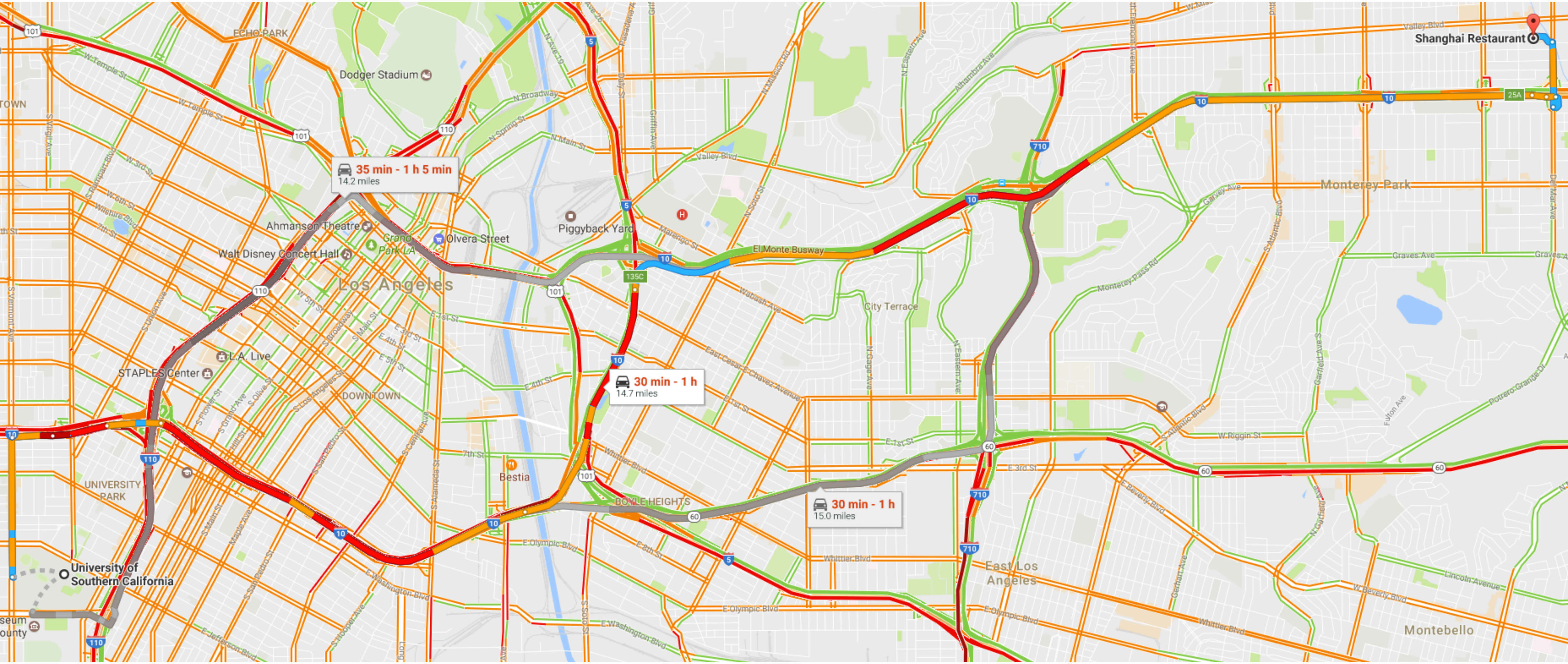
Computational Aspects of Optimal Information Revelation

Yu Cheng

May 8, 2017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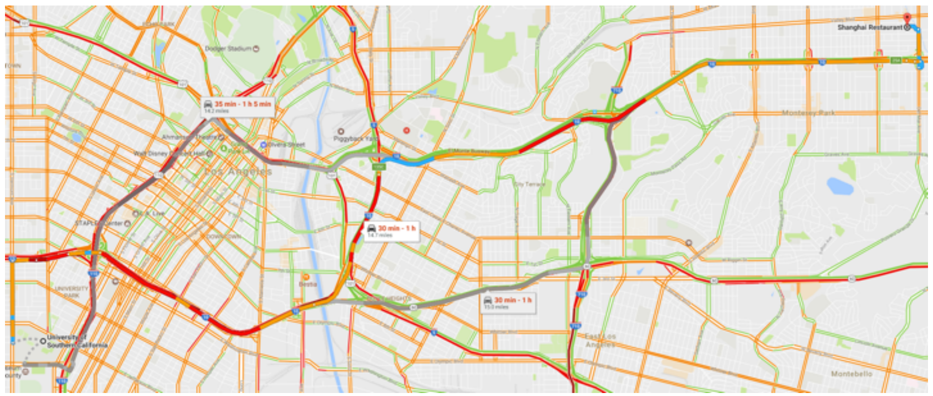
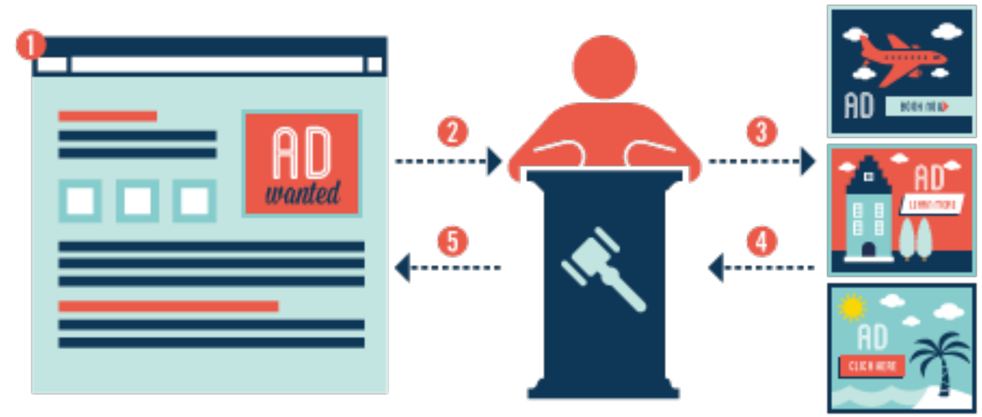
Shanghai Restaurant
University of Southern California
Los Angeles
DOWNTOWN
Dodger Stadium
ECHO PARK
Ahmanson Theatre
Walt Disney Concert Hall
Grand Park LA
Olvera Street
Piggyback Yard
L.A. Live
STAPLES Center
UNIVERSITY PARK
Bestia
BOYLE HEIGHTS
City Terrace
Monterey Park
East Los Angeles
Montebello
El Monte Busway

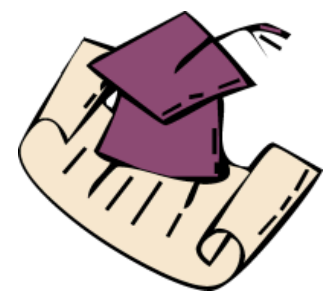
How to reveal information optimally to other strategic players?

Examples





	Cooperate	Defect
Cooperate	$\theta - 1$ / $\theta - 1$	0 / $\theta - 5$
Defect	$\theta - 5$ / 0	-4 / -4



The Signaling Problem

- Strategic interactions with uncertainty.

- Choices of the agents depend on the information available to them.

- An informed principal must choose how to reveal partial information in order to induce a desirable outcome.

Mechanism Design	Information Structure Design (Signaling, Persuasion)

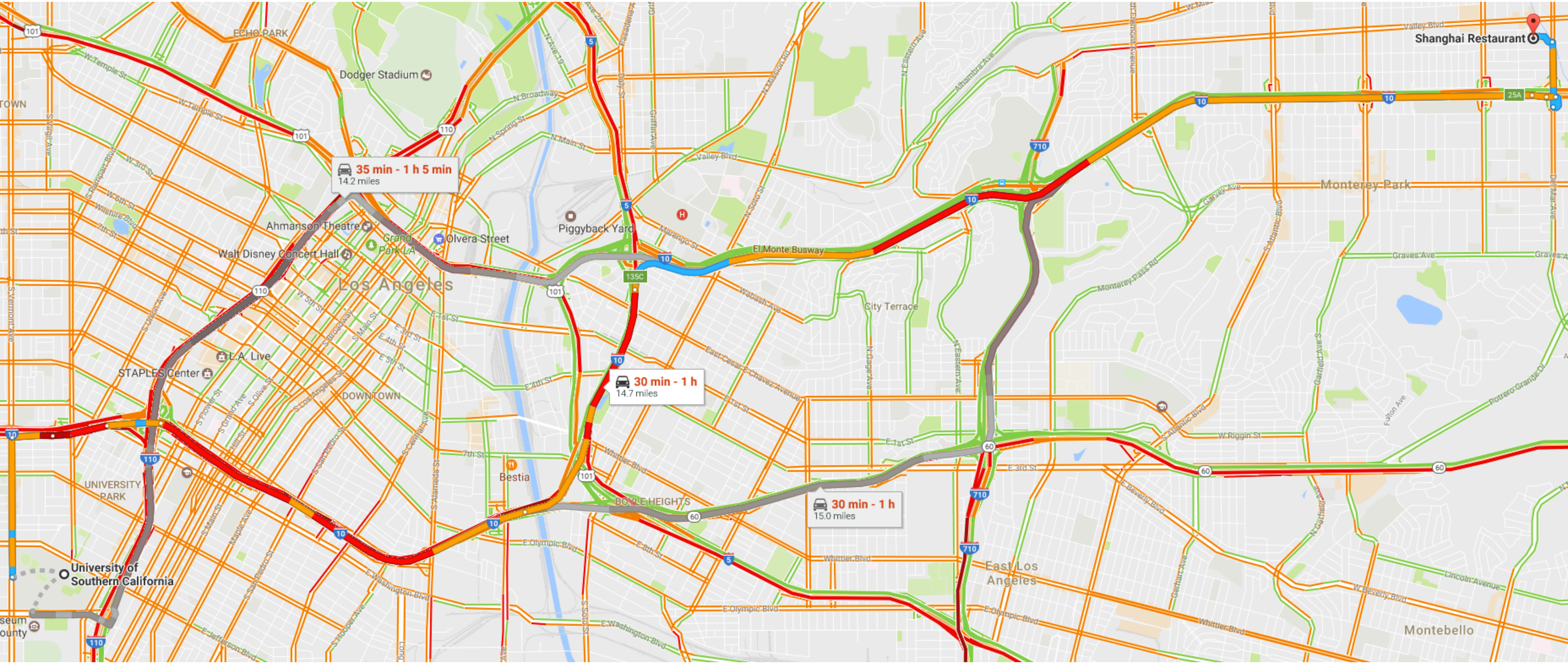
A principal interested in the game's outcome

Design allocation and payment rules	Choose what information to reveal
Incentives	Beliefs

How hard is it (computationally) to find the optimal information structure?

