Hardness of Signaling in Bayesian Games

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Joint work with

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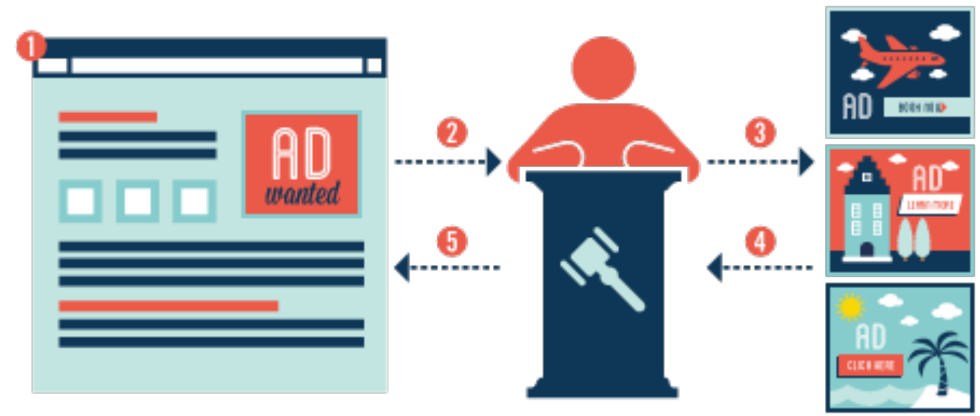
Motivation

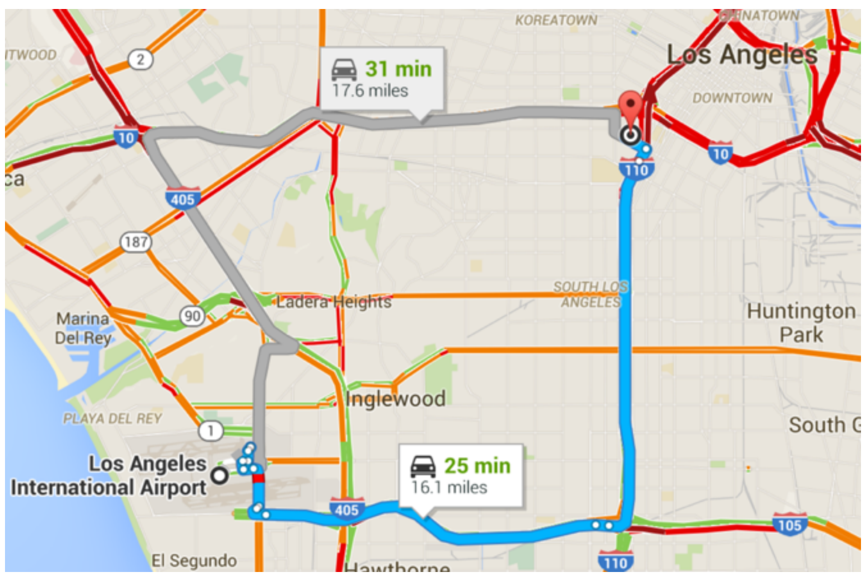
- Uncertainty in strategic interactions
- Information asymmetry

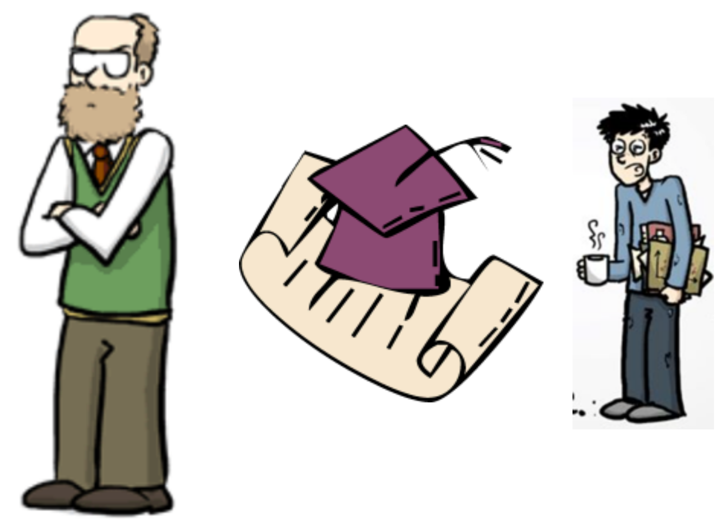
- Information revelation (Signaling):

  The act of exploiting informational advantage to
  - Affect the decisions of others
  - Induce desirable equilibrium

Signaling: Examples







Prisoner's Dilemma

	Cooperate	Defect
Cooperate	-1, -1	-5, 0
Defect	0, -5	-4, -4

Prisoner's Dilemma

$\theta \sim U\{2, 0, -2\}$

	Cooperate	Defect
Cooperate	$\theta - 1$ / $\theta - 1$	0 / $\theta - 5$
Defect	$\theta - 5$ / 0	-4 / -4

- C = Cooperate
  D = Defect
- (C, C) is a NE if $\theta \geq 1$
  (D, D) is a NE if $\theta \leq 1$
- Principal gets
  \$1 for (C, C)
  \$0 otherwise

Prisoner's Dilemma

$\theta \sim U\{2, 0, -2\}$

	Cooperate	Defect
Cooperate	$\theta - 1$ $\theta - 1$	0 $\theta - 5$
Defect	$\theta - 5$ 0	-4 -4

