Query Complexity of Approximate Nash Equilibria

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Nash Equilibrium

		Presenter	
		Put effort into presentation (E)	Do not put effort (NE)
Audience	Pay attention (A)	$2, 2$	$-8, -10$
	Do not pay attention (NA)	$0, -3$	$0, 0$

- Players cannot improve their payoffs by deviation:

(A, E)   (NA, NE)   ((1/5 A, 4/5 NA), (4/5 E, 1/5 NE))

Multi-Player Games

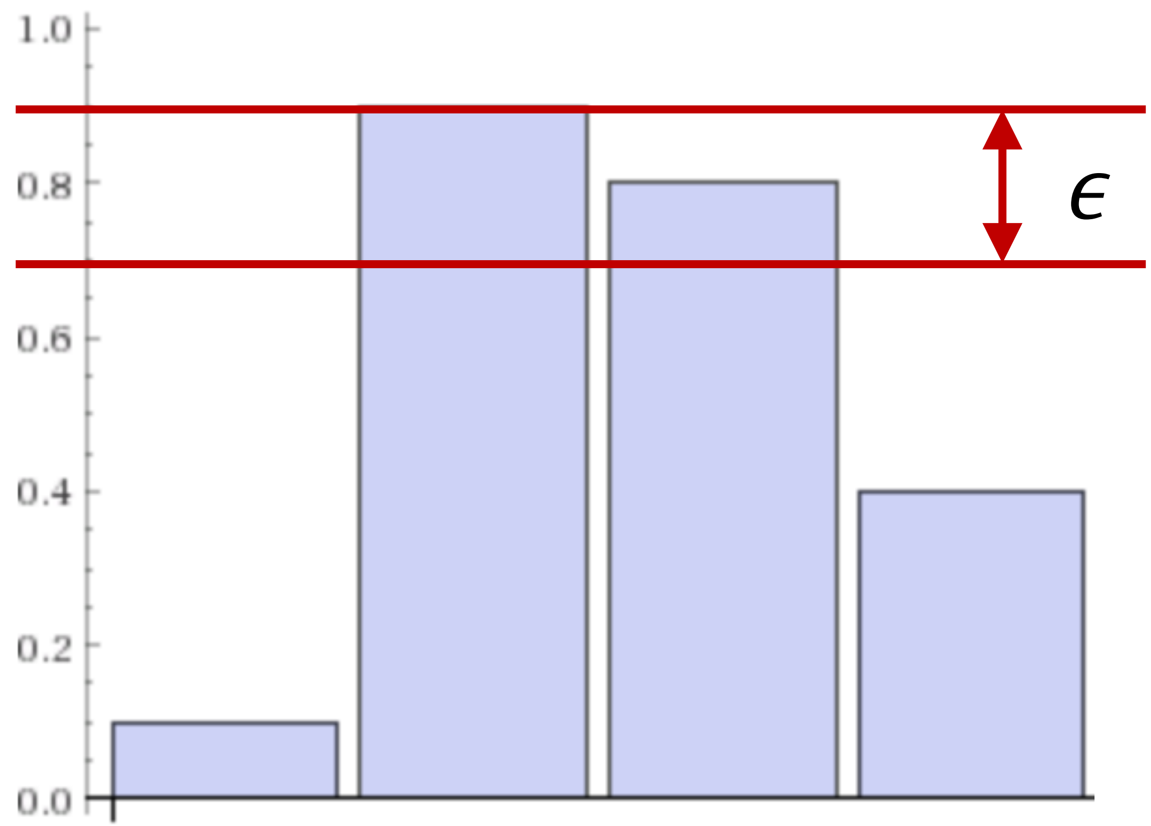
- n-player two-strategy games
  - Normal form: description size $= n2^n$

- Query complexity
  - Oracle access to payoff functions

ANE vs. WSNE

- ϵ-approximate Nash equilibrium (ANE)
  - Players can gain at most ϵ by unilateral deviation
- ϵ-well-supported Nash equilibrium (WSNE)
  - Any strategy that is used with non-zero probability by a player must be an ϵ-best response

ANE vs. WSNE



Recap

- Input: An n-player two-strategy game G (payoff oracle)
- Output: An ϵ-ANE (ϵ is a small constant in this talk)
- Goal: Minimize the number of queries
  - 2^n queries gives full information

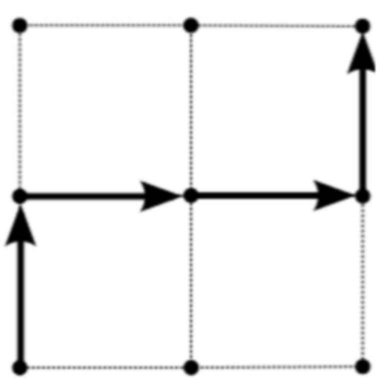
Related Work

	Solution Concept	# of Queries
[Hart and Nisan '13]	(1/2)-ANE (deterministic)	$2^{\Omega(n)}$
	exact NE	
[Babichenko '14]	ϵ-WSNE	$2^{\Omega(n)}$
Our Result	ϵ-ANE	$2^{\Omega(n/\log n)}$