This Talk

- Network Routing Games
- Normal Form Games
- Mixture Selection Framework
- Future Work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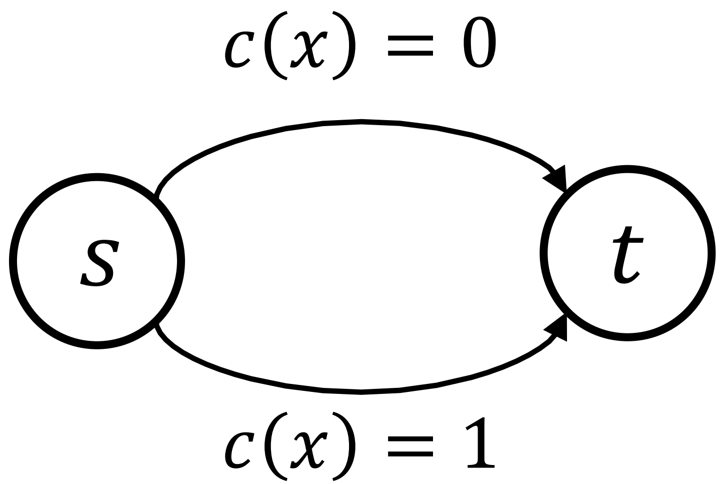
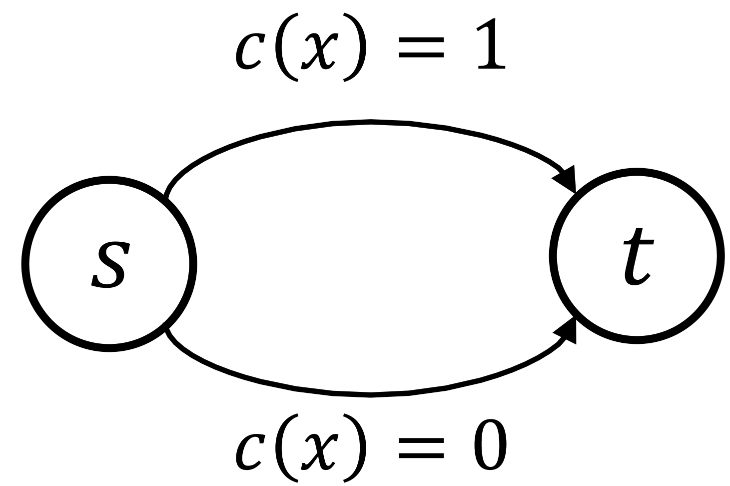
How to reveal information ~~optimally~~?
to minimize the latency
of selfish routing?

Network 1: Two Parallel Links





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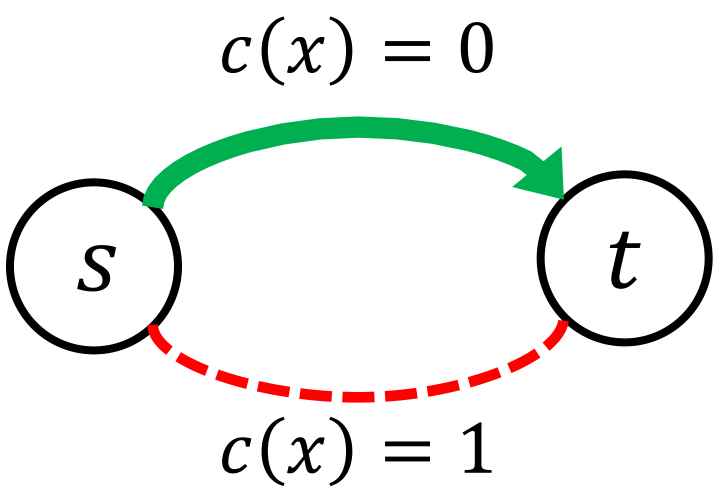
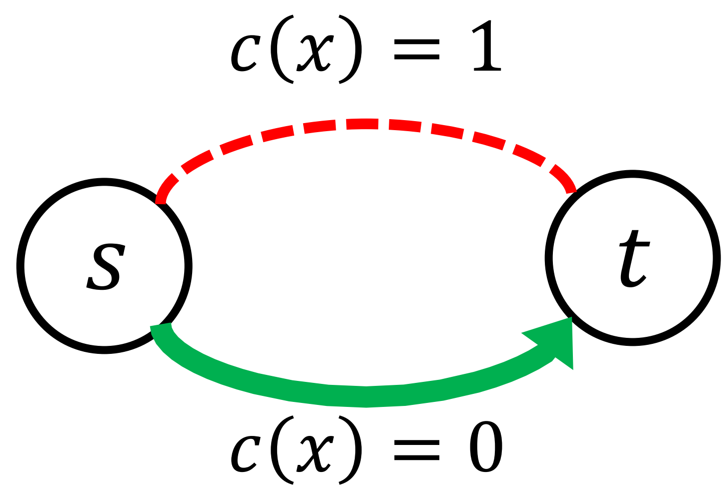
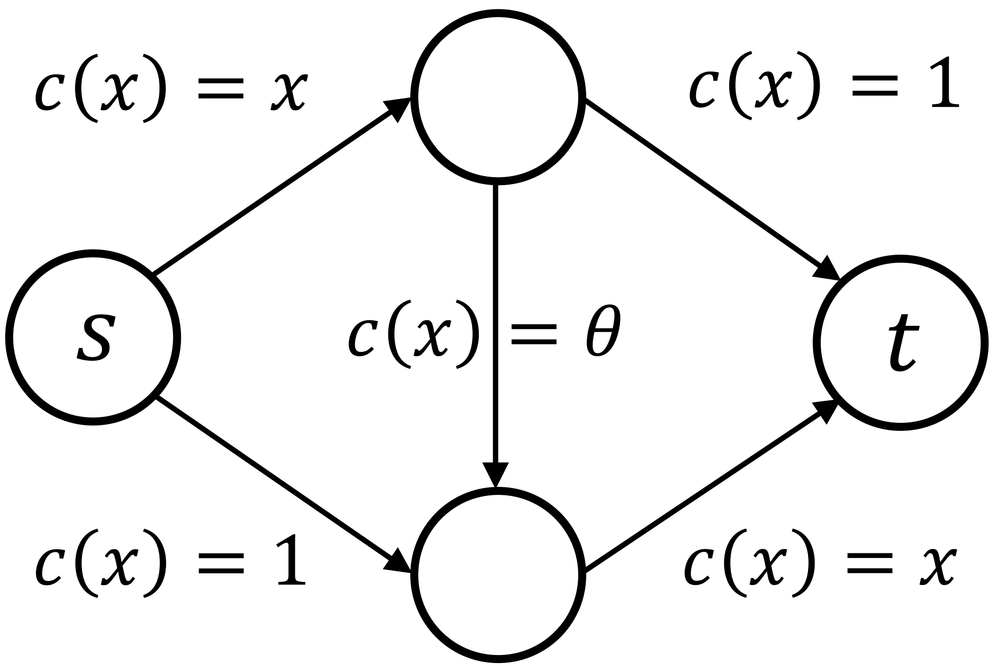


