive or negative) path from a to x—if either $a \rightarrow x$ or $a \not\rightarrow x$. The path $a \rightsquigarrow x$ is an *inference*, while the sequence of edges $((a,b_0),+),\ldots,((b_n,x),\pm)$ is the *argument* supporting $a \rightsquigarrow x$.[2]

An inheritance hierarchy Γ ***supports*** a path $a \rightsquigarrow b$, written $\Gamma \rhd a \rightsquigarrow b$, if $a \rightsquigarrow b$ is in Γ ***and it is admissible***. In this paper, we take "admissible" to be vacuous: any path in Γ is supported. This is credulous inheritance at its most general, so that *any* conflicting paths are potentially ambiguous and result in corresponding credulous extensions. Previous approaches to inheritance have used ambiguity-resolving heuristics such as shortest path (Fahlman's NETL [1]), inferential distance (Tourtetzky's TMOIS [4]), and local inconsistency elimination (TMOIS). In our view, these ambiguity-resolving heuristics do not *define* what may be a credu-

[1]According to Horty (personal communication), general skeptical inheritance is intended to offer an unambiguous alternative to credulous inheritance; computing the intersection of the credulous extensions is only one way to accomplish this.

[2]Like Horty *et. al.*, the approach described in this paper is upwards reasoning. That is, inheritance works from the focus node upwards, rather than from the root downwards. The ramifications of upwards versus downwards reasoning are discussed in [5]. Upwards reasoning seems to have the appropriate properties for skeptical inheritance.

lous extension, but rather express *preferences among* credulous extensions. Shoham [3] makes a similar argument with respect to nonmonotonic logics and models. Although this paper follows the "preference criterion" approach, taking admissibility to include these ambiguity-resolving heuristics allows the definitions below to serve for the "definitional" approach as well.[3]

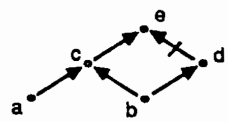

Figure 2: Unambiguous w.r.t. a, ambiguous w.r.t. b.

Ambiguity arises when two supported paths conflict. Formally, an inheritance hierarchy Γ is *ambiguous* w.r.t. a node a if there is some node $x \in V_\Gamma$ such that $\Gamma \rhd a \rightarrow x$ and $\Gamma \rhd a \not\rightarrow x$. In this case, we say that the ambiguity is *at* x. Ambiguity is always relative to a node: for example, the hierarchy in figure 2 is unambiguous w.r.t. a, but ambiguous w.r.t. b (at e). Credulous inheritance is a means of resolving these ambiguities.

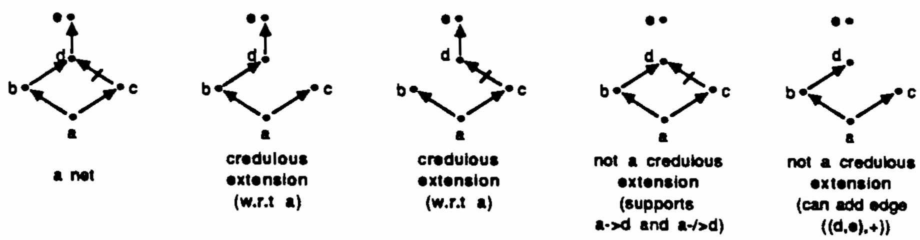

Figure 3: A network and some extensions.

A *credulous extension* of an inheritance hierarchy Γ with respect to a node a is a maximal unambiguous subhierarchy of Γ with respect to a: if $X^{\Gamma,a}$ is a credulous extension of Γ w.r.t. a, then there is no unambiguous subgraph of Γ supporting a proper superset of the paths that $X^{\Gamma,a}$ supports. An example of a network with several credulous extensions—and some non-extensions—is given in figure 3. If $X^{\Gamma,a}$ is a credulous extension of Γ w.r.t. a, a is called the *focus node* of $X^{\Gamma,a}$. If an extension $X^{\Gamma,a} \rhd a \rightarrow x$, x is *true in* $X^{\Gamma,a}$; similarly $a \not\rightarrow x$ and *false in* $X^{\Gamma,a}$.

