Of the People: Voting Is More Effective with Representative Candidates

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Is democracy more effective when the candidates for office are of the people?

- Drawn from the population, say uniformly at random.

Metric and Distortion

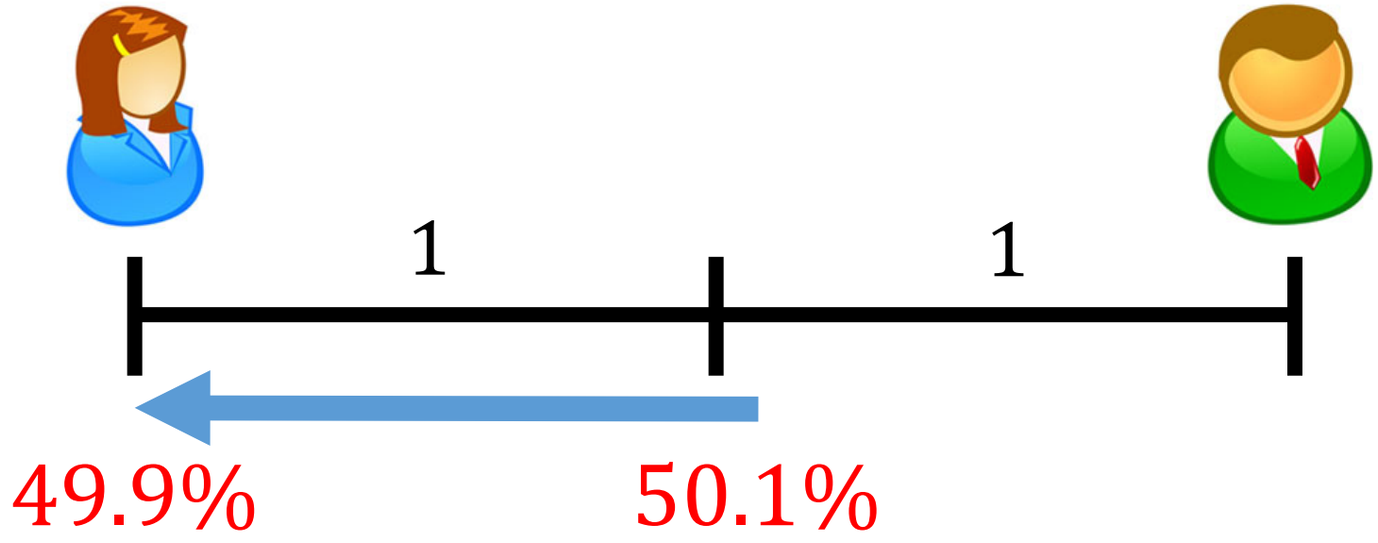
The candidates/voters are embedded within a common metric space [Black '48, Downs '57, Moulin '80, Barberà et al. '93, Merrill and Grofman '99].

Cost of a candidate $\stackrel{\text{def}}{=}$ average distance to the voters.

[Procaccia and Rosenschein '06, Caragiannis and Procaccia '11, Boutilier et al. '15, Anshelevich Bhardwaj Postl '15].

$$\text{Distortion of an election} \stackrel{\text{def}}{=} \frac{\text{cost of the winner}}{\text{cost of the optimal candidate}}.$$

Example of Distortion 3 [Anshelevich et al.]



Cost() ≈ 0.5

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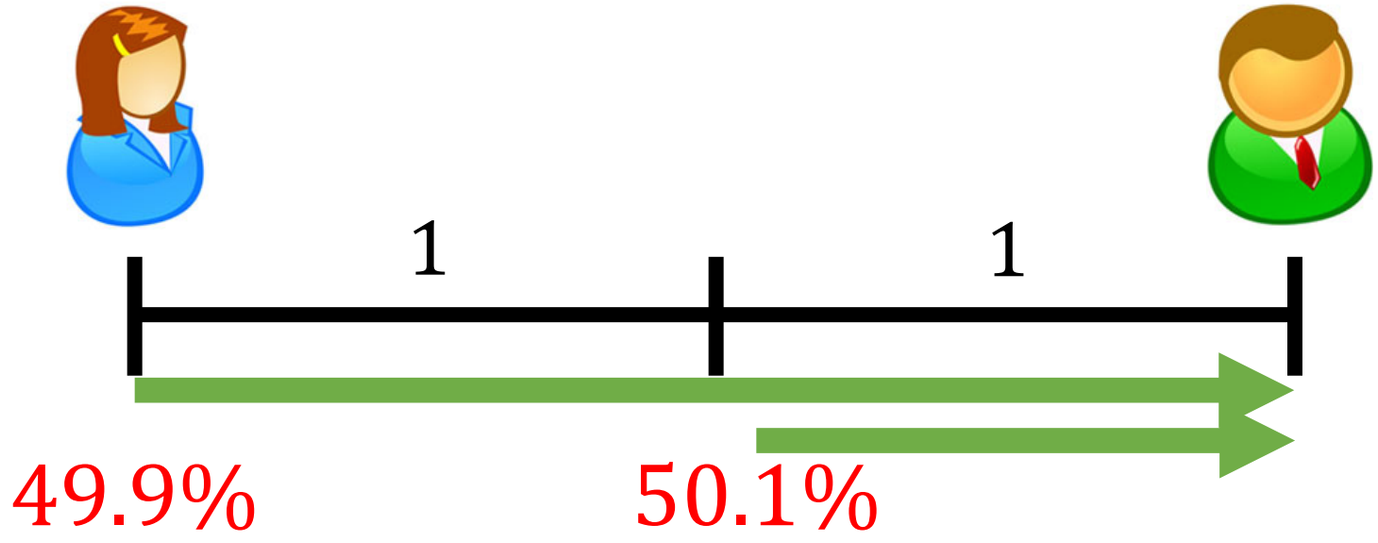