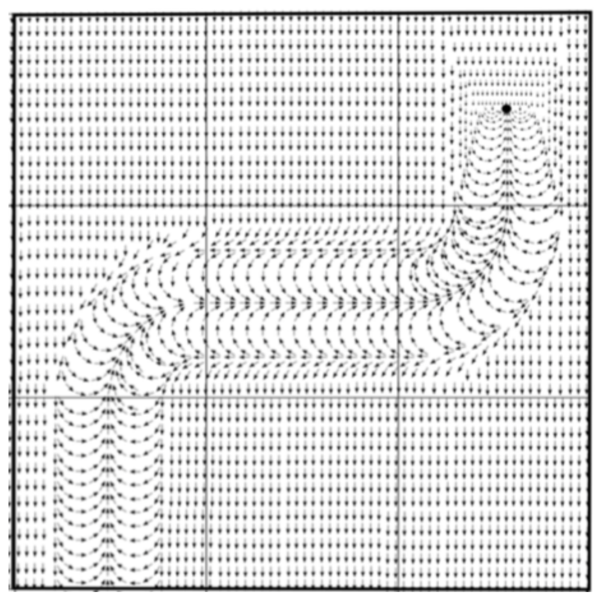
Outline

[Babichenko '14]: $\mathrm{QC}_p\big(\mathrm{WSNE}(n,\epsilon)\big) = 2^{\Omega(n)}$

- p = success probability = $2^{-\Theta(n)}$

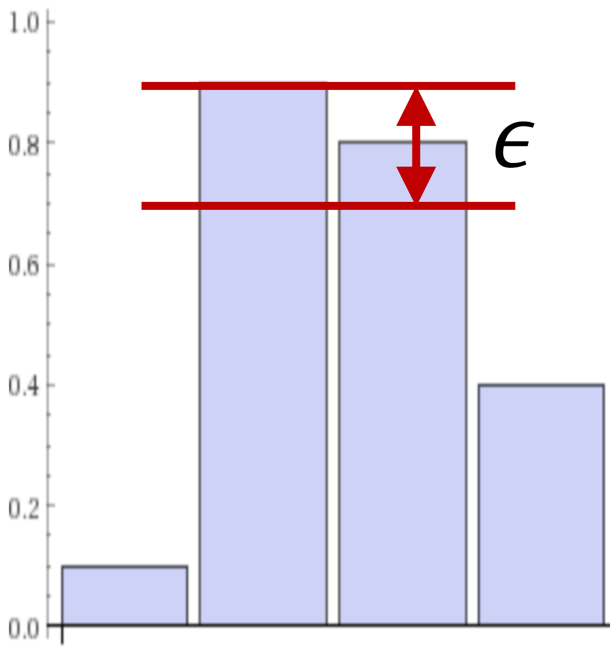


$\leq$



$\leq \mathrm{WSNE}(n,\epsilon) \leq \mathrm{ANE}(n,\epsilon)$

Challenge: WSNE vs. ANE



If everyone plays sub-optimally with probability ϵ, then in aggregate about ϵn players are playing arbitrarily, making the outcome of the game unpredictable.

Our Reduction

$$\mathrm{WSNE}(n, \epsilon) \le \sout{\mathrm{ANE}(n, \epsilon)}$$

$$\le \mathrm{ANE}(8n \log(n/\epsilon), \epsilon/8)$$

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$$\mathrm{WSNE}(n,\epsilon) \le \mathrm{ANE}(8n\log(n/\epsilon), \epsilon/8)$$

$$\mathrm{QC}_p\big(\mathrm{WSNE}(n,\epsilon)\big) = 2^{\Omega(n)}, \quad p = 2^{-\Theta(n)}$$

$$\Downarrow$$

$$\mathrm{QC}_p\big(\mathrm{ANE}(n,\epsilon)\big) = 2^{\Omega(n/\log n)}, \quad p = 2^{-\Theta(n/\log n)}$$

Our Reduction

$$\mathrm{WSNE}(n, \epsilon) \leq \mathrm{ANE}(8n \log(n/\epsilon), \epsilon/8)$$

- Given a game G, construct G':
  - Replace each player with $O(\log n)$ agents
  - Use majority voting to decide the strategy of each group
  - Payoffs are determined using payoff functions of G

Our Reduction

- In an $(\epsilon/4)$-ANE of G'
  - If strategy 0 is not ϵ-best response for player $i \Longrightarrow$

    Each agent in group i can put probability at most $1/4$ on 0
  - $O(\log(n/\epsilon))$ Players + Tail bounds

    $\Longrightarrow \Pr[$Majority of group $i =$ strategy $0] \leq \epsilon/n$

- Set all $\leq \epsilon/n$ probabilities to zero

Open Problems

- $2^{\Omega(n/\log n)}$?
  - $2^{\Omega(n)}$ [Rubinstein '16]

- Is there a reduction with $O(n)$ players?
  - Fault-tolerant computation: Simulate an n-gadget circuit with "cheap" gadgets that malfunction with probability ϵ