- Reveal no information
  - Always (D, D)
  - Principal gets \$0
- Reveal full information
  - (C, C) when $\theta = 2$
  - (D, D) when $\theta = 0, -2$
  - Principal gets \$1/3

Prisoner's Dilemma

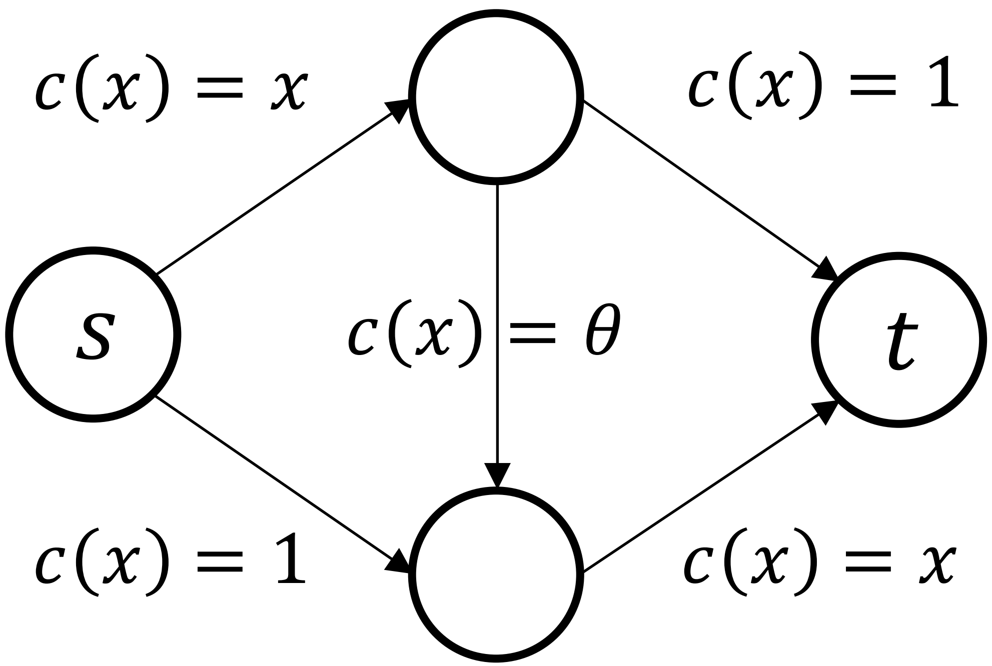
$\theta \sim U\{2, 0, -2\}$

	Cooperate	Defect
Cooperate	$\theta - 1$ $\theta - 1$	0 $\theta - 5$
Defect	$\theta - 5$ 0	-4 -4

- Optimal signaling scheme
  - **High** $\theta = 0, 2$
  - **Low** $\theta = -2$

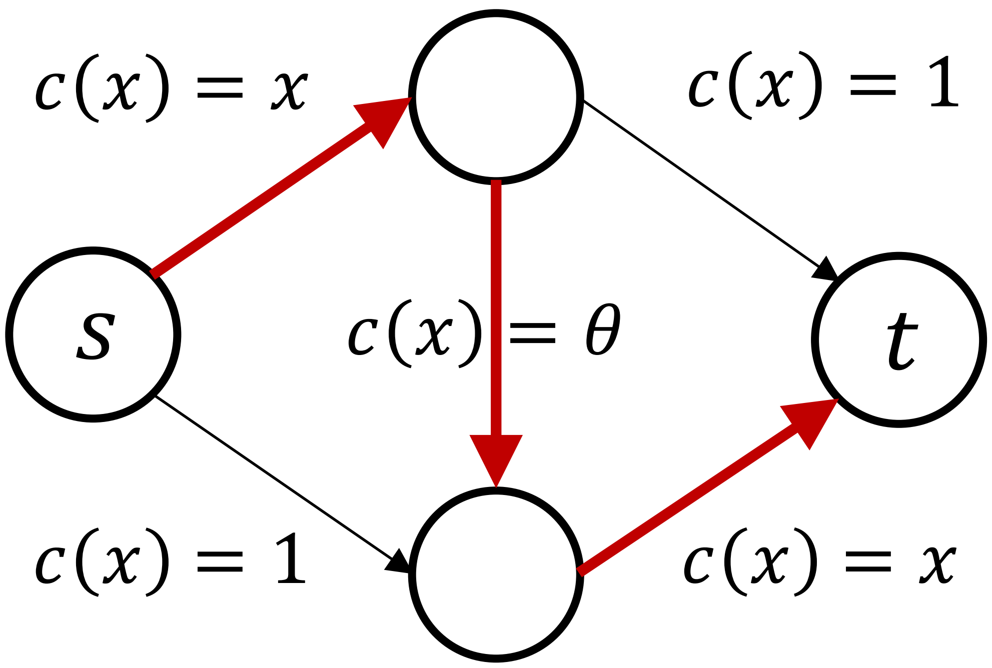
- $E[\theta \mid \textbf{High}] = 1$
- $E[\theta \mid \textbf{Low}] = -2$
- Player play (C, C) when they receive **High**, so principal gets **$**$2/3$