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Cost() ≈ 1.5

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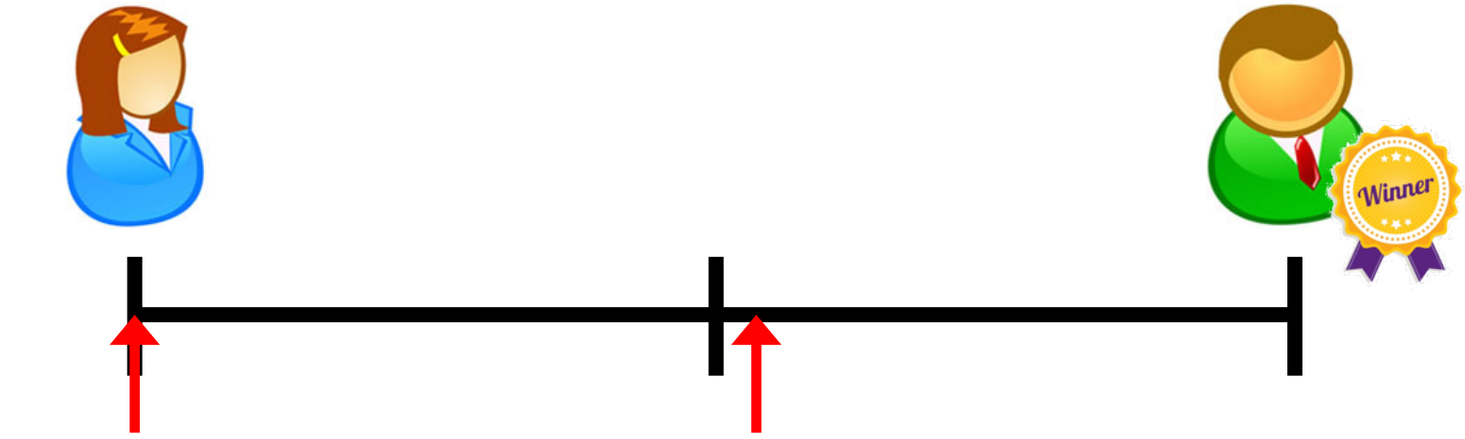
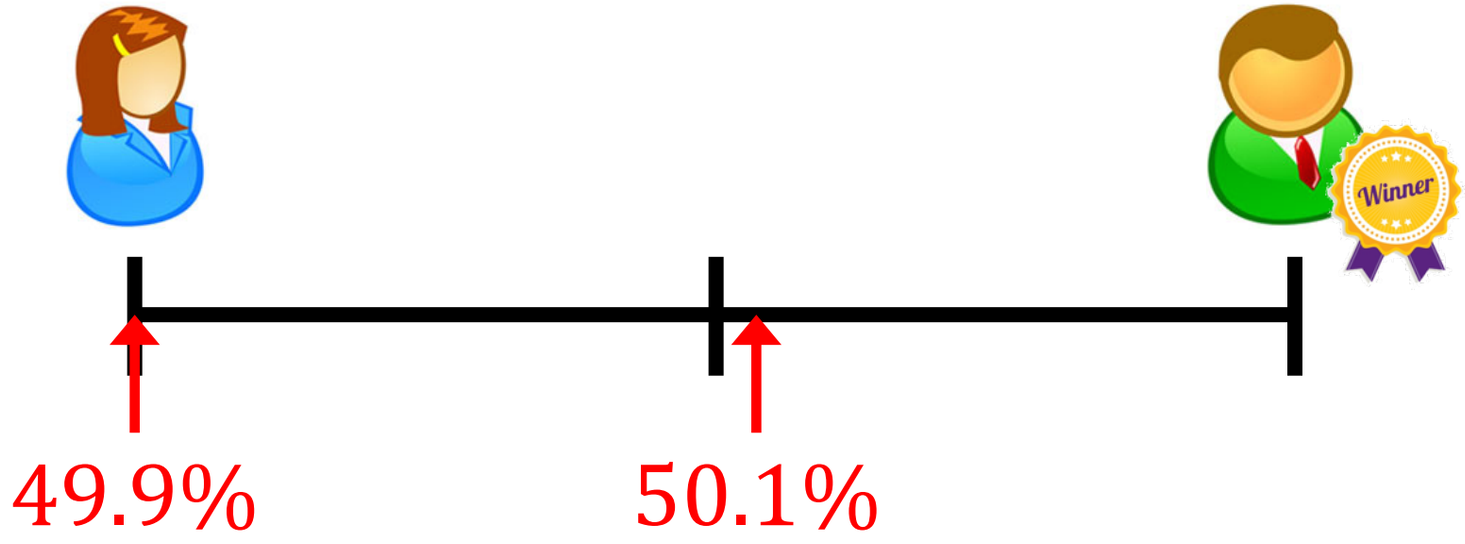


 wins the election, 50.1% against 49.9%.