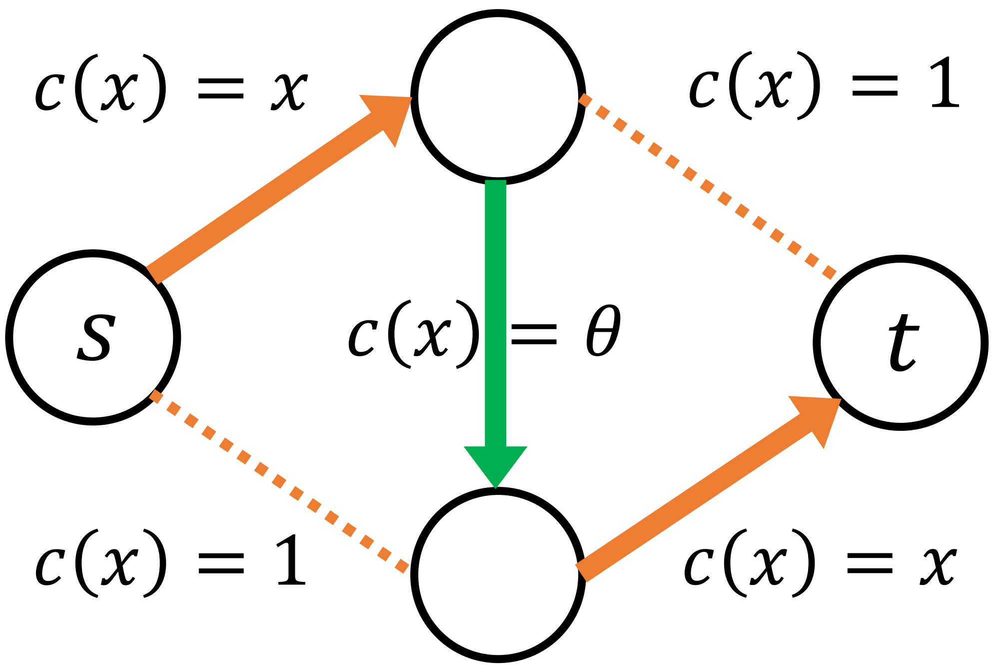


Network 2: Braess's Paradox



$\theta \sim U\{0, \infty\}$

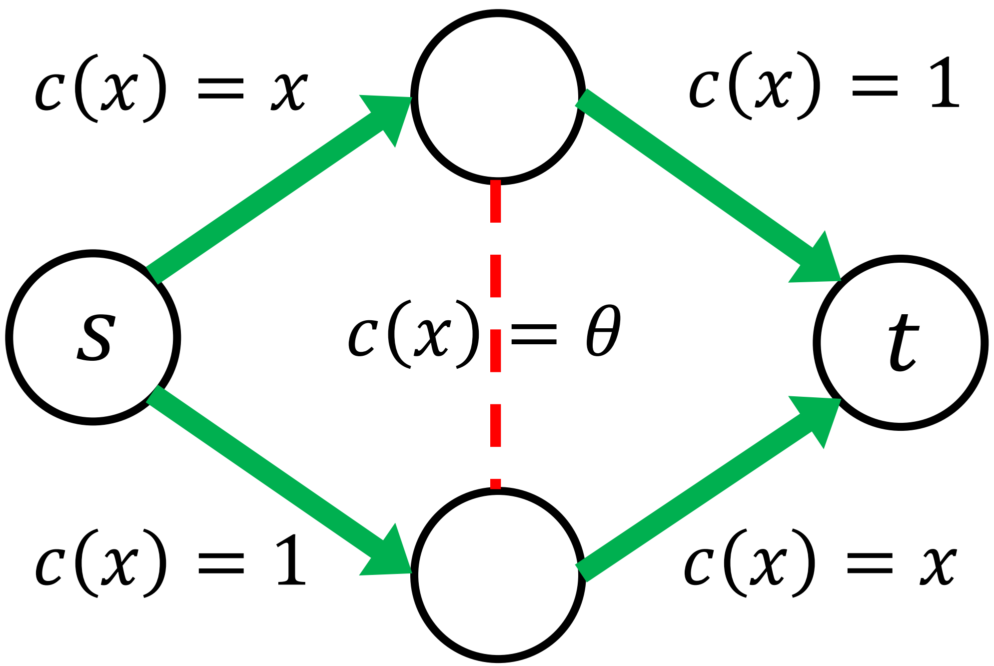
Network 2: Braess's Paradox



$\theta \sim U\{0, \infty\}$

- When $\theta = 0$,
  - Cost $= 2$.

Network 2: Braess's Paradox



$\theta \sim U\{0, \infty\}$

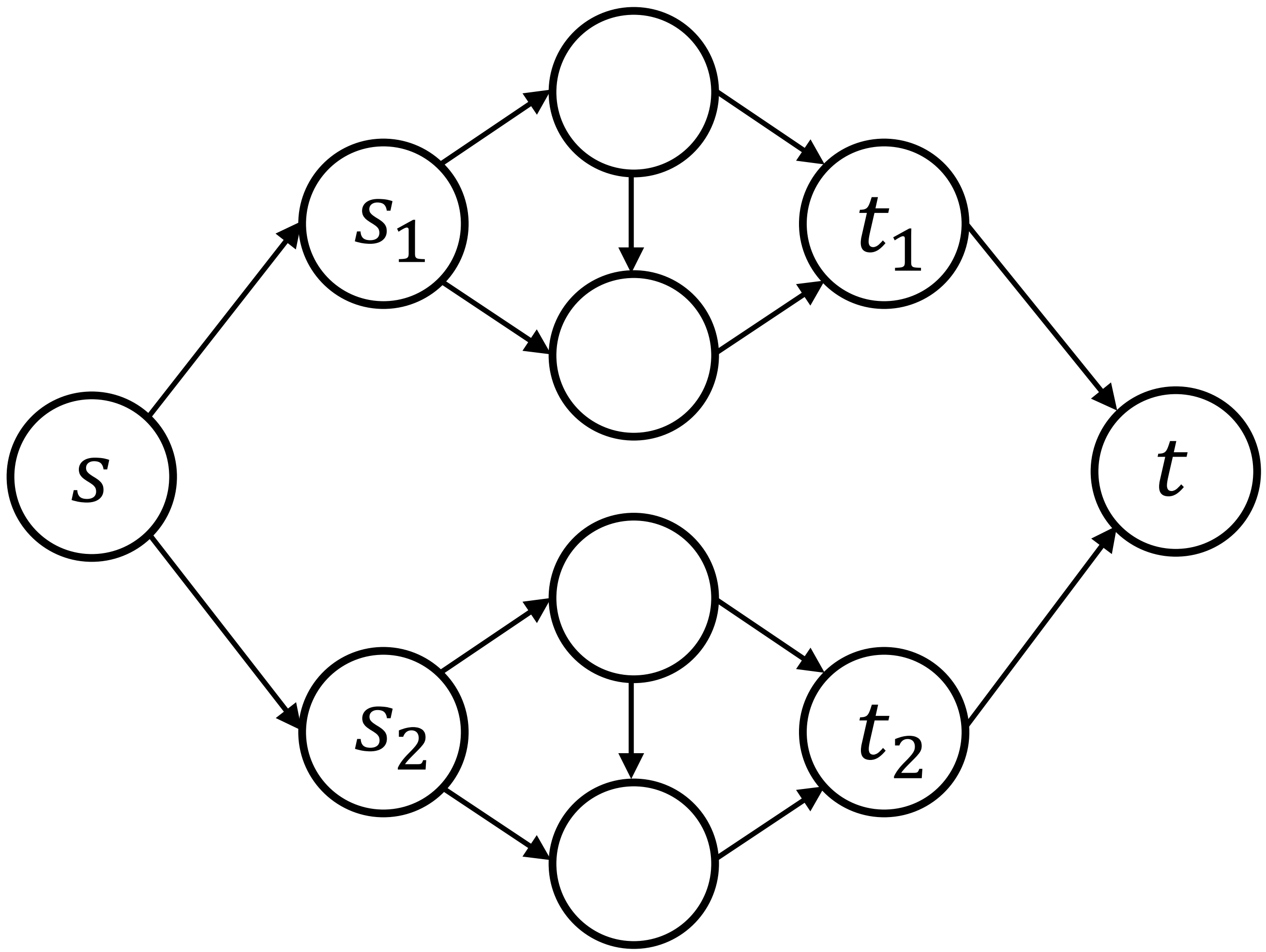
- When $\theta = 0$,
  - Cost = 2.
- When $\theta \geq 0.5$,
  - Cost = 1.5.
- Optimal:
  reveal no information.

Our Results [BCKS '16]

- NP-hard to get multiplicative $(4/3 - \epsilon)$ approximation.
  - Even for single commodity and linear latencies.