Braess's Paradox



$\theta \sim U\{0,1\}$

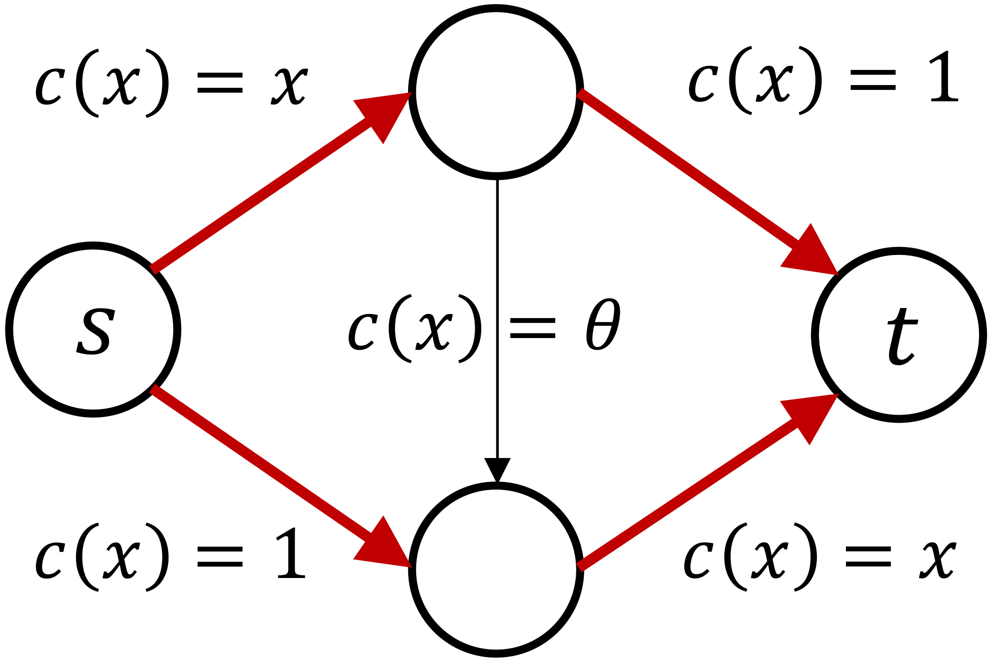
Braess's Paradox



$\theta \sim U\{0,1\}$

- When $\theta = 0$
  - Cost = 2

Braess's Paradox



$\theta \sim U\{0, 1\}$

- When $\theta = 0$
  - Cost $= 2$
- When $\theta \geq 0.5$
  - Cost $= 1.5$
- Optimal:
  reveal no information

How hard is it
to reveal information optimally?

Previous Work

- Optimal information structure can be intricate
  - [Blackwell '51] [Akerlof '70] [Hirshleifer '71] [Spence '73] [Milgrom and Weber '82] [Lehrer et al. '10] [Abraham et al. '13] [Bergemann et al. '13] [Alonso and Câmara '14] …
- Computational complexity of (approximate) optimal signaling
  - [Emek et al. '12] [Milterson and Sheffet '12] [Guo and Deligkas '13] [Dughmi '14] [Cheng et al. '15] …

Signaling Problem

- Payoffs depend on state of nature Θ
- Players know a common prior λ of Θ
- An informed principal knows the realization of Θ
  - Public signals Σ
  - Commits to a signaling scheme $\varphi: \Theta \longrightarrow \Sigma$
- Players Bayes update based on the signal, and play a NE

Bayesian Games

- Two-player zero-sum games
  - Goal: maximize row player's utility

- Network routing games (non-atomic)
  - Goal: minimize latency of Nash flow

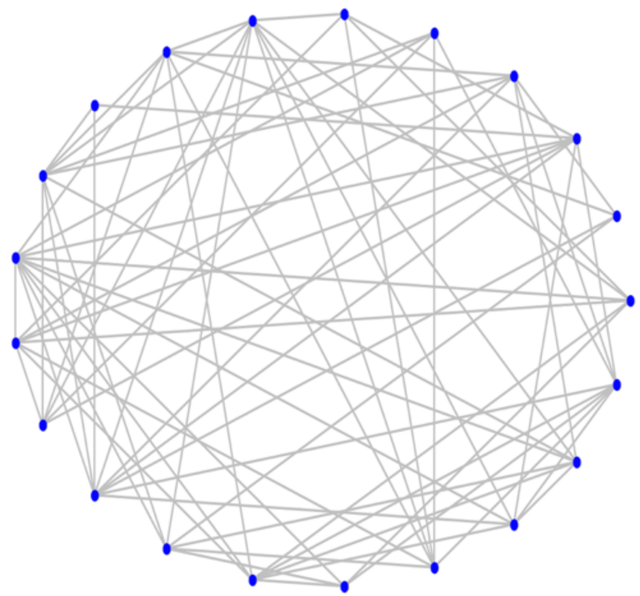
Both admit poly-time computable equilibria $\Rightarrow$ can study the signaling problem bereft of equilibrium computation concerns