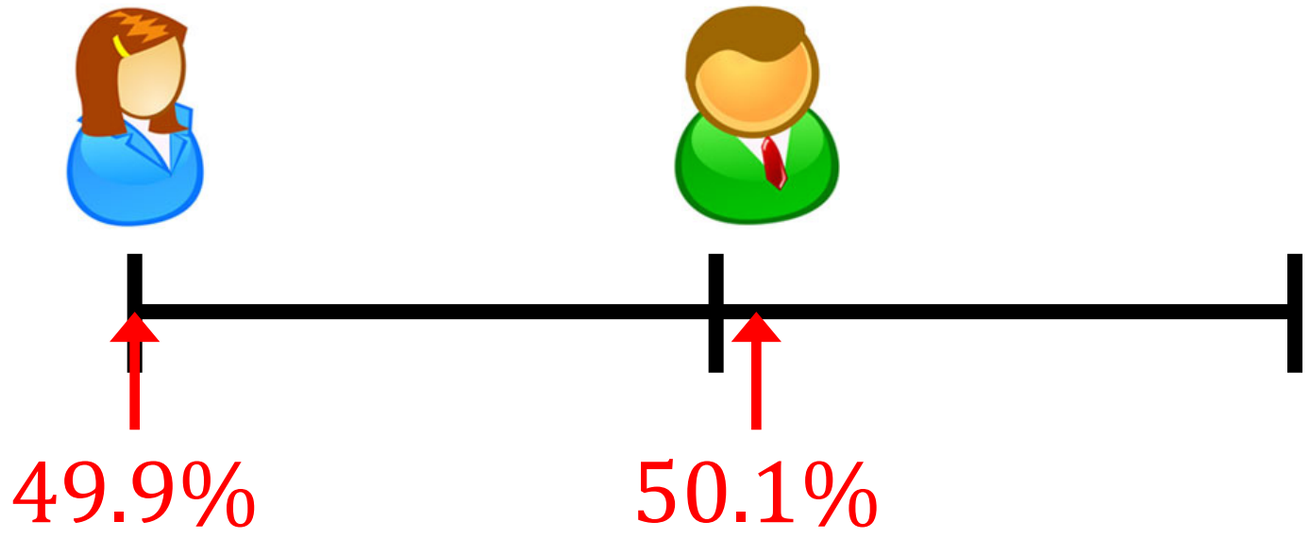
$$\text{Distortion} \stackrel{\text{def}}{=} \frac{\text{cost}(\,)}{\text{cost}(\,)} \approx 3$$

Example of Distortion 3 [Anshelevich et al.]



Reason for high distortion:  is not **of the people.**

Of the People



Had we drawn two candidates from the population,

the winner would always be the socially optimal choice.

Of the People $\Rightarrow$ Smaller Distortion?

Does social welfare improve when candidates are i.i.d. from the population of voters?

Focus on:

- Two candidates.
- Majority rule.

We assume the candidates are drawn i.i.d from a distribution $\boldsymbol{p}$ supported on the metric space.

Our Results

Given candidates drawn from $\boldsymbol{p}$, we study the expected distortion.

- $\boldsymbol{p}$ is arbitrary [non-representative].
- $\boldsymbol{p}$ is uniform over the voters [representative].

	Representative	Non-Representative
Line Metric	$4 - 2\sqrt{2} \approx 1.1716$	2
General Metric	$[1.5, 2 - \frac{1}{652})$	2

Our Results

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Takeaway Message

Voting is more effective with representative candidates.

Exact improvement depends on the complexity of the metric space.