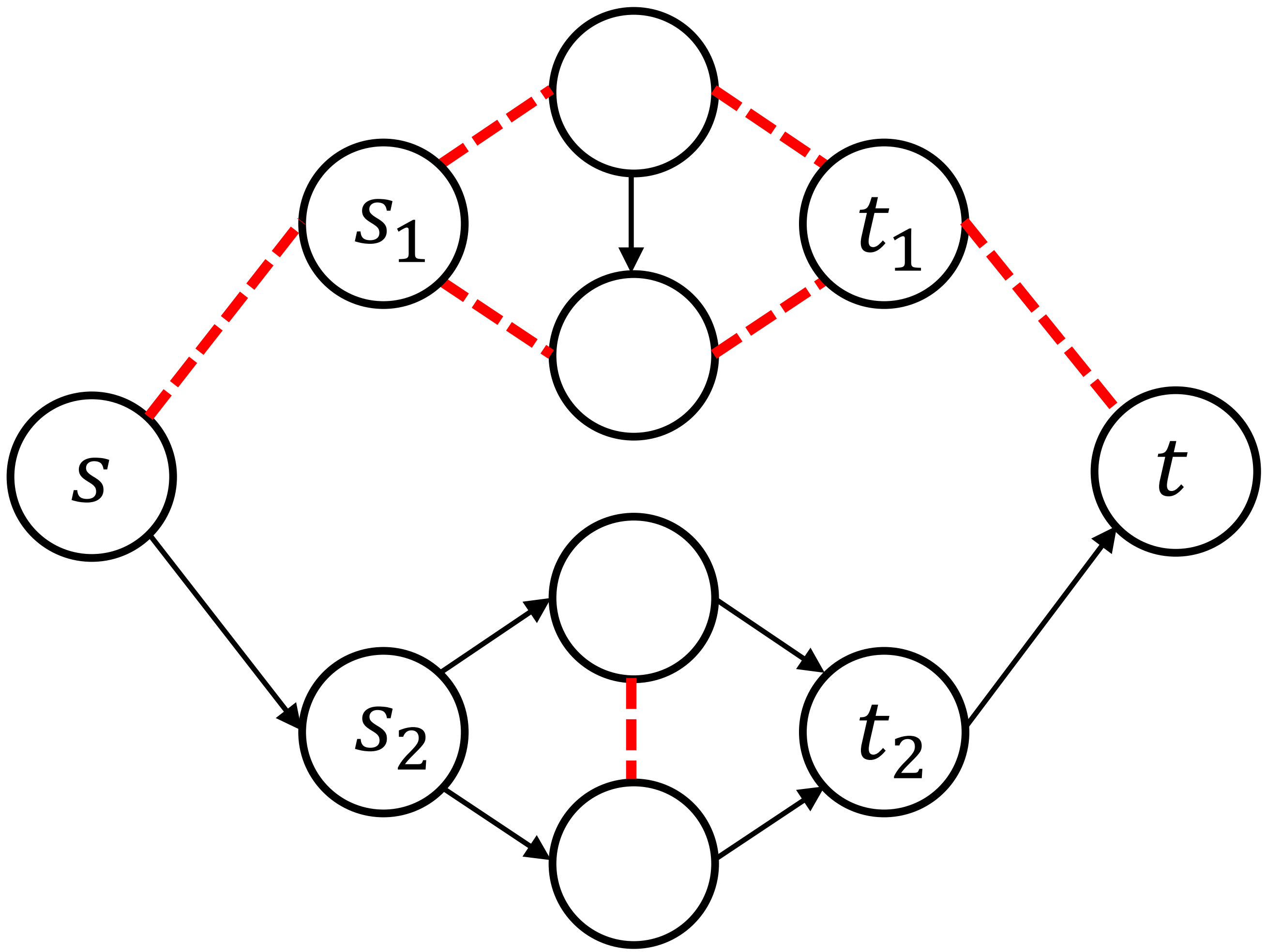
- Full-revelation = price of anarchy.
  - $4/3$ is tight for linear latencies.

Proof Outline



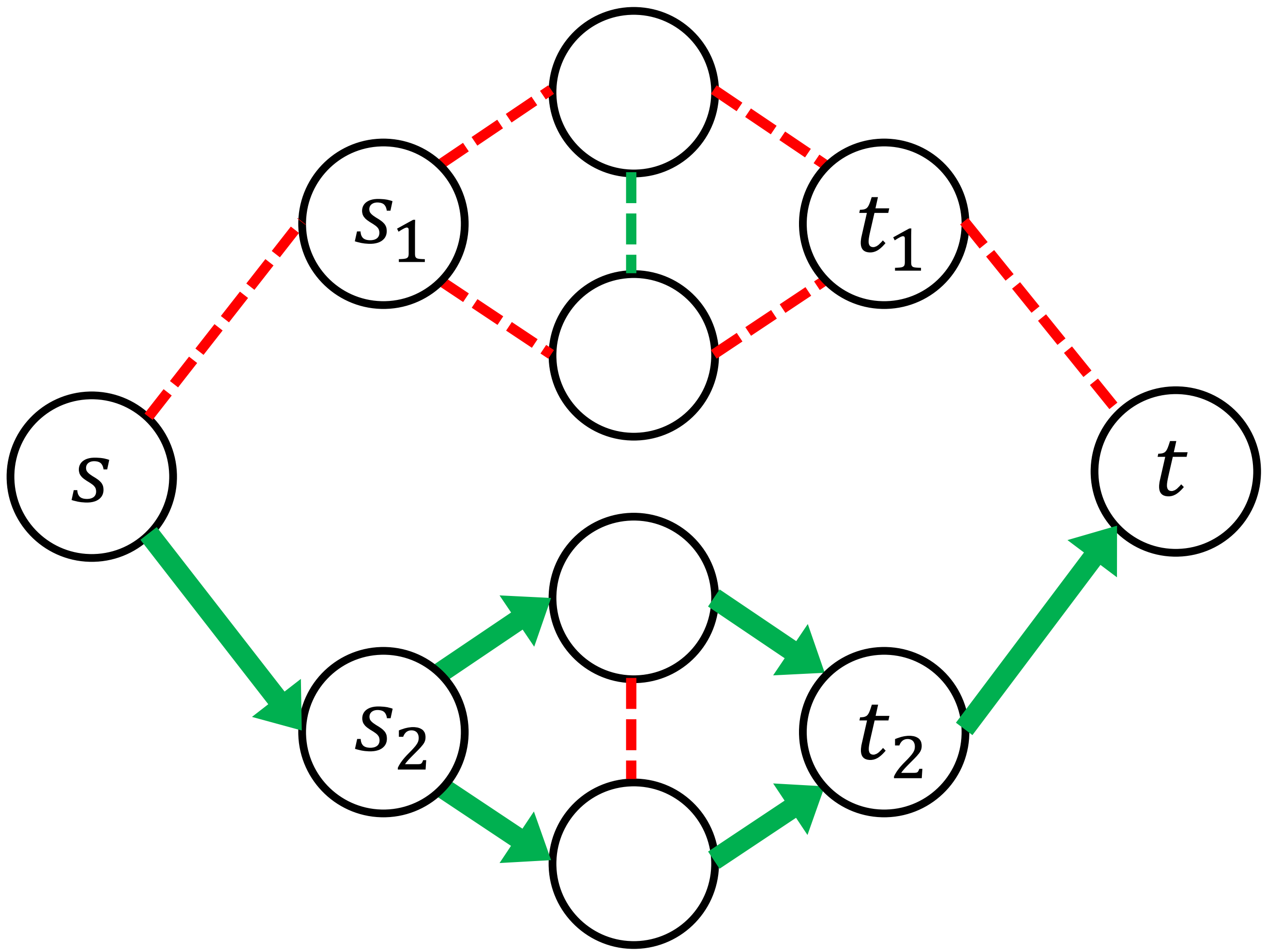
- Exactly one of the link is broken with probability $\sum p_e = 1$.
- What is the optimal signaling scheme?

Proof Outline



- Partition the links into two disjoint sets.
- Reveal which set contains the broken link.

Proof Outline



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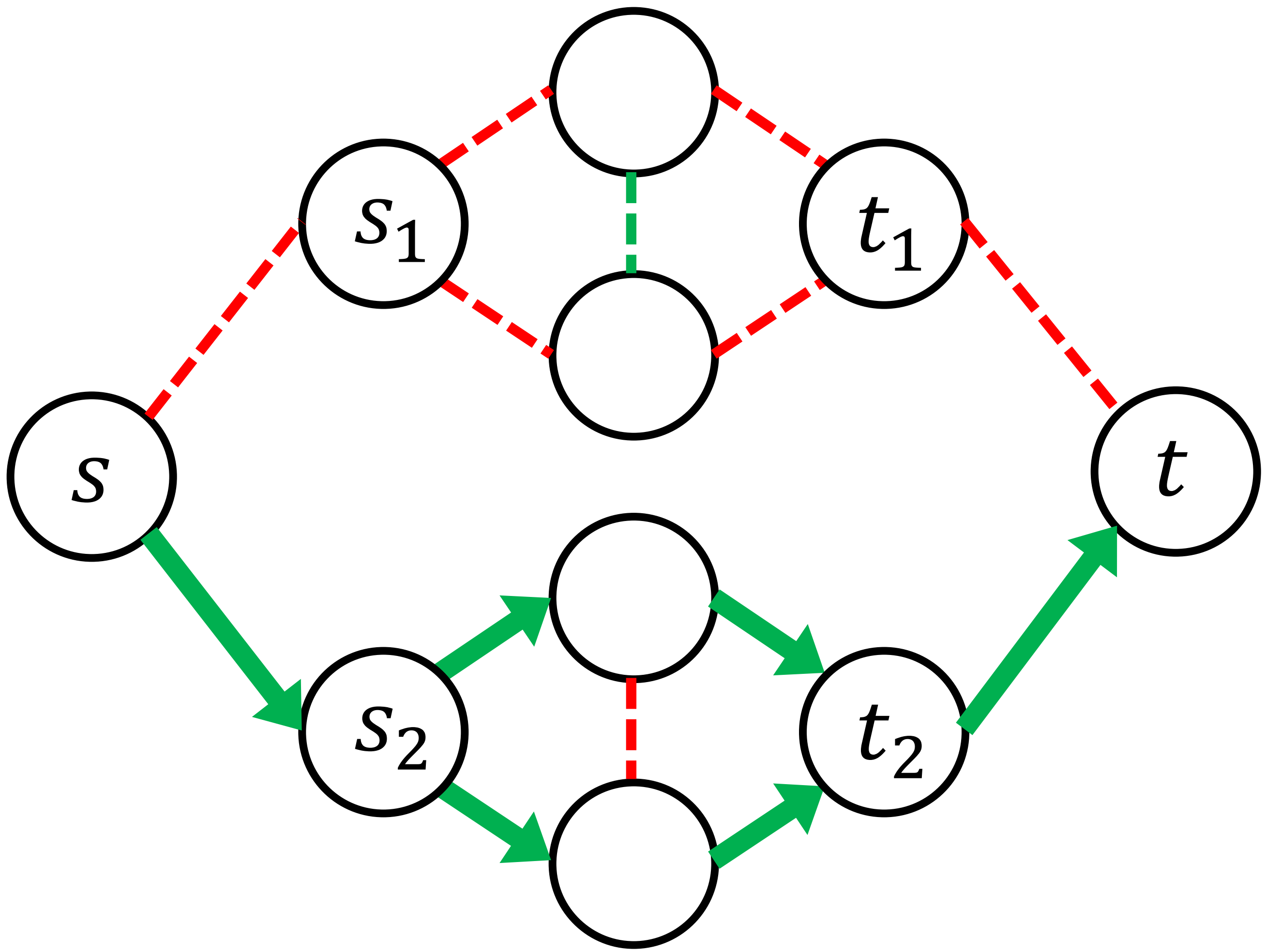


- Optimal signaling is as hard as optimal network design.
- NP-hard [CDR '06].

