On the Distortion of Voting with Multiple Representative Candidates







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Voting and Social Choice

• *n* candidates: *A*, *B*, *C*

- Population of voters: Each ranks all candidates A > B > C
- Voting rule selects a winner based on voters' preferences

Can we compare voting rules **quantitatively**?





If the voting rule is Plurality (each voter casts one vote)





Fix a voting rule.

Candidates drawn i.i.d. from voter population.



Model

- Metric Space
 - Voters rank closer candidates higher
- Representative Candidates
 - Candidates drawn i.i.d. from the voter distribution.

Can we bound the expected distortion?

Our Results

- A clean and tight characterization of **positional scoring rules** that have **constant distortion**.
 - Candidates receive points based on their rank position on each ballot and the candidate with the most points overall wins.
 - Independent of the number of candidates and the metric space.

Positional Scoring Rules

- Plurality: (1, 0, ..., 0)
- Veto: (1, ..., 1, 0)
- Borda: $(n 1, n 2, ..., 1, 0) \Rightarrow (1, \frac{n-2}{n-1}, ..., \frac{1}{n-1}, 0)$
- *k*-Approval: (1, ..., 1, 0, ..., 0)
- Dowdall: $(1, 1/2, 1/3, \dots, 1/n) \Rightarrow \cdots$

Example: Plurality



Limit Scoring Rule







Our Results

Voting Rule	E [Distortion]
Plurality	$\Theta(n)$
Dowdall	$\Theta(n)$
k-Approval, $k = O(1)$	$\Theta(n)$
Borda	Θ(1)
k-Approval, $k = n/2$	Θ(1)
Veto	$\Theta(n)$

Our Results

Voting Rule	E [Distortion] (This paper)	Worst-Case [Anshelevich et al.]
Plurality	$\Theta(n)$	$\Theta(n)$
Dowdall	$\Theta(n)$	
k-Approval, $k = O(1)$	$\Theta(n)$	
Borda	$\Theta(1)$	$\Theta(n)$
k-Approval, $k = n/2$	Θ(1)	
Veto	$\Theta(n)$	\sim

Our Contributions

- Clean and tight characterization.
- Metric Space + Representative Candidates ⇒
 Allow us to distinguish voting rules that feels the same under classic axioms, or even under worst-case metric voting.
- Average-case vs. worst-case voting

Open Questions

- What can we say for voting rules that are not positional?
- How robust are the results to other notions of cost?
- Voters rank the candidates by perceived location, but the cost is evaluated by actual location.
 Can we bound the distortion now?