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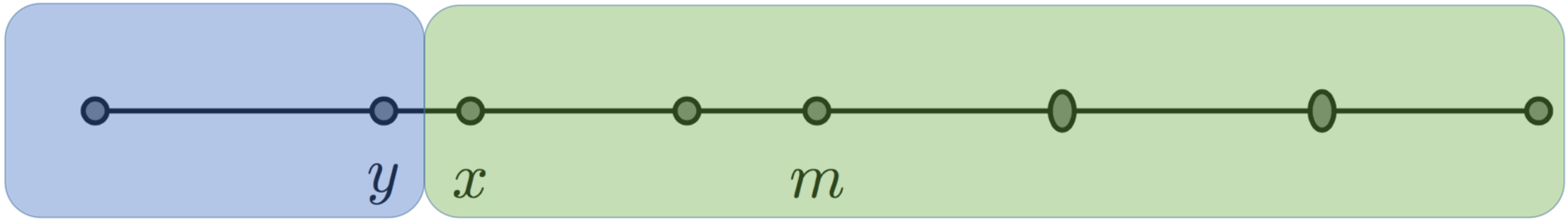
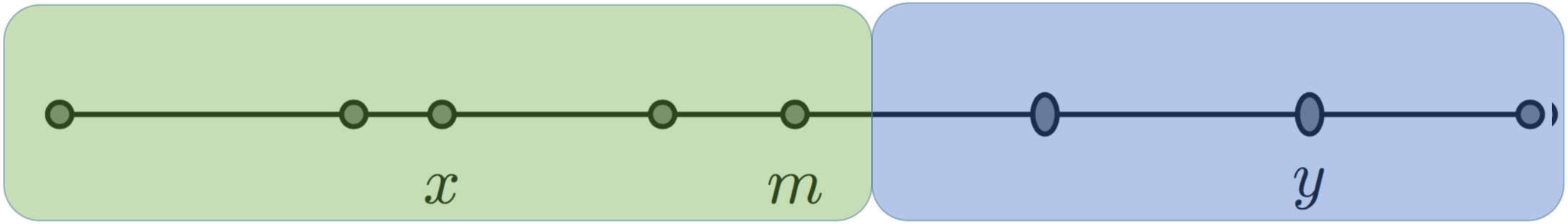
Voting on the Line: Structural Results



The median voter m.

1) The candidate closer to the median m wins the election.



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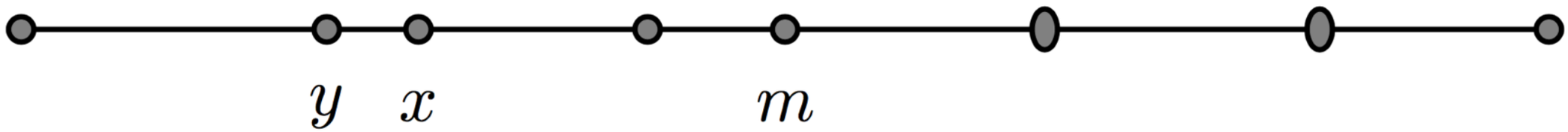
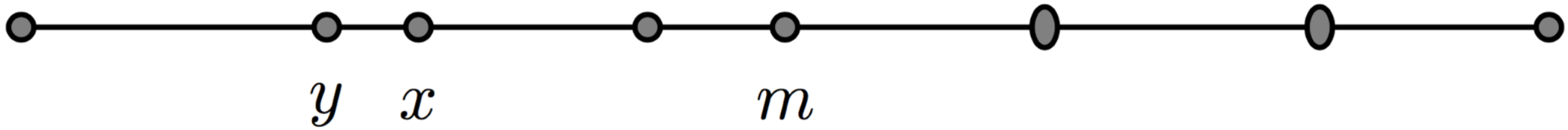


1) The candidate closer to the median m wins the election.

2) For two candidates x, y on the same side of the median m, the one closer to m has smaller social cost.

Intuition: More than half of the population need to first get to x before they can get to y.

Voting on the Line: Structural Results



1) The candidate closer to the median m wins the election.
2) For two candidates x, y on the same side of the median m, the one closer to m has smaller social cost.