In general, there may be many credulous extensions of Γ. If Γ is ambiguous w.r.t. a, there will be several extensions of Γ w.r.t. a. $\mathcal{X}(\Gamma, a)$ denotes the set of credulous extensions of Γ w.r.t. a. Further, $\mathcal{X}(\Gamma, a)$—the set of credulous extensions

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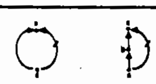

The current definition yields two credulous extensions for each of these networks: one in which $a \rightarrow x$, and another in which $a \not\rightarrow x$. If we restrict admissibility so that a path $a \rightsquigarrow x = ((a, x), \pm)$ is not admissible in Γ whenever $((a, x), \mp)$ is in Γ_E (and vice versa), the hierarchy in (i) has a single credulous extension w.r.t. a, supporting no paths. Using the definition of inferential distance given in [2], (ii) has a single credulous extension w.r.t. a, supporting $a \rightarrow b$ and $a \not\rightarrow x$.

w.r.t. a—may differ from $\mathcal{X}(\Gamma, b)$: ambiguities which arise in determining what a is may never arise in determining what b is. However, $\mathcal{X}(\mathcal{X}(\Gamma, a), b) = \mathcal{X}(\mathcal{X}(\Gamma, b), a)$: computing the credulous extensions of a hierarchy w.r.t. multiple focii is independent of the order in which they are considered.[4]

Credulous extensions involve making arbitrary choices. *Skeptical* inheritance is intended to select only those inferences that are not arbitrary. These are precisely the inferences which hold in any credulous extension, or possible state of the world. Formally, an inheritance hierarchy Γ skeptically permits a path $a \leadsto x$, written $\Gamma \mid\!\sim a \leadsto x$, if for every credulous extension $X^{\Gamma,a} \in \mathcal{X}(\Gamma, a)$, $X^{\Gamma,a} \mid\!\!> a \leadsto x$. $\mid\!\sim$ defines *ideally skeptical inheritance.*

Unfortunately, there is no "skeptical extension" corresponding to the definition of ideally skeptical inheritance. The closest we can come is $\Omega(\Gamma, a)$, the subgraph of Γ containing those edges that are in every credulous extension of Γ w.r.t. a: $\Omega(\Gamma, a) = \langle V_\Gamma, \bigcap_{X^{\Gamma,a} \in \mathcal{X}(\Gamma,a)} E_{X^{\Gamma,a}} \rangle$. $\Omega(\Gamma, a)$ supports exactly those arguments, or sequences of edges, that are in every credulous extension. However, there may be inferences $a \leadsto x$ that are supported in every credulous extension of Γ w.r.t. a, but have different supporting arguments in different extensions: some extensions may contain one sequence of edges from a to x, while other extensions contain a different sequence of edges from a to x. Hierarchies with this property are discussed in section 4. This means that, $\Omega(\Gamma, a)$—and any other "skeptical extension"—can support at most a subset of the inferences ideally skeptical inheritance admits; capturing the complete set of inferences requires reasoning by cases.

3 Ambiguity blocking inheritance

The first attempt at skeptical inheritance was taken by Horty *et. al.* in [2]. They argue that an ambiguous line of reasoning should not be allowed to interfere with other lines of reasoning. Thus, their definition of skeptical inheritance bars any path for which there is an uncontested counterargument. Essentially, their algorithm says:

> Starting from the focus node a, if a node x is ambiguous w.r.t. a in Γ, eliminate all edges into and out of x. When the entire hierarchy has been scanned, the remaining edges form a new network, $\mathrm{B}(\Gamma, a)$, which is unambiguous w.r.t. a. This is the ***ambiguity blocking*** skeptical extension of Γ; Horty *et. al.*'s algorithm concludes that a network Γ admits $a \leadsto x$ exactly when $\mathrm{B}(\Gamma, a) \mid\!\!> a \rightarrow x$.

While ambiguity blocking inheritance seems sound, it results in some anomalous conclusions. Consider, for example, figure 4. Ambiguity blocking inheritance on $\Gamma$with focus node a determines that e is ambiguous w.r.t. a, so it eliminates all

[4]It might therefore seem logical to define a credulous extension independently of any particular node; this would simply be a maximal subgraph with no ambiguity w.r.t. any node. Unfortunately, this is not terribly useful, as disambiguating one node (say, b in figure 2) can require eliminating perfectly good paths for another node (e.g. the path $a \rightarrow e$).