This Talk

- Network Routing Games
- Normal Form Games
- Mixture Selection Framework
- Future Work

Prisoner's Dilemma

	Cooperate	Defect
Cooperate	-1 -1	0 -5
Defect	-5 0	-4 -4

Prisoner's Dilemma [Dughmi'14]

$\theta \sim U\{2, 0, -2\}$

		Cooperate	Defect
	Cooperate	$\theta - 1$ $\theta - 1$	0 $\theta - 5$
	Defect	$\theta - 5$ 0	-4 -4

- C = Cooperate,
  D = Defect.
- (C, C) is a NE if $\theta \geq 1$,
  (D, D) is a NE if $\theta \leq 1$.
- Principal gets

  \$1 for (C, C),

  \$0 otherwise.

Prisoner's Dilemma [Dughmi'14]

$\theta \sim U\{2, 0, -2\}$

	Cooperate	Defect
Cooperate	$\theta - 1$ $\theta - 1$	0 $\theta - 5$
Defect	$\theta - 5$ 0	-4 -4

- Reveal no information:
  - Agents always play (D, D).
  - Principal gets \$0.
- Reveal full information:
  - (C, C) when $\theta = 2$,
  - (D, D) when $\theta = 0, -2$.
  - Principal gets \$1/3.

Prisoner's Dilemma [Dughmi'14]

$\theta \sim U\{2, 0, -2\}$

	Cooperate	Defect
Cooperate	$\theta - 1$ $\theta - 1$	0 $\theta - 5$
Defect	$\theta - 5$ 0	-4 -4

- Optimal signaling scheme:
  - **High** $\theta = 0, 2$,
  - **Low** $\theta = -2$.
- $E[\theta \mid \textbf{High}] = 1$,
  $E[\theta \mid \textbf{Low}] = -2$.
- Agents play (C, C) when they receive **High**, so principal gets **$**2/3.

Our Results

- Bayesian Zero-Sum Games.

  $|\Theta| = \#\text{strategies} = n$.

  Principal's payoff $=$ Row player's payoff.

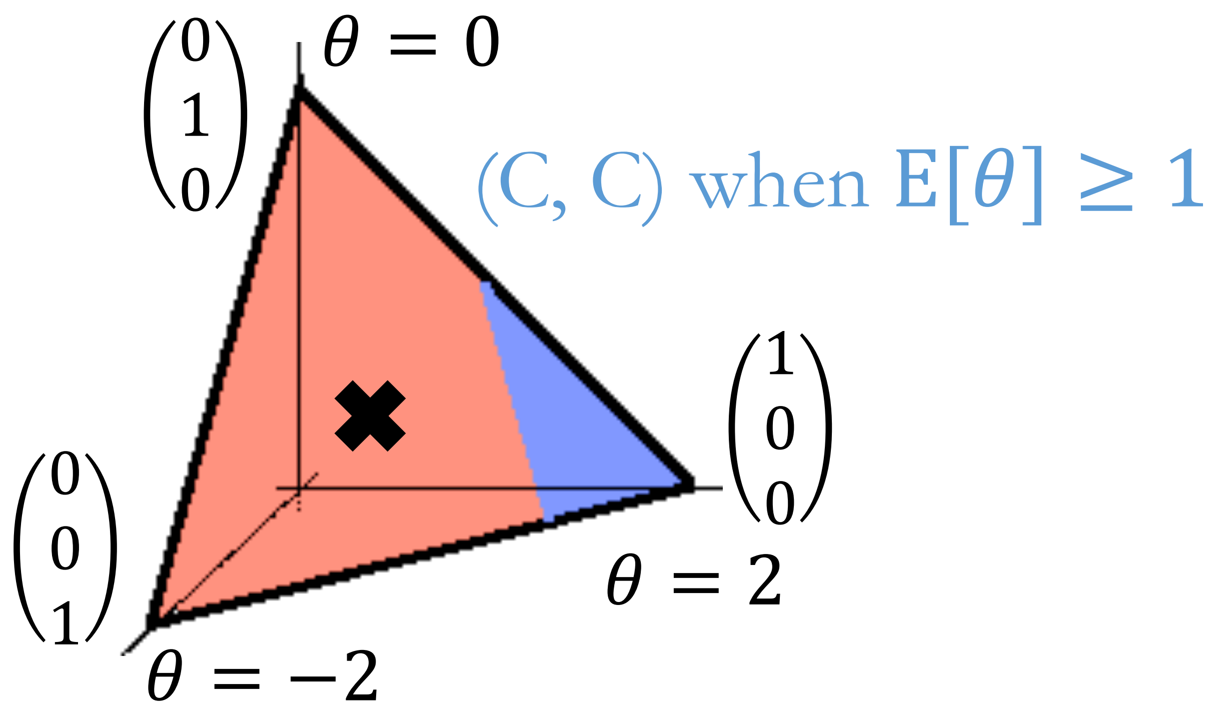
- There is a Quasi-PTAS: We can compute an ϵ-optimal signaling scheme in time $n^{O(\log n/\epsilon^2)}$ [CCDEHT '15].

Prior Decomposition

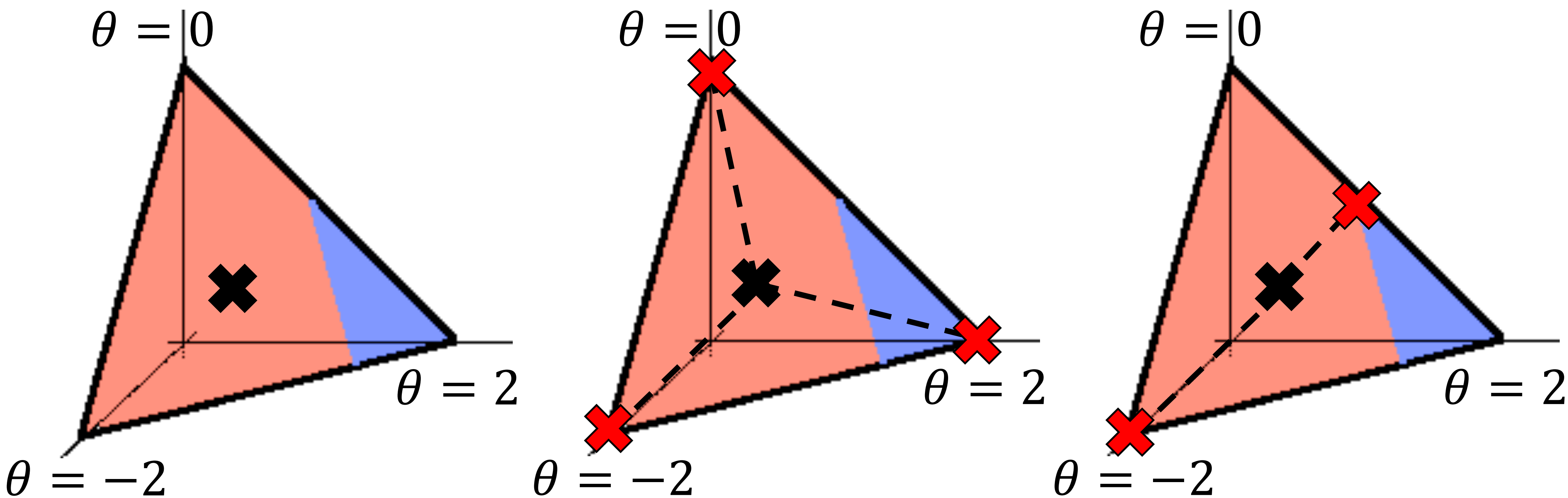
$\theta \sim U\{2, 0, -2\}$

	C	D
C	$\theta - 1$, $\theta - 1$	0, $\theta - 5$
D	$\theta - 5$, 0	-4, -4

Posterior $\mu \in \mathbb{R}^3$: $\mu_1 = \Pr[\theta = 2], \ldots$



Prior Decomposition

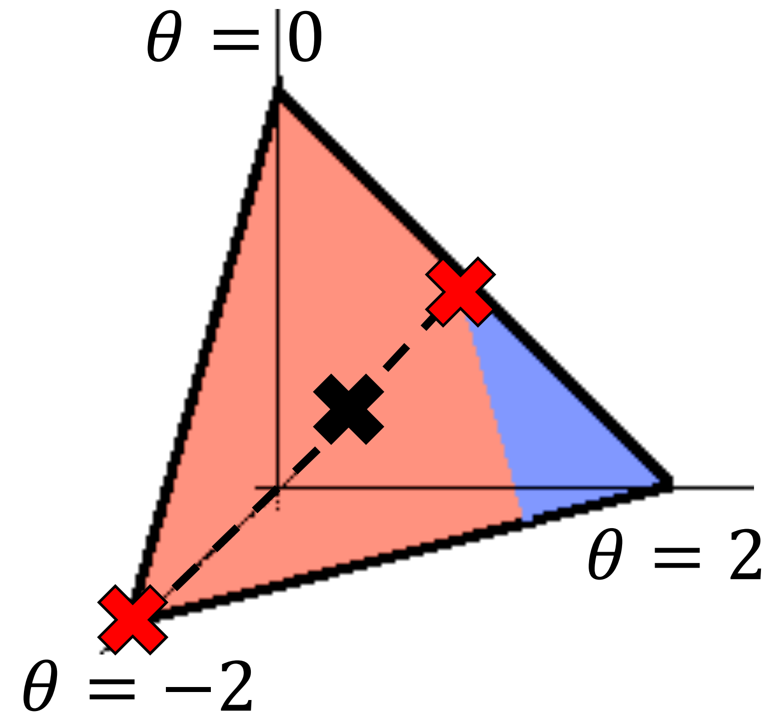


Prior Decomposition

$$\mu_1 = \begin{pmatrix} 1/2 \\ 1/2 \\ 0 \end{pmatrix} \quad \mu_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \lambda = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix} = \frac{2}{3}\mu_1 + \frac{1}{3}\mu_2$$