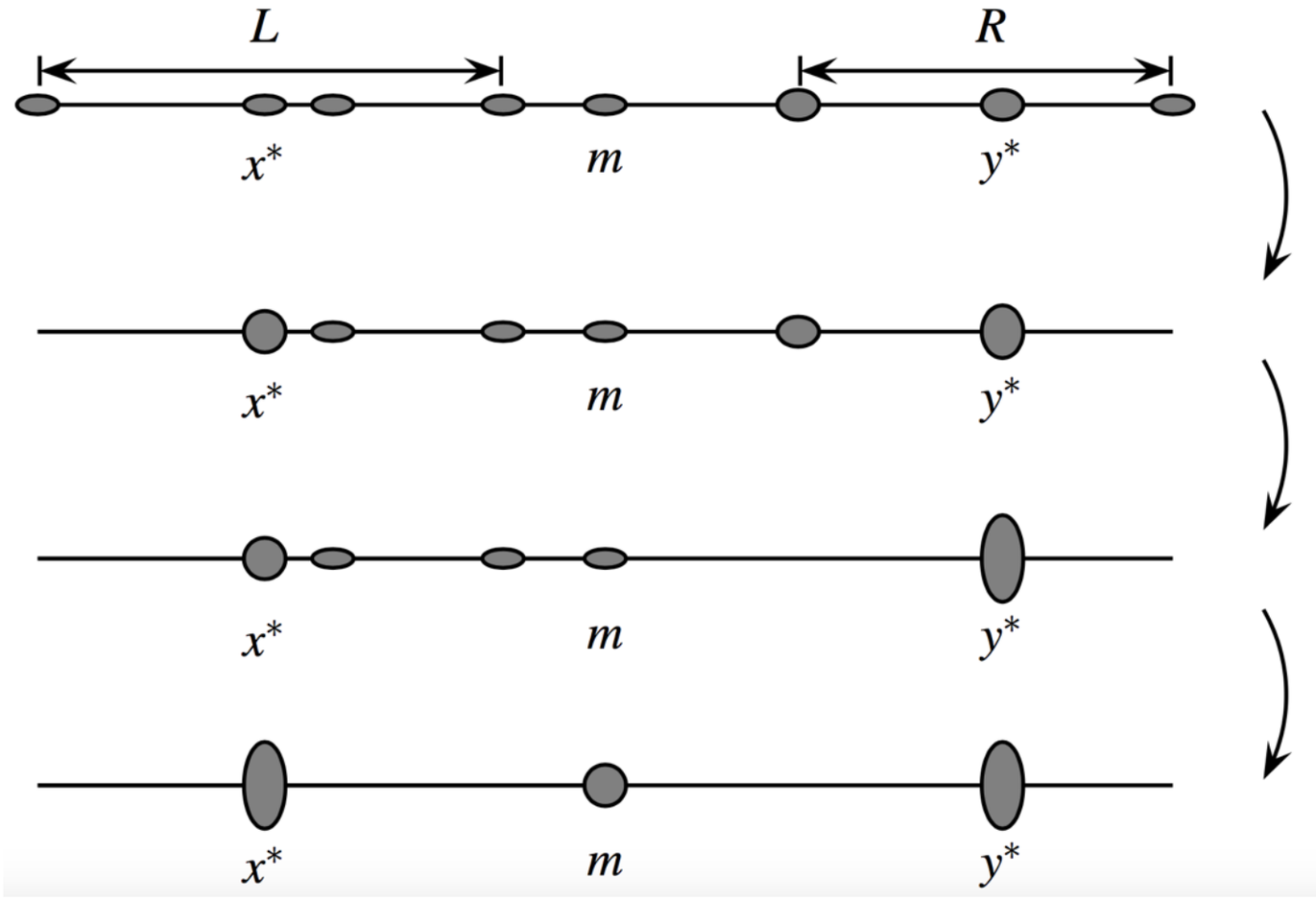
(1) + (2) $\Longrightarrow$ If both candidates are on the same side of m, then majority voting elects the socially better candidate.

Voting on the Line (≈ 1.17)

Given any instance with support size larger than 3,
we can reduce its support to 3 using a series of operations,
without decreasing the distortion.

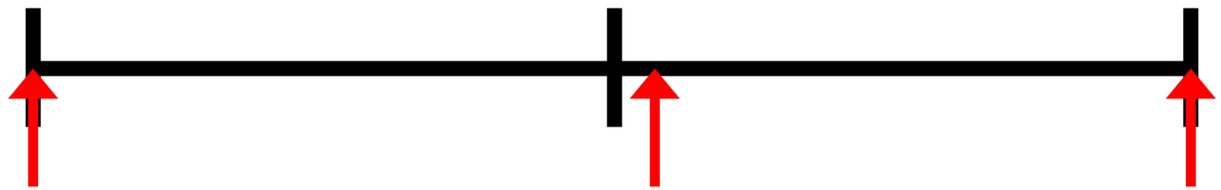
When shifting the probabilities, we use a global argument to show that the operation increases the distortion **on average**.

Voting on the Line (≈ 1.17)



Voting on the Line (≈ 1.17)

For support 3 distributions, we can optimize the locations and probabilities of these 3 points.



$p_1 \approx 49.9\%$ $p_2 \approx 29.3\%$ $p_3 \approx 20.7\%$

$$\text{Distortion} = 2p_1p_3 \cdot \frac{cost_3}{cost_1} + (1 - 2p_1p_3) \cdot 1 \approx 1.17.$$