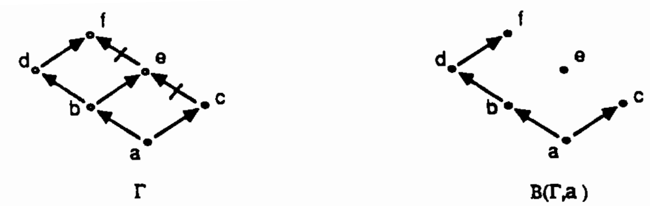

Figure 4: Applying ambiguity blocking inheritance to Γ w.r.t. a yields $\mathrm{B}(\Gamma, a)$.

edges to and from e. In particular, it eliminates the edge $((e, f), +)$, making f unambiguous w.r.t. a: $\mathrm{B}(\Gamma, a) \rhd a \rightarrow f$. This is certainly one possibility. But it is also possible that $a \rightarrow e$; and if $a \rightarrow e$, it is ambiguous whether $a \rightarrow f$—that is, a might not be an f. It is certainly not safe to assume from the ambiguity at e that the path $a \rightarrow f$ is always true. But this is precisely what ambiguity blocking inheritance does.[5]

A more severe anomaly follows from this first. Ambiguity blocking inheritance computes a kind of "parity" on the number of ambiguities in a path. According to Horty *et. al.*, the network in figure 5 is skeptical as to whether a is-a e or an i, but supports the conclusions that a is-a g and a j. Similarly, this net is skeptical about whether b or f is-a j, but allows the paths from a and d to j. Unlike the first anomaly, this result calls into question the intuitiveness of ambiguity blocking inheritance. In any case, ambiguity blocking inheritance is ***promiscuous***: there are inferences $a \rightsquigarrow x$ such that $\mathrm{B}(\Gamma, a) \rhd a \rightsquigarrow x$ but $\Gamma \mid\!\not\vdash a \rightsquigarrow x$.

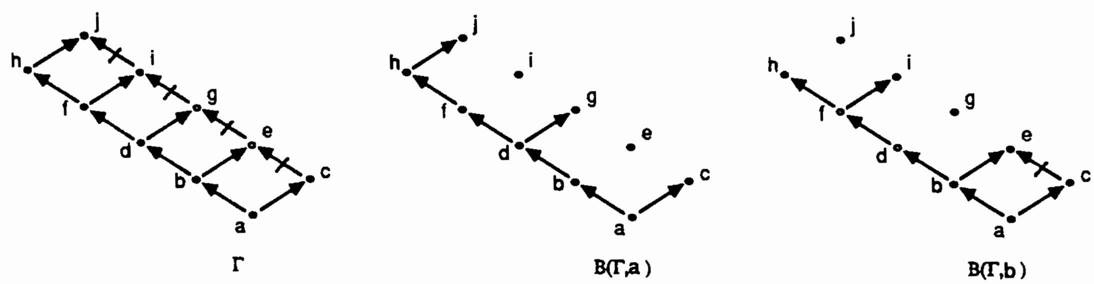

Figure 5: Ambiguity blocking inheritance computes "parity": a is not an e, is a g, is not an h, is a j,..... Also, a is a j, b is not a j, d is a j, f is not a j,.....

4 Ambiguity propagating inheritance

Unlike ambiguity blocking inheritance, *ambiguity propagating* skeptical inheritance allows ambiguous lines of reasoning to proceed. An argument thus cannot be certain unless there are *no* counterarguments; ambiguity blocking inheritance considers only *unambiguous* counterarguments. The details of an algorithm for ambi-

[5] Horty *et. al.*originally pointed out this difference between ambiguity blocking inheritance and ideally skeptical inheritance in [2] and b[5].

guity propagating inheritance are given in the appendix. This algorithm computes $\Pi(\Gamma, a)$, the ambiguity propagating skeptical extension of Γ w.r.t. a, in time $O(E_\Gamma)$:

> Starting from the focus node a, if a node x is ambiguous w.r.t. a in Γ, rather than eliminating all edges to and from x, retain x but mark it as ambiguous, and continue inheriting. Although paths to and from x won't be in $\Pi(\Gamma, a)$, they can still act as counterarguments and prevent other nodes from being unambiguous. $\Pi(\Gamma, a)$ is the subgraph of Γ with those edges $((x, y), \pm) \in E_\Gamma$ such that x and y are both unambiguous w.r.t. a.

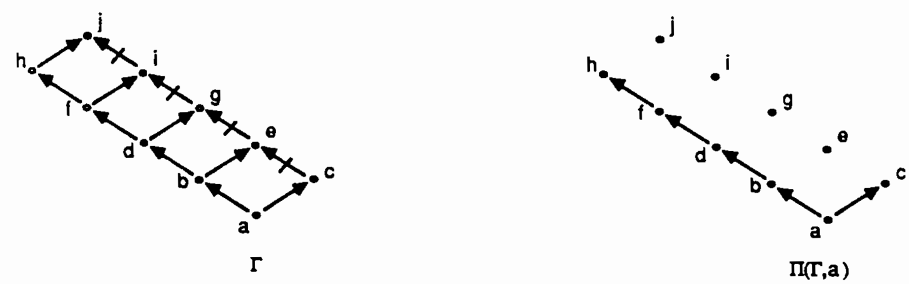

Figure 6: Ambiguity propagating inheritance draws no conclusions about whether a is-a e or g or i or j.