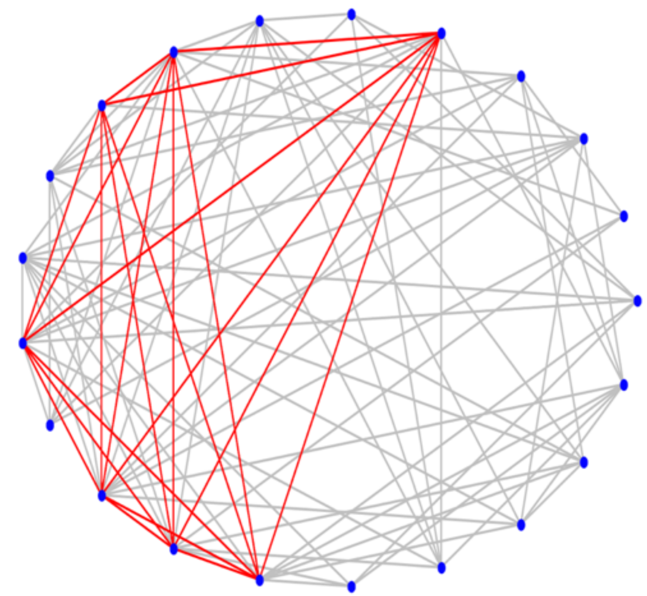
Previous Results

- Zero-sum games

FPTAS	PTAS	Quasi-PTAS
✗ [Dughmi '14] Planted-Clique hard		

Planted Clique Conjecture





- No poly-time algorithm that recovers a planted k-clique from $G(n, 1/2)$ with constant success probability for $k = o(\sqrt{n})$ and $k = \omega(\log n)$

Previous Results

- Zero-sum games

FPTAS	PTAS	Quasi-PTAS
✗ [Dughmi '14] Planted-Clique hard	?	✓ [Cheng et al. '15]

Our Results

- Zero-sum games

FPTAS	PTAS	Quasi-PTAS
✗ [Dughmi '14] ~~Planted-Clique hard~~ NP-hard	~~?~~ Planted-Clique hard	✓ [Cheng et al. '15]

[Rubinstein] proved ETH-hardness for PTAS (unlikely to be NP-hard)

Our Results

- Zero-sum games

FPTAS	PTAS
NP-hard	Planted-Clique hard

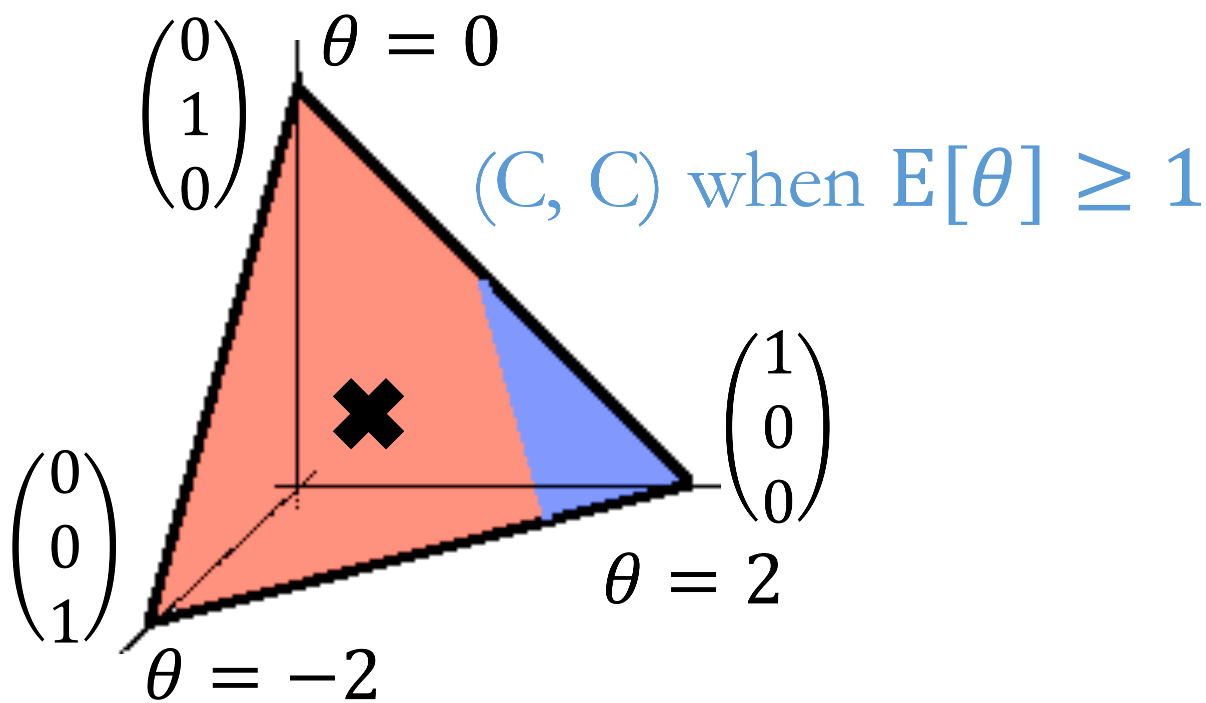
- Network routing games
  - NP-hard to get multiplicative approximation better than 4/3, even for single commodity and linear latencies
  - Full-revelation achieves approximation = price of anarchy, so 4/3 is tight for linear latencies