$$OPT = \frac{2}{3} f(\mu_1) + \frac{1}{3} f(\mu_2) = \frac{2}{3}$$

$$OPT = f^+(\lambda) = \begin{array}{ll} \max & \sum p_i f(\mu_i) \\ s.t. & \sum p_i \mu_i = \lambda \end{array}$$



Quasi-PTAS

- Optimize over all $O(\log n/\epsilon^2)$-sparse signals.
- Given a signal / posterior distribution μ.

$$\boxed{A_\mu} = \mu_1 \boxed{A_1} + \mu_2 \boxed{A_1} + \ldots + \mu_n \boxed{A_n}$$

Quasi-PTAS

$$\boxed{A_\mu} = \mu_1 \boxed{A_1} + \mu_2 \boxed{A_1} + \ \ldots + \mu_n \boxed{A_n}$$

- $\tilde{\mu}=$ sample $O(\log n/\epsilon^2)$ times from μ.
- Standard tail bound + union bound

$$\Longrightarrow \max_{i,j} \left| A_{\tilde{\mu}} - A_\mu \right|_{i,j} \le \epsilon \text{ with probability } 1-\epsilon.$$

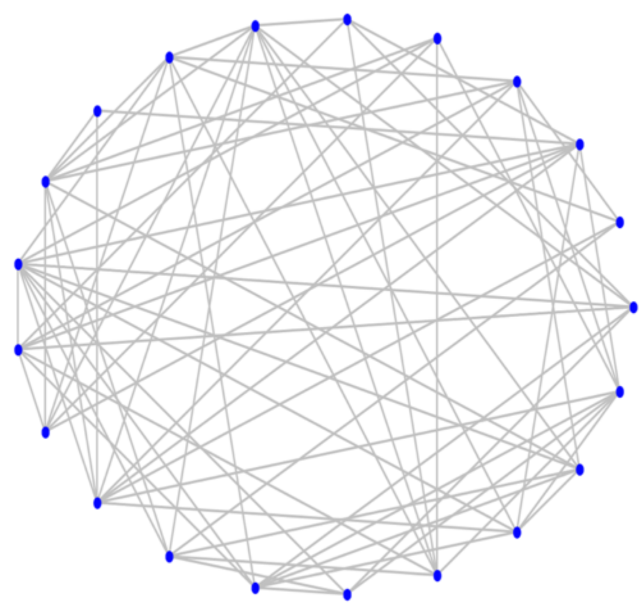
- Value of $A_{\tilde{\mu}}$ is $O(\epsilon)$-close to value of A_μ.

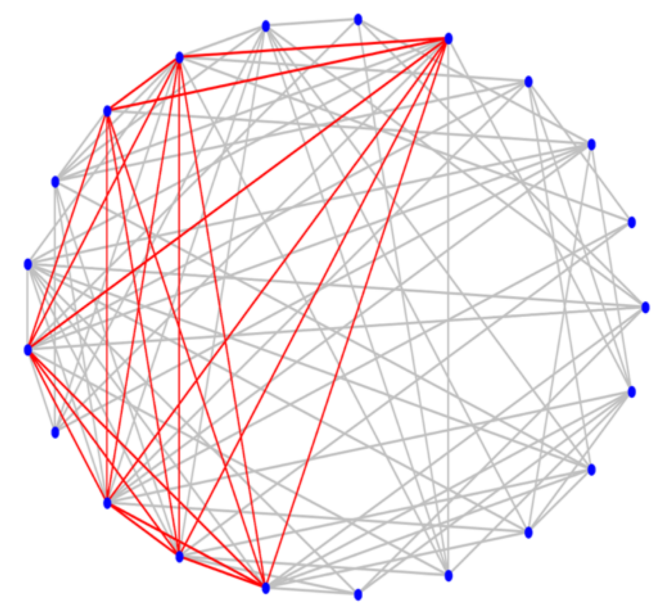
Our Results

- There is a Quasi-PTAS: We can compute an ϵ-optimal signaling scheme in time $n^{O(\log n/\epsilon^2)}$ [CCDEHT '15].

- Tight assuming the Planted Clique Conjecture [BCKS '16] or the Exponential Time Hypothesis [R '16][CK '16].

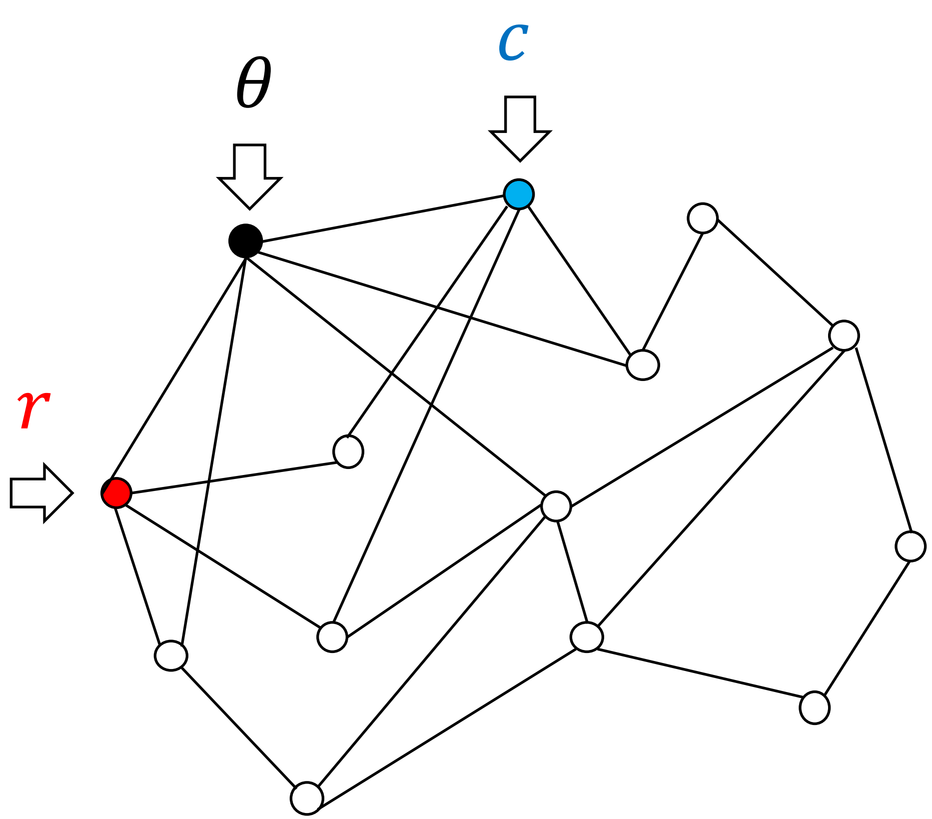
Planted Clique Conjecture





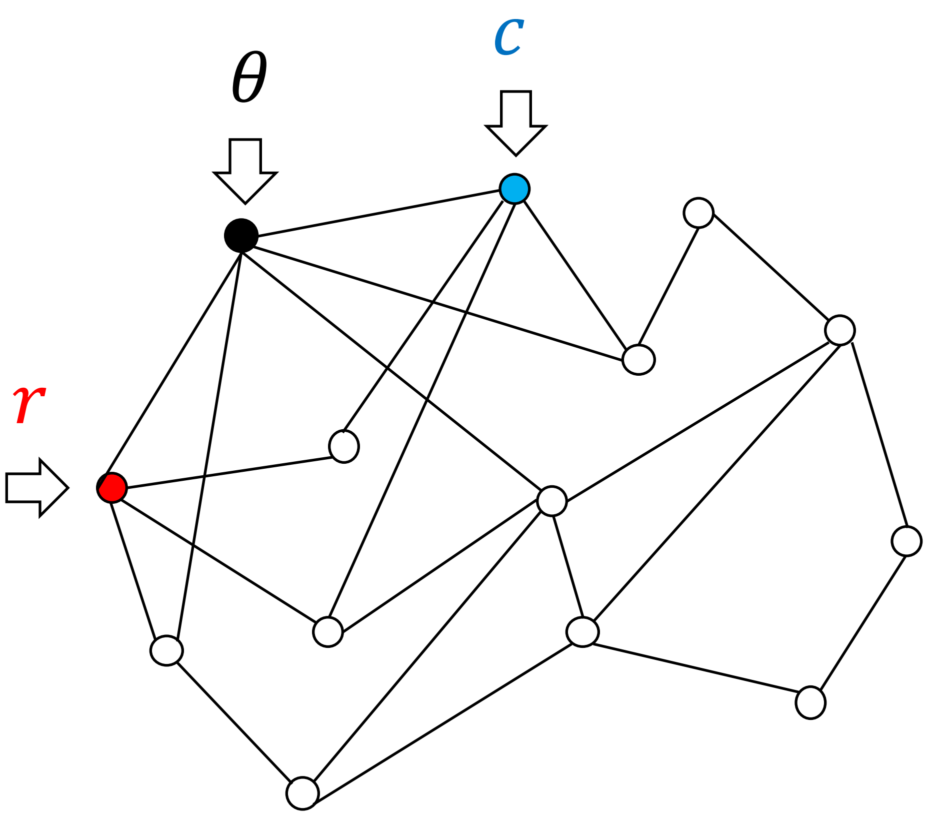
- No poly-time algorithm that recovers a planted k-clique from $G(n, 1/2)$ with constant success probability for $k = o(\sqrt{n})$ and $k = \omega(\log n)$

