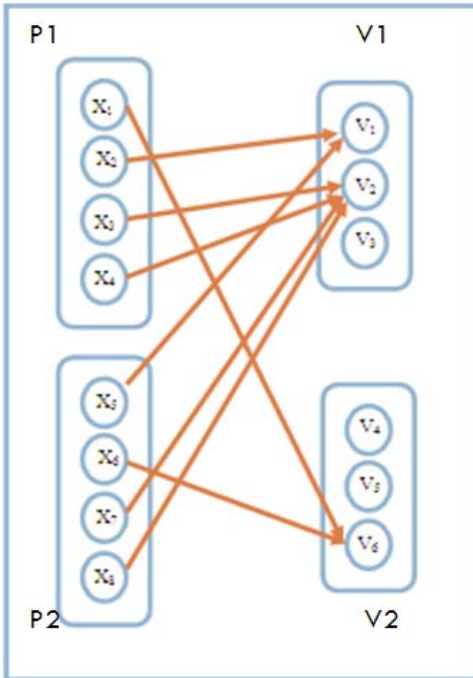


STATIC STRUCTURAL SYMMETRY BREAKING (SSSB) FOR PIECEWISE SYMMETRIC VARIABLES AND VALUES

Example: Consider a constraint satisfaction problem with variables $X = (X_1 \dots X_8)$ and domains $D(X) = \{v_1 \dots v_6\}$. Assume that there are;

- I. Two variable partitions $P_1 = (X_1, \dots, X_4)$ and $P_2 = (X_5, \dots, X_8)$.
- II. Two value partitions $V_1 = (v_1, v_2, v_3)$ and $V_2 = (v_4, v_5, v_6)$.



We define the *signature* of a value v to be the tuple that counts for each variable partition, the number of variables x that are assigned to the value v [Flener et al, 2006].

For instance, under the assignment that is shown on the left the signatures for each value are;

- I. $v_2 \in V_1$ has signature (2,2)
- II. $v_1 \in V_1$ has signature (1,1)
- III. $v_6 \in V_2$ has signature (1,1)

We can break variable symmetry for each partition by requiring that variables with smaller indices take smaller or equal values. Further, we exploit the signature abstraction to break value symmetries. We require that smaller values have lexicographically larger or equal signatures within each value partition. We use global cardinality constraint (GCC) to count the number of times a value is taken in each variable partition. In our example, SSSB introduces the following constraints:

- I. $X_1 \leq X_2 \leq X_3 \leq X_4$ and $X_5 \leq X_6 \leq X_7 \leq X_8$
- II. $\text{GCC} ([X_1, \dots, X_4], [v_1, \dots, v_6], [\text{count}_{v_1}^1, \dots, \text{count}_{v_6}^1])$
- III. $\text{GCC} ([X_5, \dots, X_8], [v_1, \dots, v_6], [\text{count}_{v_1}^2, \dots, \text{count}_{v_6}^2])$
- IV. $(\text{count}_{v_1}^1, \text{count}_{v_1}^2) \geq_{\text{LEX}} (\text{count}_{v_2}^1, \text{count}_{v_2}^2) \geq_{\text{LEX}} (\text{count}_{v_3}^1, \text{count}_{v_3}^2)$
- V. $(\text{count}_{v_4}^1, \text{count}_{v_4}^2) \geq_{\text{LEX}} (\text{count}_{v_5}^1, \text{count}_{v_5}^2) \geq_{\text{LEX}} (\text{count}_{v_6}^1, \text{count}_{v_6}^2)$

Cornflower Library offers SSSB method as an extension to Ilog Solver. The user can also specify the partitioning order of the signatures.