Prior Decomposition

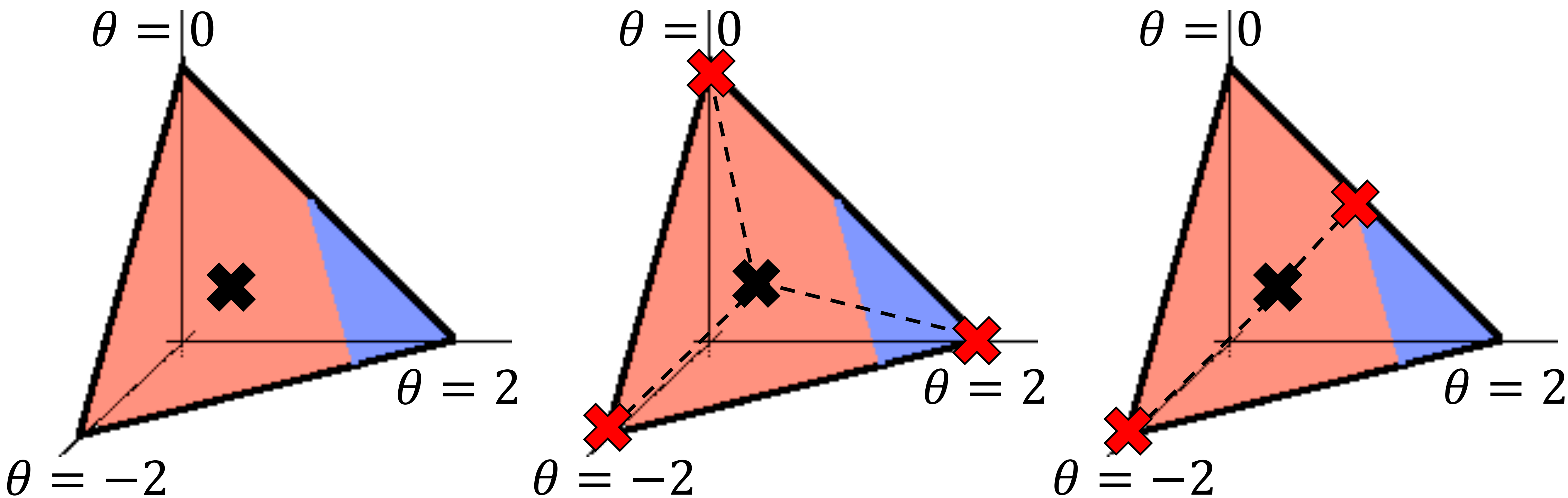
$\theta \sim U\{2, 0, -2\}$

	C	D
C	$\theta - 1$, $\theta - 1$	$\theta - 5$, 0
D	0, $\theta - 5$	-4, -4

Posterior $\mu \in \mathbb{R}^3$: $\mu_1 = \Pr[\theta = 2], \ldots$



Prior Decomposition



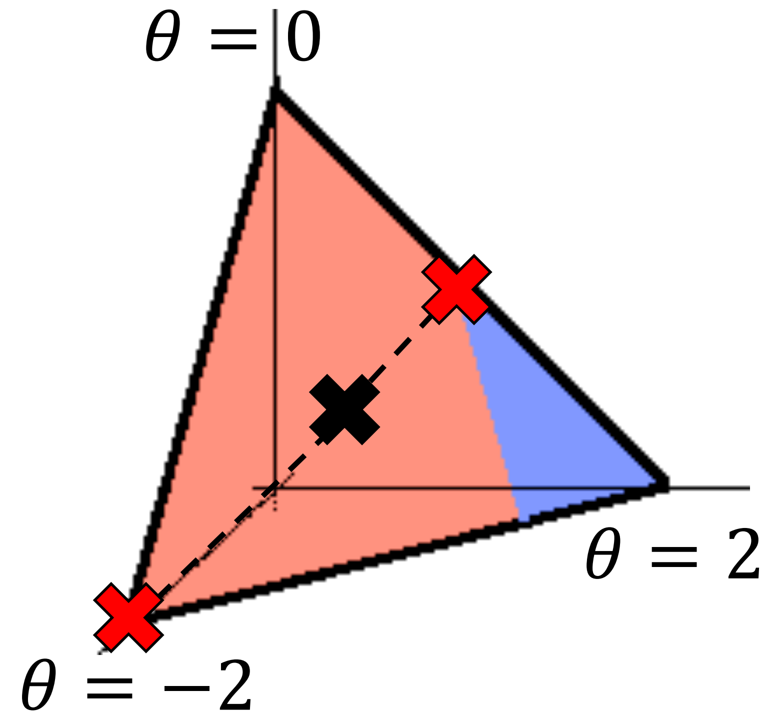
Prior Decomposition

$$\mu_1 = \begin{pmatrix} 1/2 \\ 1/2 \\ 0 \end{pmatrix} \quad \mu_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \lambda = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix} = \frac{2}{3}\mu_1 + \frac{1}{3}\mu_2$$