Security Games on Graphs [Dughmi'14]



- Given $G = (V, E)$,
  - State of nature $\theta \sim uni(V)$,
  - **Row** picks $r \in V$,
  - **Col** picks $c \in V$.
- Objective (zero-sum):
  - **Row** wants to be adjacent to θ,
  - **Col** wants to catch **Row** or θ.

Security Games on Graphs [Dughmi'14]



Row's payoff $= 1$

Nature picks $\theta \in V$,

Row and **Col** pick $r, c \in V$.

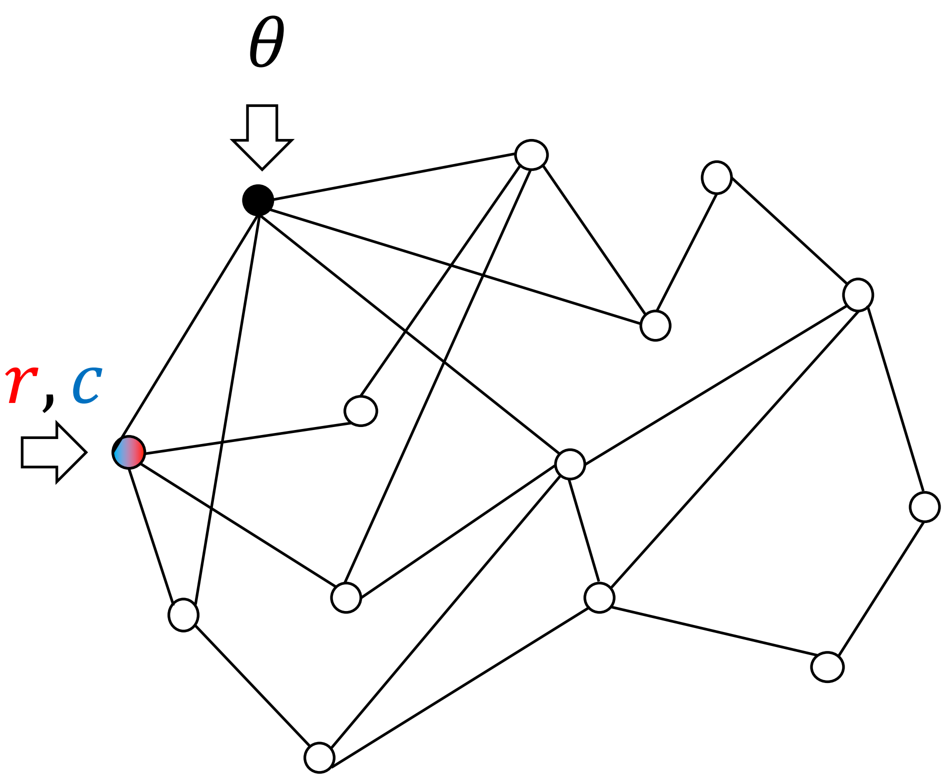
Row's payoff:

$+1$ if $(\theta, r) \in E$,

-1 if $\mathrm{c} = \theta$,

-1 if $\mathrm{c} = \mathrm{a}$.

Security Games on Graphs [Dughmi'14]



Row's payoff $= 1 - 1 = 0$

Nature picks $\theta \in V$,

Row and **Col** pick $r, c \in V$.

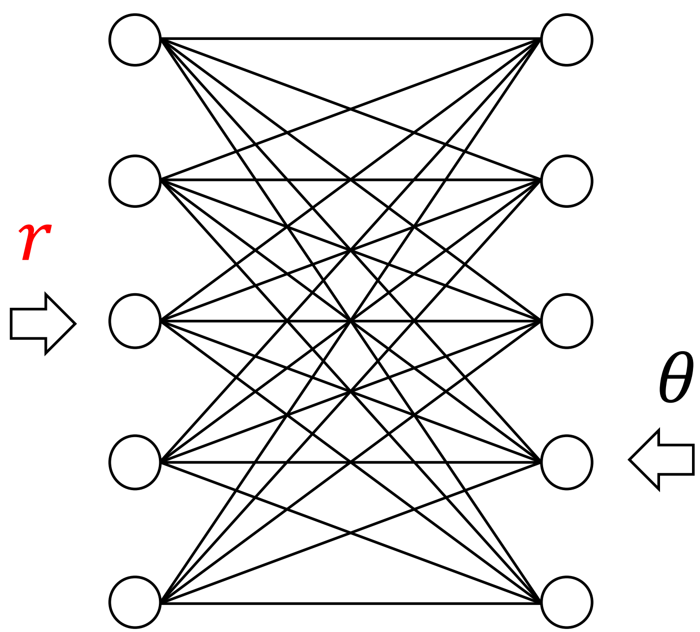
Row's payoff:

$+1$ if $(\theta, r) \in E$,

-1 if $\mathrm{c} = \theta$,

-1 if $\mathrm{c} = \mathrm{a}$.

Security Games on Graphs [Dughmi'14]

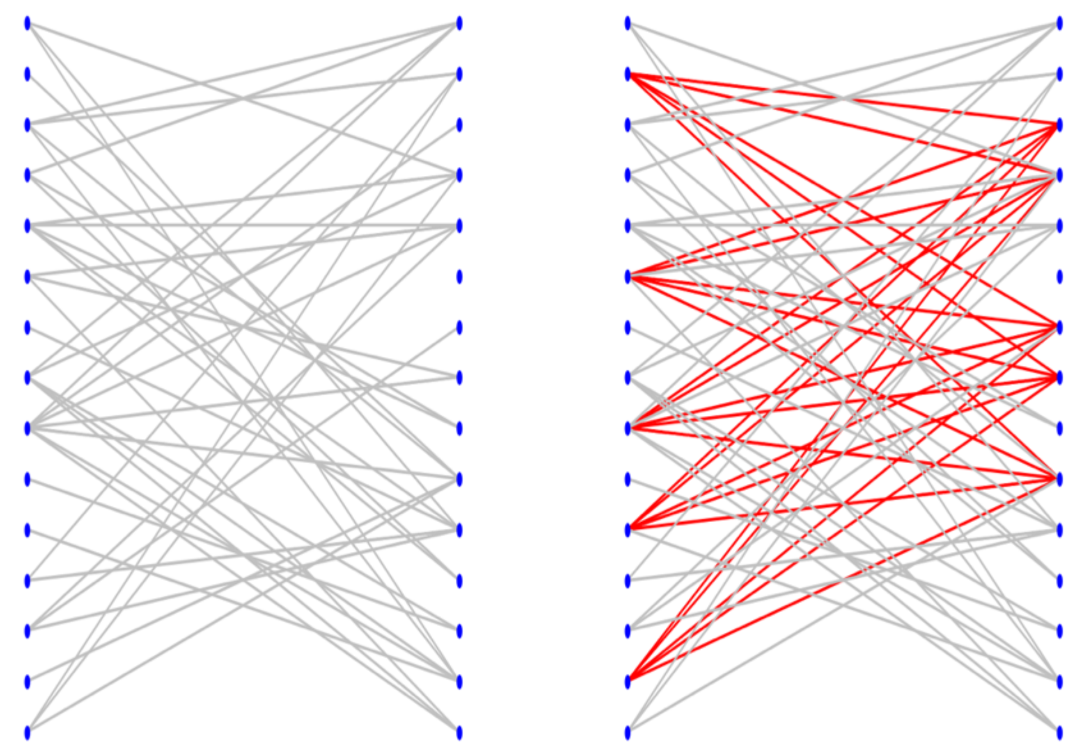


- Asymmetry of payoffs.
- Principal reveals

  $\theta \in L$ or $\theta \in R$.
- **Row** chooses uniformly from the other side.
  - Always have $(\theta, r) \in E$.
  - Hard for **Col** to catch.

Security Games on Graphs [Dughmi'14]

- Cliques are good for **Principal** and **Row**.
- Optimal Signaling ≈ partitions the graph into disjoint dense subgraphs.