For example, the cascading ambiguities of figures 4 and 5, which gave ambiguity blocking inheritance difficulty, present no problem for ambiguity propagating inheritance. Figure 6 gives the ambiguity propagating extension for the network in figure 5.

In fact, $\Pi(\Gamma, a)$ never supports extra paths. $\Pi(\Gamma, a)$ is a subset of $\Omega(\Gamma, a)$, the "best approximation" to a skeptical extension, with the further property that $\Pi(\Gamma, a)$ and $\Omega(\Gamma, a)$ support $a \rightsquigarrow x$ for exactly the same nodes x. $\Pi(\Gamma, a)$ and $\Omega(\Gamma, a)$

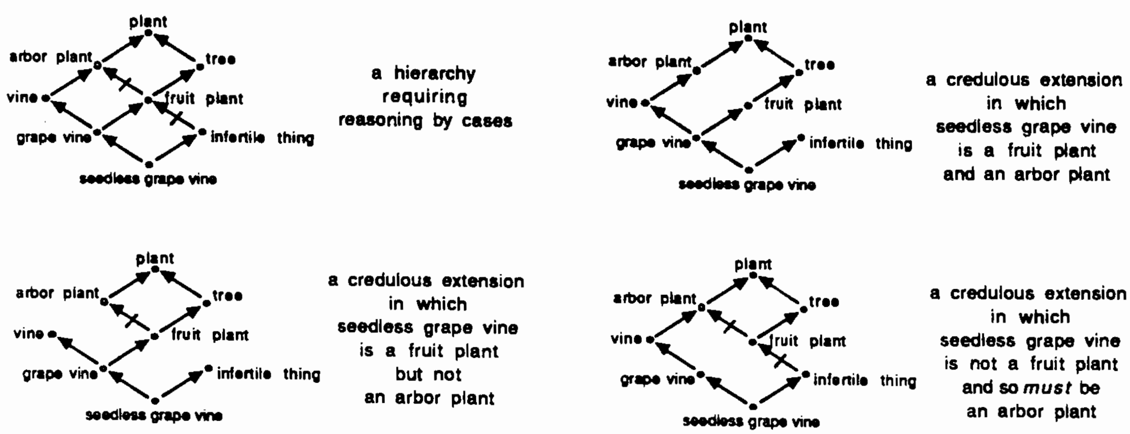

Figure 7: Whether a seedless grape vine is a fruit plant, or an arbor plant, it is certainly a plant!

both support those arguments, or sequences of edges, w.r.t. a that are true in all credulous extensions of Γ w.r.t. a. But there are some *inferences* $a \rightsquigarrow x$ that are supported by every credulous extension of Γ w.r.t. a, but have different supporting arguments in different extensions. In these circumstances, we need to do reasoning by cases.

For example, consider the hierarchy in figure 7. Every credulous extension w.r.t. *seedless grape vine* supports the inference *seedless grape vine*$\rightarrow$*plant*, so ideally skeptical inheritance concludes that *seedless grapes* are *plants*. Suppose, for example, that a *seedless grape vine* is a *fruit plant*; then it is a *plant*. Suppose that it is *not* a *fruit plant*; then it is unambiguously an *arbor plant*, and therefore a *plant*. In any state of the world, no matter how we resolve the ambiguities of the taxonomy, a *seedless grape vine* is a *plant*.

If we wish to determine what is true in all possible worlds, we cannot avoid the overhead of reasoning by cases. There are facts which are true in all credulous extensions, but which have no justification in the intersection of those extensions. This is why we can't generate a "skeptical extension"—no set of edges of Γ from *seedless grape vine* to *plant* is in every credulous extension, so no such path can be in the "skeptical extension." We can only detect these situations by, in effect, computing all of the credulous extensions. Since this might be quite expensive, ambiguity propagating inheritance may be the best tractable solution to the problem of computing the intersection of the credulous extensions.