$$OPT = \frac{2}{3} f(\mu_1) + \frac{1}{3} f(\mu_2) = \frac{2}{3}$$

Signaling: $\max \ \sum p_i f(\mu_i)$
$s.t. \ \sum p_i \mu_i = \lambda$

Best posterior: $\max \ f(\mu)$



Zero-Sum Games: No FPTAS

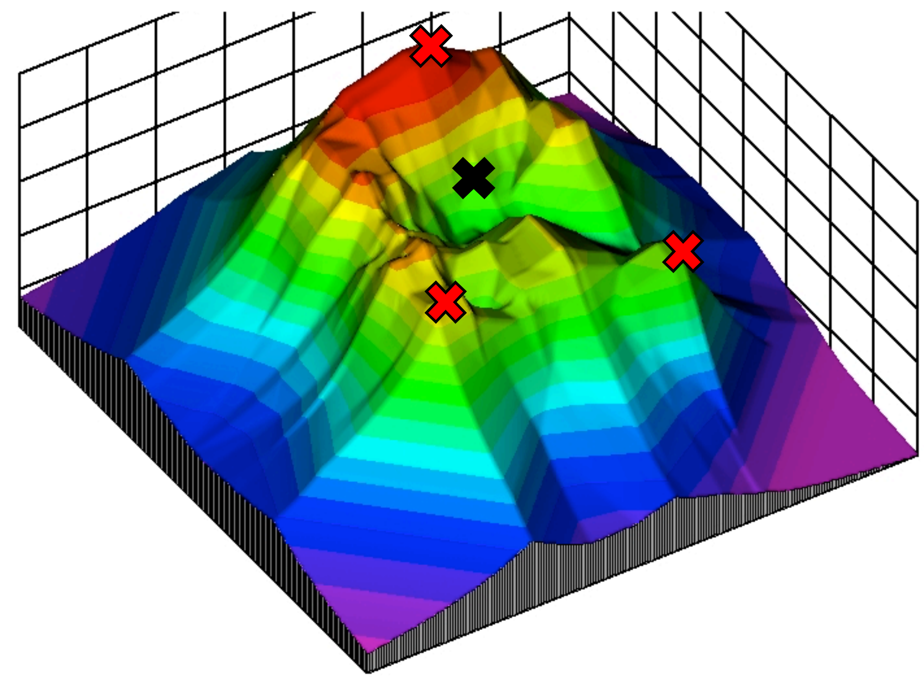
- Algorithm for optimal signaling ⇒ best posterior
  - Hardness of best posterior ⇒ hardness of signaling

- Finding an ϵ-best posterior distribution is NP-hard when $\epsilon = \text{poly}(1/n)$ (much easier to show)

Optimization and Membership Oracle

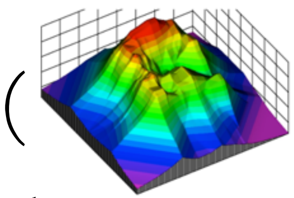
- $f: \Delta_\Theta \to [0,1]$ maps posterior to principal's utility
- Let f^+ be the minimum concave function such that $f^+ \geq f$

$$f^+(\lambda) = \max \sum p_i f(\mu_i) \quad s.t. \quad \sum p_i \mu_i = \lambda$$



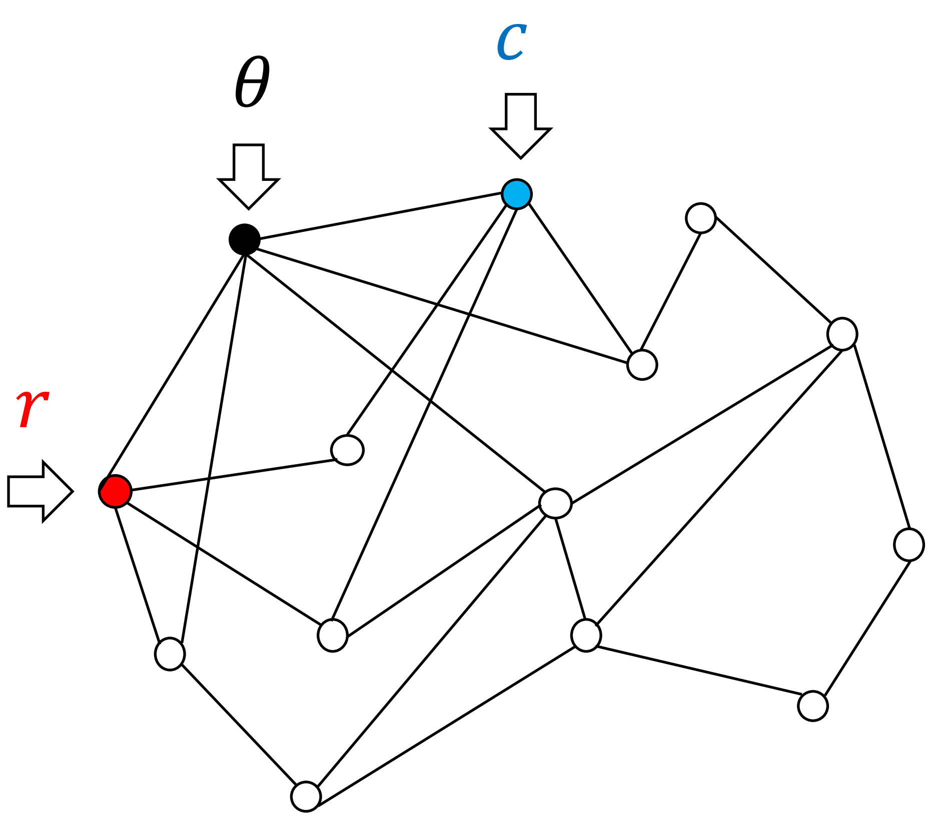
- Signaling $\Leftrightarrow$ value oracle for f^+

Optimization and Membership Oracle

- Goal: best posterior $\max_{\mu} f(\mu) = \max_{\mu} f^+(\mu)$
  - Consider $K = \{(x, y) : y \le f^+(x)\} = conv($  $)$
  - Signaling $\Leftrightarrow$ value oracle for f^+ $\Leftrightarrow$ membership oracle for K
  - $\max_{\mu} f^+(\mu) = \max_{(x,y)\in K} y$ $\Leftrightarrow$ optimization over K