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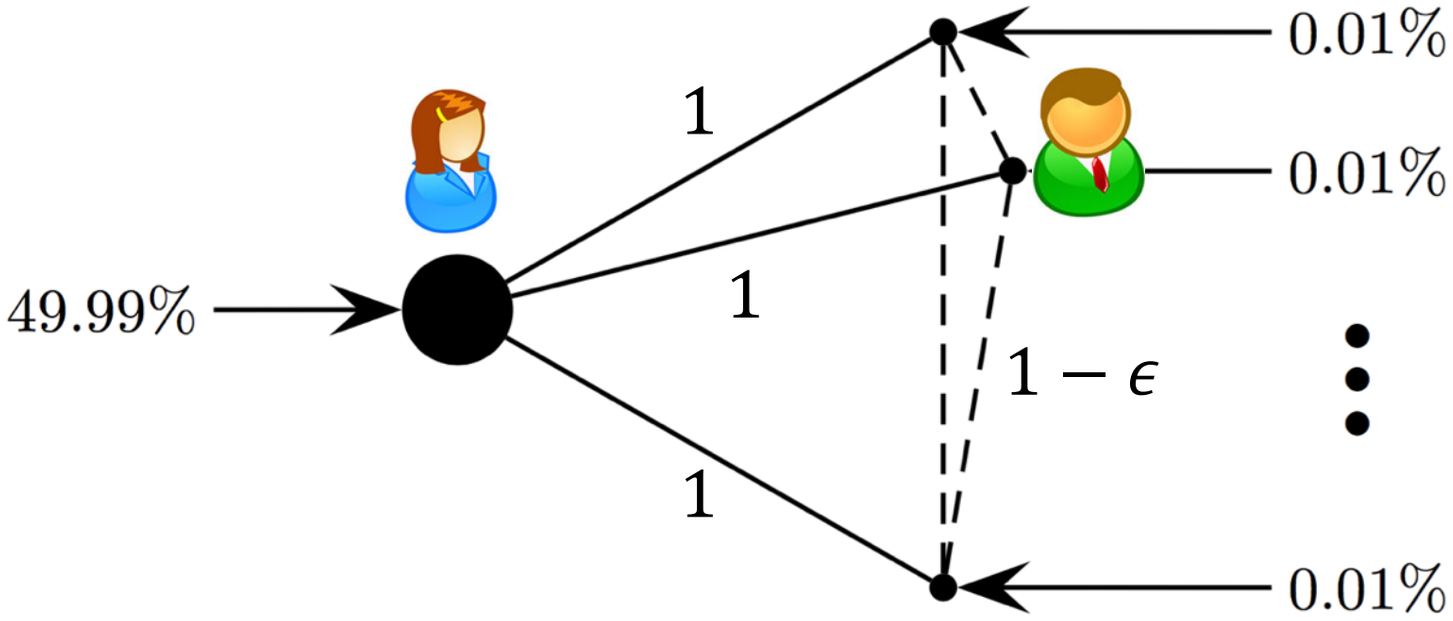
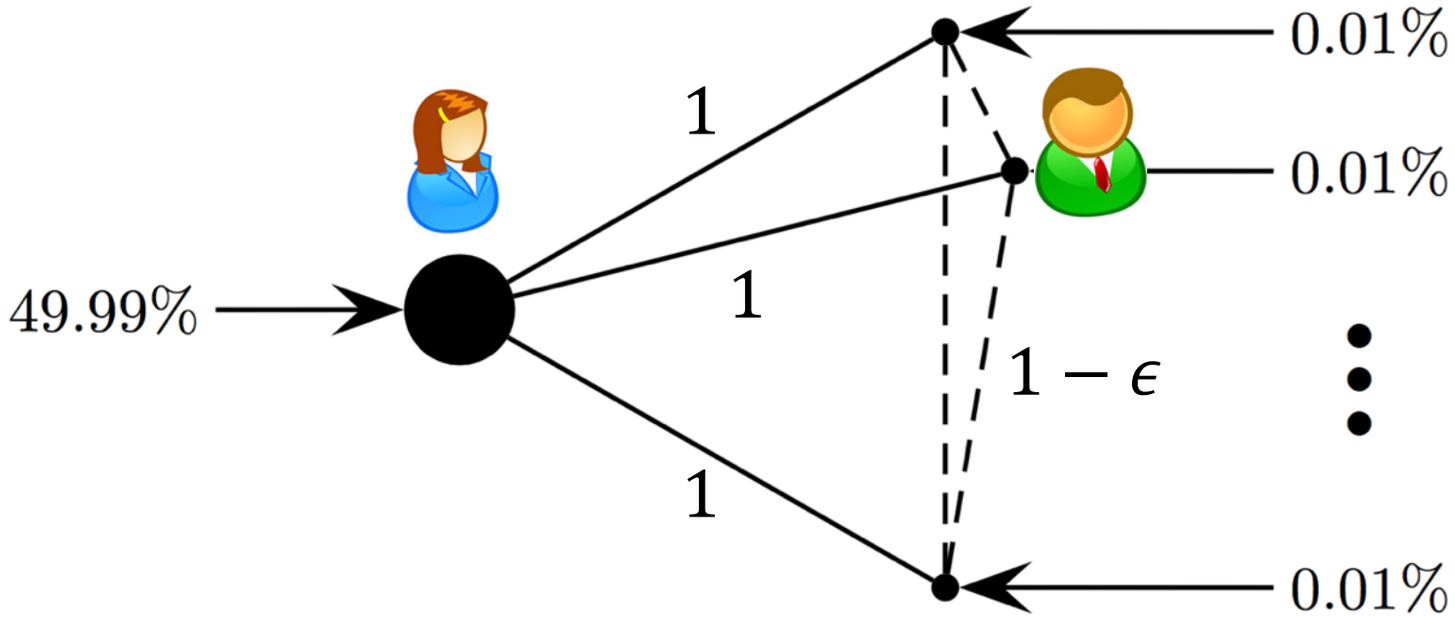
General Metric Space (≥ 1.5)



Cost( ) ≈ 0.5

Cost( ) ≈ 1

General Metric Space (≥ 1.5)