5 Ideally skeptical inheritance

Ideally skeptical inheritance, applied to an inheritance hierarchy Γ, computes those inferences $a \rightsquigarrow x$ that have some supporting argument in every credulous extension of Γ w.r.t. a. We compute ideally skeptical inheritance by keeping track of the conditions under which a path $a \rightsquigarrow x$ has support. If $a \rightsquigarrow x$ always has support, then it is true in every credulous extension of Γ.

Let $C_{\mathbf{pos}}$ be the "positive children" of x—the nodes $p_i \in V_\Gamma$ with positive edges $((p_i, x), +) \in E_\Gamma$—and let $C_{\mathbf{neg}}$ be the "negative children" of x. Then if none of $C_{\mathbf{pos}}$ is true (i.e., not $a \rightarrow p_i$), x can't possibly be true: it has no support at all. If at least one of $C_{\mathbf{pos}}$ is true, but none of $C_{\mathbf{neg}}$ is true, then x is true: its true positive children provide it with a supporting argument. If at least one of $C_{\mathbf{pos}}$ is true, but at least one of $C_{\mathbf{neg}}$ is true, too, then x is ambiguous: it has some support, but there's also a counterargument.

We can phrase this as a constraint satisfaction problem:

a: The focus node is *always* true: $\Gamma \mathrel{|\!\!\sim} a \rightarrow a$.

$\overline{\bigvee_i C_{\mathbf{pos}_i}} \supset \overline{x}$: x can't be true if none of its positive children is.

$((\bigvee_i C_{\mathbf{pos}_i}) \wedge \overline{\bigvee_i C_{\mathbf{neg}_i}}) \supset x$: If x has support but no counterargument, it's true.

There is no constraint corresponding to the third condition; if x is ambiguous, it is unconstrained.

If we conjoin the second and third constraints, we get the biconditional

$$x \equiv (((\vee_i C_{\mathrm{pos}_i}) \wedge (\overline{\vee_i C_{\mathrm{neg}_i}})) \vee ((\vee_i C_{\mathrm{pos}_i}) \wedge (\vee_i C_{\mathrm{neg}_i}) \wedge x))$$

This is the condition under which $\Gamma \mid\!\sim a \rightarrow x$. In the following algorithm for computing ideally skeptical inheritance, we label x with a formula corresponding to this constraint. The notation $[\![x]\!] = [y]$ means that the propositional variable y is the label of the node named x. The usual rules w.r.t. connectives, $\top$, and $\perp$, apply.

The label of x describes exactly those conditions under which the focus node a is-a x. If x is labeled $\top$, it is thus always true—$\Gamma \mid\!\sim a \rightarrow x$. If x is labeled $\perp$, its label can never be satisfied, so there is no credulous extension of Γ w.r.t. a that supports $a \rightarrow x$. If a node x is labeled φ, x is true ($a \rightarrow x$) in exactly those credulous extensions where φ is true. For example, in the labeling of figure 1, [[anti-military]] = [pacifist]; that is, Nixon is anti-military in exactly those extensions in which he is also a pacifist.

To determine the properties of node a:[6]

1. Label $[\![a]\!] = [\top]$.
2. Label all leaves (other than a, if a is a leaf) $[\perp]$.
3. For a generic node, x, with positive children C_{pos_i} and negative children C_{neg_j}, label

$$[\![x]\!] = [((\bigvee_i [\![C_{\mathrm{pos}_i}]\!]) \wedge (\overline{\bigvee_i [\![C_{\mathrm{neg}_i}]\!]}) \wedge \top) \vee ((\bigvee_i [\![C_{\mathrm{pos}_i}]\!] \vee \perp) \wedge (\bigvee_i [\![C_{\mathrm{neg}_i}]\!] \vee \perp) \wedge x))]$$

Figure 8 gives the skeptical labeling of the hierarchy of figure 7.

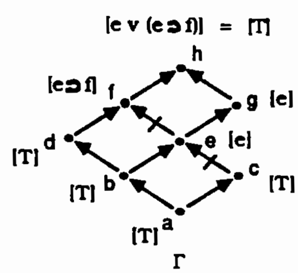

Figure 8: a is an x whenever the label of x has truth value $\top$.

It is an open problem whether ideally skeptical inheritance—computing the intersection of the credulous extensions—is $\mathcal{NP}$-hard.[7] The problem bears some relation to Boolean satisfiability, but the class of formulae that can appear as labels of nodes in Γ is

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[6] This algorithm computes the conditions under which $a \rightarrow x$. The condition under which $a \not\rightarrow x$ is given by $(\vee_i [\![C_{\mathrm{neg}_i}]\!]) \wedge (\overline{\vee_i [\![C_{\mathrm{pos}_i}]\!]})$.