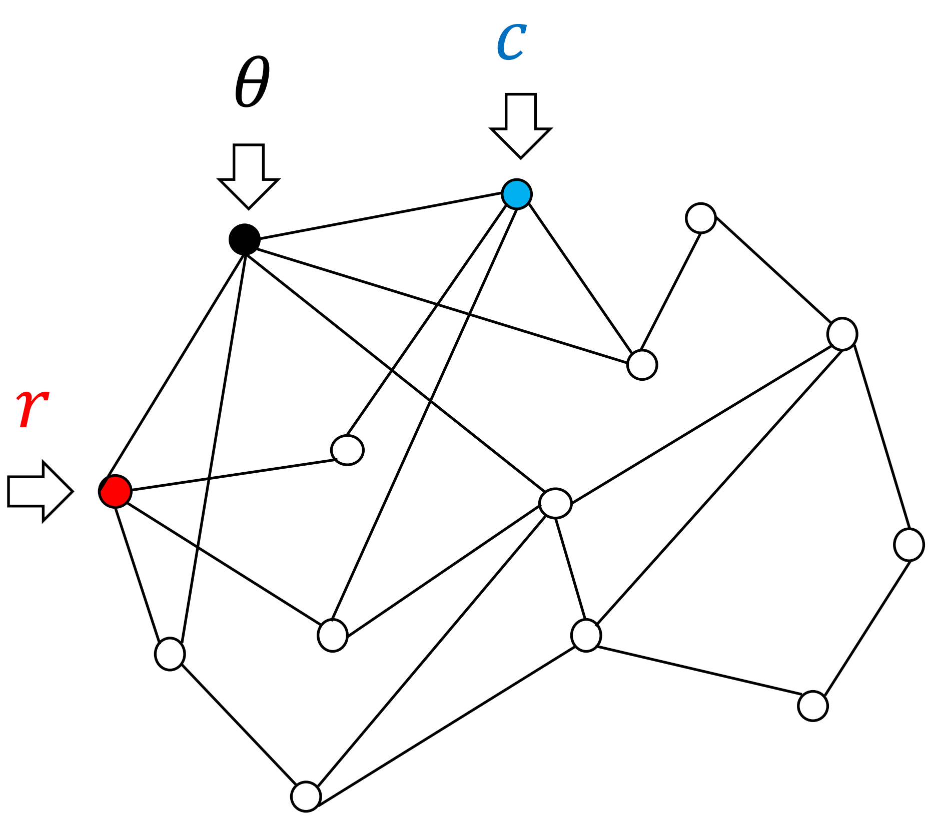
- Membership oracle $\Rightarrow$ Separation oracle $\Rightarrow$ Optimization
  - (ϵ/n)-hardness of best posterior $\Rightarrow$ ϵ-hardness of signaling

Security Games on Graphs [Dughmi '14]



- Given $G = (V, E)$
  - State of nature $\theta \sim uni(V)$
  - **Row** picks $r \in V$
  - **Col** picks $c \in V$
- Objective (zero-sum):
  - **Row** wants to be adjacent to θ
  - **Col** wants to catch **Row** or θ

Security Games on Graphs [Dughmi '14]



Row's payoff $= 1$

- Given $G = (V, E)$
- $\theta, r, c \in V$
- **Row**'s payoff

$$+1 \text{ if } (\theta, r) \in E$$
$$-1 \text{ if } \mathrm{c} = \theta$$
$$-1 \text{ if } \mathrm{c} = \mathrm{a}$$

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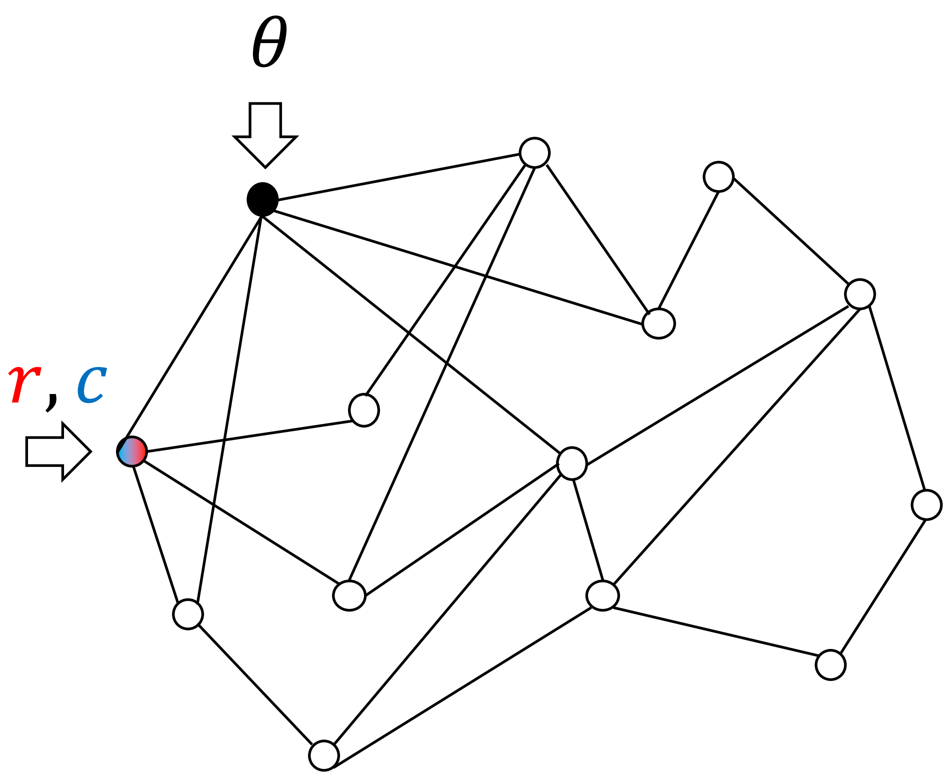