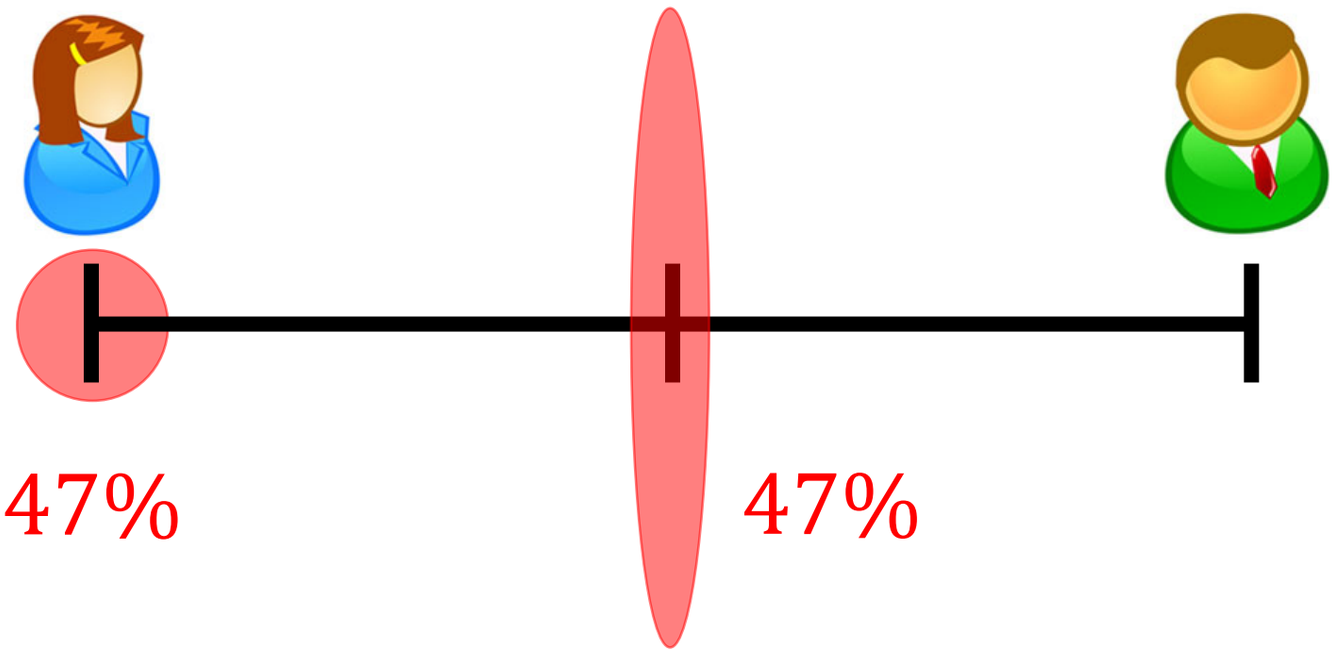


With probability $1/2$, the distortion ≈ 2

$\Longrightarrow$ Expected distortion ≈ 1.5.

General Metric Space ($\leq 2 - \frac{1}{652}$)

- If the expected distortion is sufficiently close to 2, there is a pair of candidates whose distortion is close to 3; we show that then the instance must have special structure.



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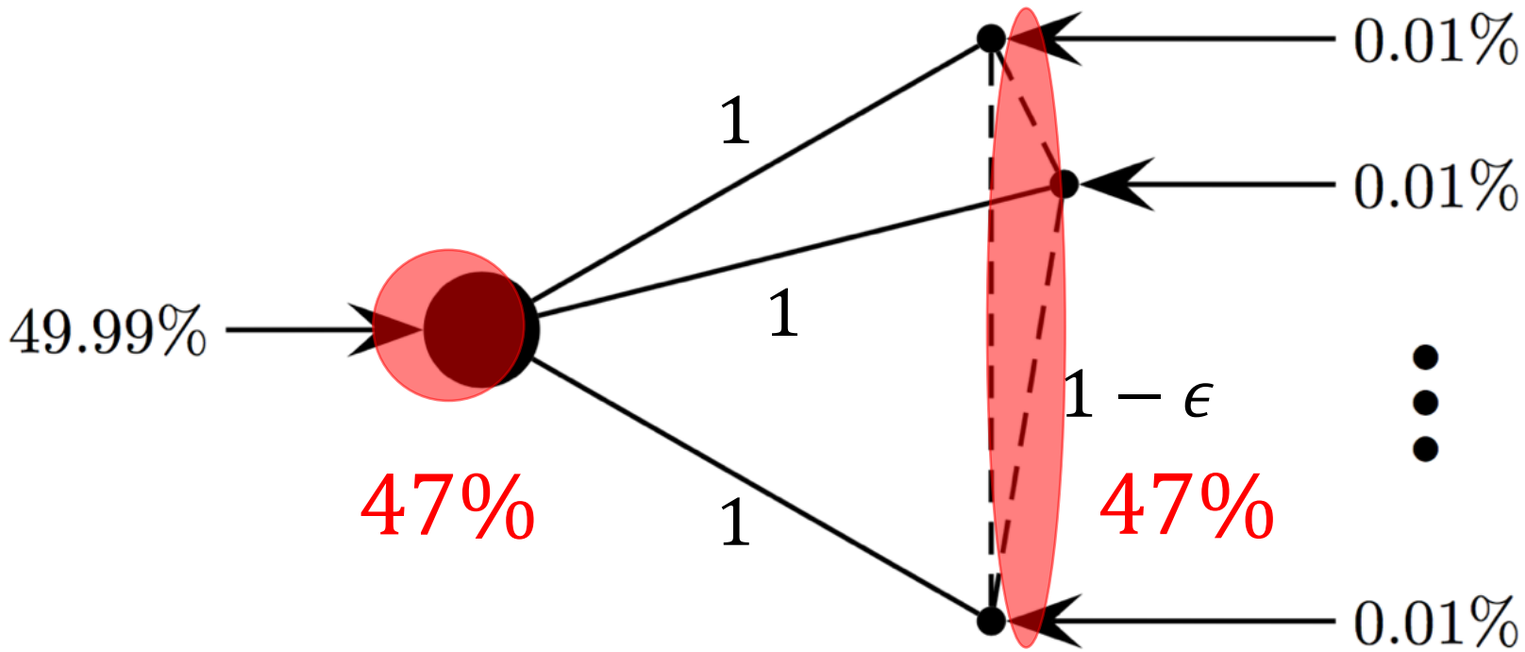