[7] Actually, the problem of determining whether a path $a \rightsquigarrow x$ is in $\Omega(\Gamma,\ )$would be in co-$\mathcal{NP}$; the relevant reduction might come from non-tautology of boolean formulae to the exclusion of a path.

somewhat restricted. In any case, the complexity of such an algorithm is in terms of the ambiguity, rather than the size, of the network. While it is possible to construct networks with high degrees of ambiguity—indeed, several pathological examples are given here—it seems likely that in practice, most networks will have a much lower degree of ambiguity relative to their size. Further, it may be possible to exploit locality of ambiguity, since it is interconnectivity that results in increases in complexity.

6 Acknowledgements

Leora Morgenstern, David Touretzky, Jeff Horty, and the (individual and collective) members of Brown University AI group, provided much-appreciated assistance in this work.

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A Computing ambiguity propagating inheritance

The algorithm given here is one of a family which compute ambiguity propagating inheritance. It computes the complete set of paths supported by ambiguity propagating inheritance—$\Pi(\Gamma, a)$, for each node $a \in V_\Gamma$—in a single pass requiring time $O(|V_\Gamma||E_\Gamma|)$ for an inheritance hierarchy $\Gamma = \langle V_\Gamma, E_\Gamma \rangle$. Whenever possible, it draws conclusions about the truth of a node in *all* extensions; if this is not possible, it attempts to reach conclusions about *some* extensions; otherwise, it draw no conclusions about a node.

The three truth values used by this algorithm are $\top$ (true), $\perp$ (false), and $+$ (ambiguous). Each node in the hierarchy Γ is labeled with a set of truth values. If a is the focus node and x receives the label...

$\{\top\}$: Every credulous extension of Γ supports $a \rightarrow x$.

$\{\top, +\}$: There is some credulous extension $X^{\Gamma,a}$ of Γ such that $X^{\Gamma,a} \triangleright a \rightarrow x$.

$\{\top, +, \perp\}$: There is some credulous extension $X_1^{\Gamma,a}$ of Γ w.r.t. a such that $X_1^{\Gamma,a} \triangleright a \rightarrow x$; there is another credulous extension $X_2^{\Gamma,a}$ of Γ w.r.t. a such that $X_2^{\Gamma,a} \triangleright a \not\rightarrow x$.

$\{+, \perp\}$: There is some credulous extension $X^{\Gamma,a}$ of Γ such that $X^{\Gamma,a} \triangleright a \not\rightarrow x$.

$\{\perp\}$: Every credulous extension of Γ supports $a \not\rightarrow x$.

$\{\}$: No credulous extension of Γ supports either $a \rightarrow x$ or $a \not\rightarrow x$.

In this marking scheme, $+$ never appears alone in a label.

The notation $[\![x]\!]$ denotes the label of node x. A node x is *possible* if $\top \in [\![x]\!]$, and *definite* if $[\![x]\!] = \{\top\}$.

Let Γ be an inheritance hierarchy. To compute the ambiguity propagating skeptical extension of Γ w.r.t. a focus node a:

- Mark Γ recursively, marking each node only after all of its children have been marked.
- Mark $[\![a]\!] = \{\top\}$; mark all (other) leaves, and any children of the focus node, $\{\}$.
- To mark a node x:

  If there is a possible node p with positive edge $((p, x), +) \in E_\Gamma$, and a possible node n with negative edge $((n, x), -) \in E_\Gamma$, mark $[\![x]\!] = \{\top, +, \perp\}$

  else if there is a definite node p with positive edge $((p, x), +) \in E_\Gamma$, mark $[\![x]\!] = \{\top\}$

  else if there is a possible node p with positive edge $((p, x), +) \in E_\Gamma$, mark $[\![x]\!] = \{\top, +\}$

  else if there is a definite node n with negative edge $((n, x), -) \in E_\Gamma$, mark $[\![x]\!] = \{\perp\}$

  else if there is a possible node n with negative edge $((n, x), -) \in E_\Gamma$, mark $[\![x]\!] = \{+, \perp\}$

  else mark $[\![x]\!] = \{\}$.

The complete set of inferences supported by Γ under ambiguity propagation can be computed in parallel by using a separate set of truth values for each node, i: $\{\top_i, +_i, \bot_i\}$. Each node i is initially marked with $[i] = \{\top_i\}$. Labels are propagated according to the algorithm given above, save that only truth values with the same subscript can interact. Labels with a given subscript are computed for each node only if they are mentioned in one of that node's children.