Row’s payoff $= 1 - 1 = 0$

- Given $G = (V, E)$
- $\theta, r, c \in V$
- **Row**’s payoff

$$+1 \text{ if } (\theta, r) \in E$$
$$-1 \text{ if } \mathrm{c} = \theta$$
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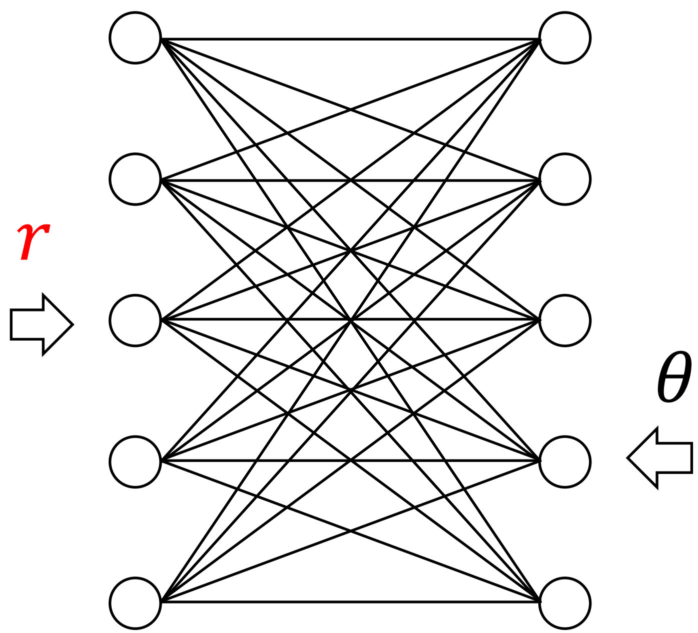
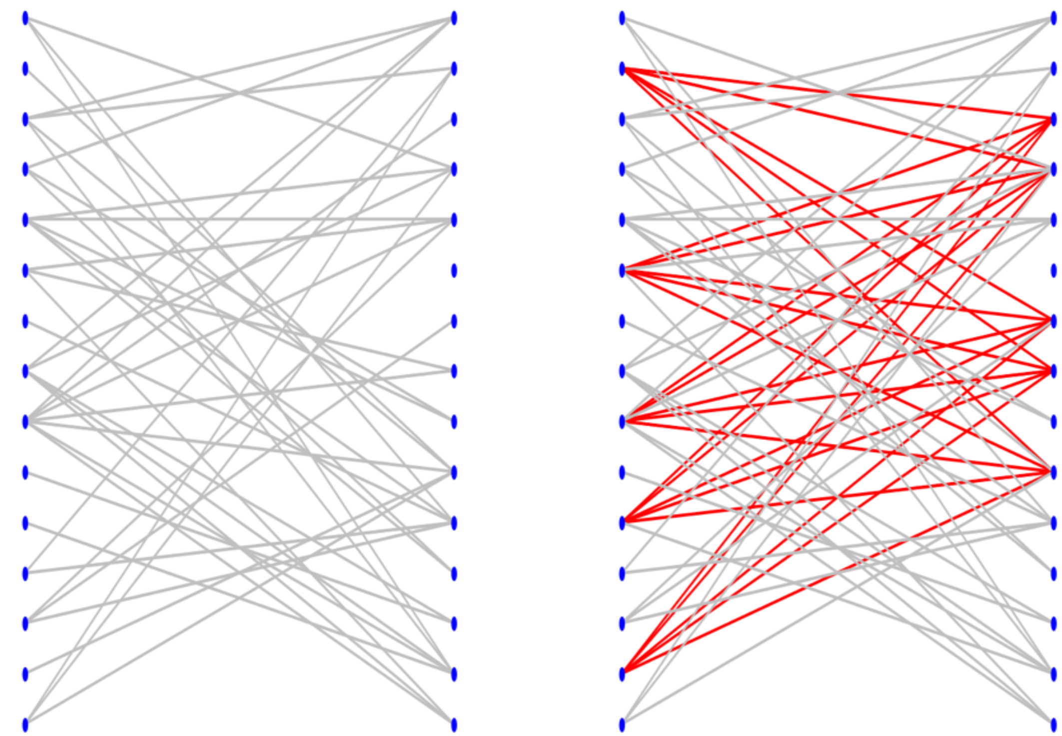


- Asymmetry of payoffs
- Principal reveals
  $\theta \in L$ or $\theta \in R$
- **Row** chooses uniformly from the other side
  - Always have $(\theta, r) \in E$
  - Hard for **Col** to catch

Security Games on Graphs [Dughmi '14]

- Cliques are good for **Principal** and **Row**

- $\max_{\mu} f(\mu) \geq 1 - \frac{1}{k}$ iff G has a $k \times k$ bipartite clique
  - NP-hard



Zero-Sum Games: No FPTAS

- Membership oracle ⇒ Separation oracle ⇒ Optimization
  - Hardness of optmization ⇒ Hardness of testing membership
  - FPTAS version works as well (shallow cut ellipsoid)

- Powerful technique to prove hardness
  - Exploit the equivalence of separation and optimization

Open Problems

- PTAS for membership ⇔ PTAS for optimization?
  - We know FPTAS for membership ⇔ FPTAS for optimization
- Poly-time (additive) constant-approximations for signaling in zero-sum games
  - Currently, only quasi-PTAS is known [Cheng et al. ’15]
- Private signals

Thanks!

Q & A