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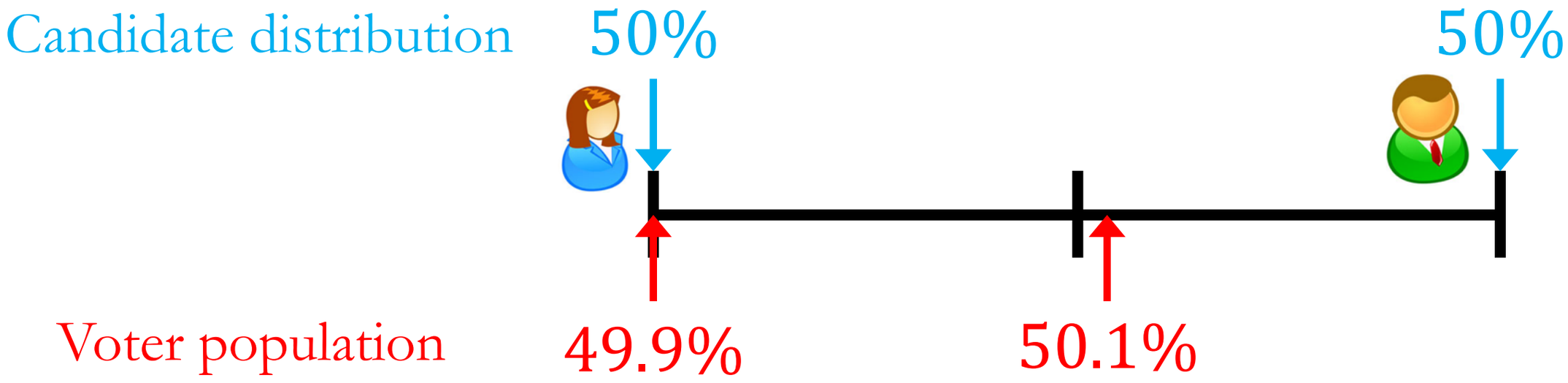
Non-Representative (≥ 2)



With probability $1/2$, we get a distortion of ≈ 3.

So expected distortion $\approx (1/2) \cdot 3 + (1/2) \cdot 1 = 2$.

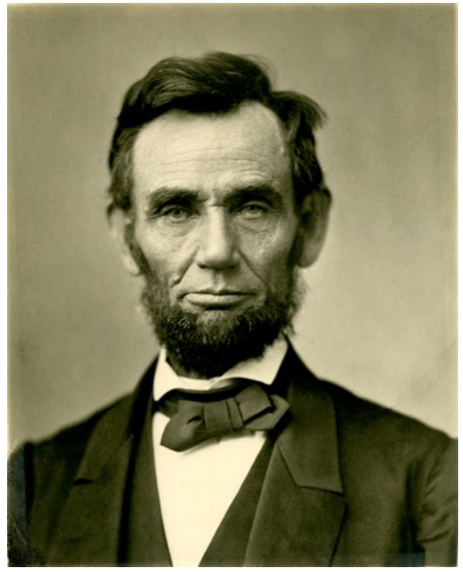
Non-Representative (≤ 2)

[Anshelevich et al.] Any election between a pair of candidates has distortion is at most 3.

If we draw two candidates whose social costs are within a factor of 3, we assume the worse candidate wins.

Ignore the metric space and treat the costs as arbitrary numbers, optimize the probability distribution over costs.

Conclusion



. . .AND THAT GOVERNMENT OF THE PEOPLE,
BY THE PEOPLE, FOR THE PEOPLE,
SHALL NOT PERISH FROM THE EARTH.
Abraham Lincoln

- For two candidates and a majority election:

Government by the people
is better for the people
if it is also of the people.

Open Questions

	Representative	Non-Representative
Line Metric	1.1716	2
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Maximum expected distortion in general metric spaces.

Open Questions

	Representative	?	Non-Representative
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How gracefully does the distortion degrade as the voter and candidate distributions become more and more dissimilar?

Open Questions

	Representative	?	Non-Representative
Line Metric	1.1716		2
?			2
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How does the distortion depend on the metric space?

Open Questions

More than two candidates.