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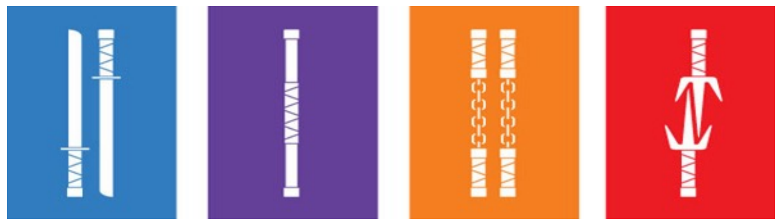
Second Price Auctions [EFGLT '12] [MS '12]



1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

- Full information:
  One buyer bids 1,
  revenue = 0.
- No information:
  Everyone bids 1/4,
  revenue = 1/4.
- Optimal: revenue = 1/2.

Second Price Auctions [EFGLT '12] [MS '12]

$$A = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$f\begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{pmatrix} = 1/4$$

$$f\begin{pmatrix} 1/2 \\ 1/2 \\ 0 \\ 0 \end{pmatrix} = 1/2$$

Mixture Selection [CCDEHT '15]

$$OPT = f^+(\lambda)$$
$$= \max \quad \sum p_i f(\mu_i)$$
$$s.t. \quad \sum p_i \mu_i = \lambda$$

$$f_{\text{zerosum}}(\mu) = \max_x \min_y (x^T A^\mu y).$$

$$f_{\text{auction}}(\mu) = \max{}_2(A\mu).$$

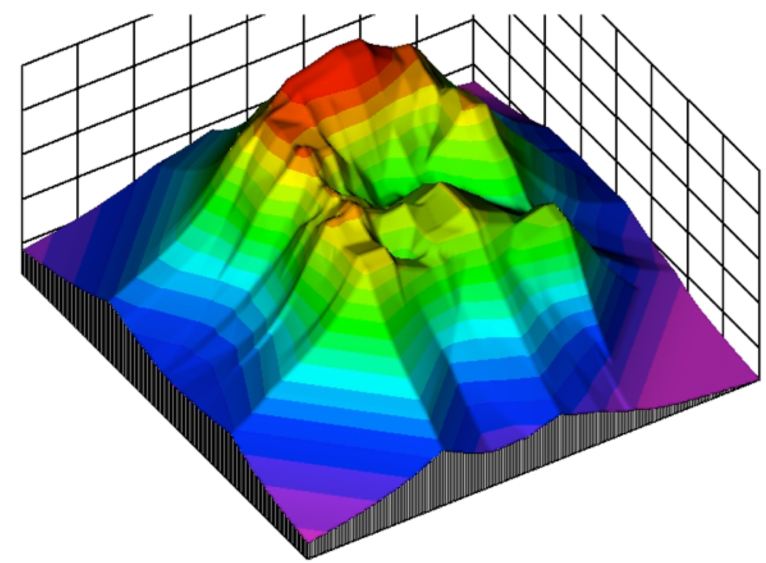
$$f_{\text{voting}}(\mu) = \cdots$$

- An optimization problem naturally arises in signaling.
- Optimal algorithm for all under one algorithmic framework.

Mixture Selection [CCDEHT ’15]

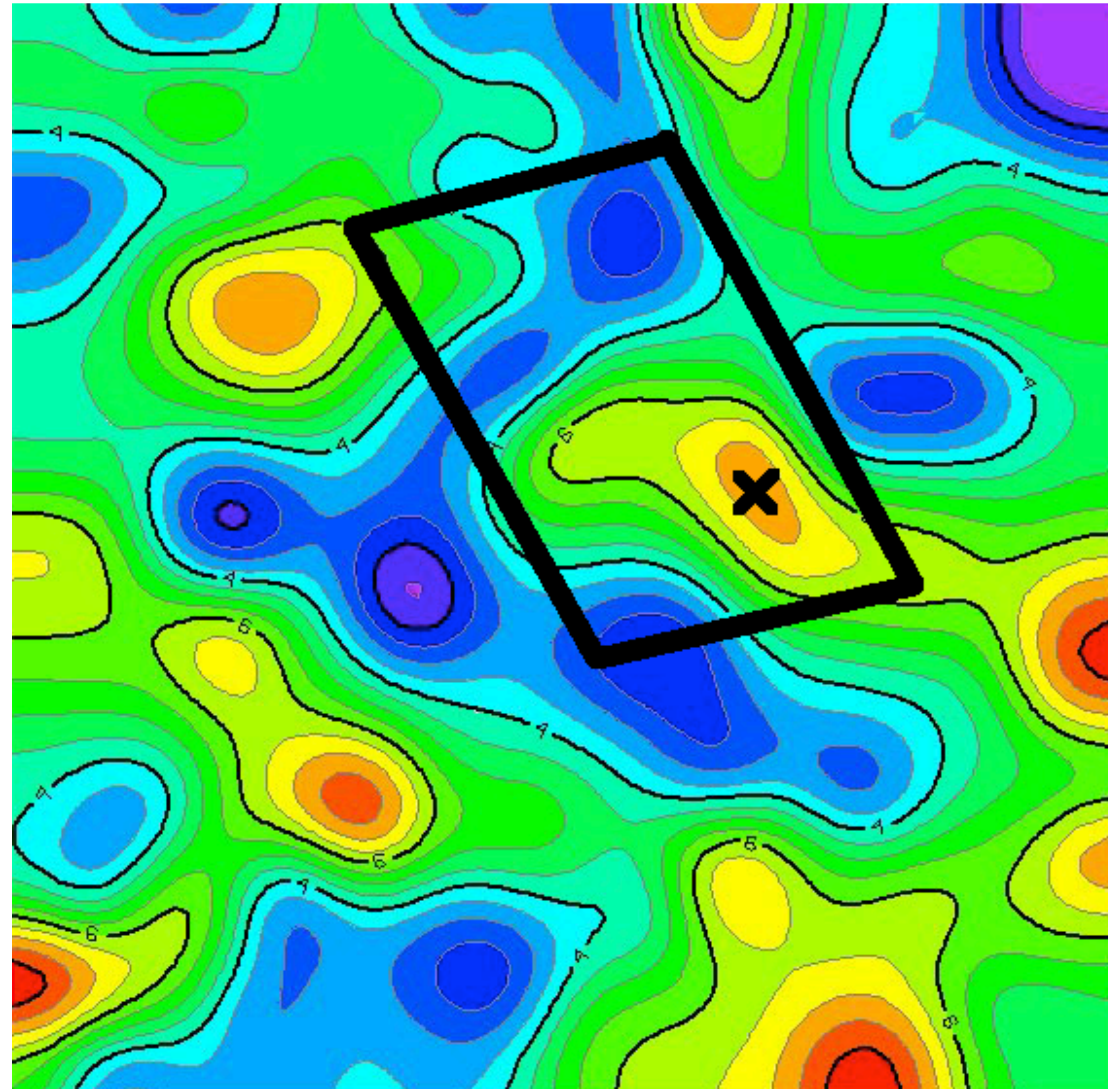
- Parameter: A function $g: [0,1]^n \to [0,1]$.

- Input: A matrix $A \in [0,1]^{n \times m}$.



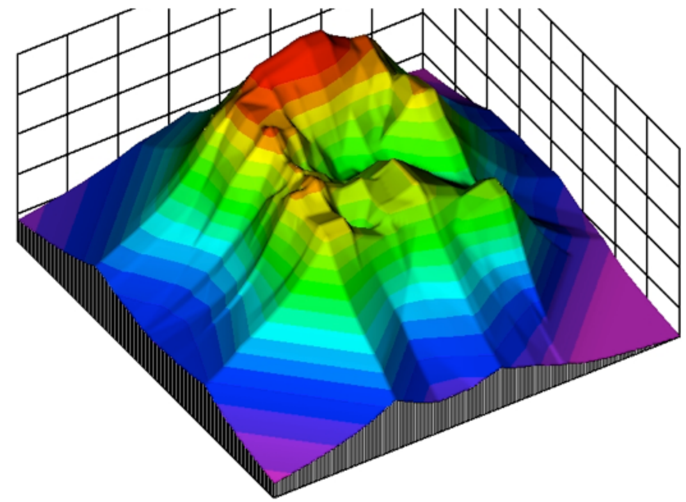
- Goal: $\max_{x \in \Delta_m} g(Ax)$.

$$\max_{x \in \Delta_m} g(Ax)$$

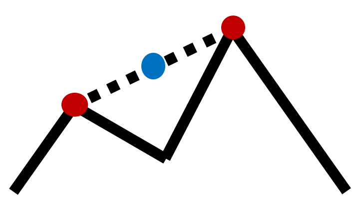


Optimal Signaling

Mixture Selection



$$\max \ \sum p_i g(A\mu_i)$$
$$s.t. \ \sum p_i \mu_i = \lambda$$



$$\max \ g(Ax)$$

Mixture Selection Framework

Two "smoothness" parameters that tightly control the complexity of Mixture Selection and Optimal Signaling.

- α-Lipschitz in L_∞: $|g(v_1) - g(v_2)| \le \alpha \cdot \|v_1 - v_2\|_\infty$
- β-Noise stable: An adversary corrupts a random subset of v, $g(v)$ changes by at most $\beta\epsilon$ if no individual coordinate is corrupted with marginal probability more than ϵ.

Our Results [CCDEHT '15]

- Main theorem: If g is α-Lipschitz and β-stable, then there is an algorithm for ϵ-optimal signaling with runtime

$$m^{O\left((\alpha/\epsilon)^2 \log(\beta/\epsilon)\right)} \cdot T_g$$

Problem	α-Lipschitz	β-Stable	Runtime
Signaling in Normal Form Games	2	poly(n)	Quasi-PTAS
Signaling in Probabilistic Auctions	1	2	PTAS
Persuading Voters	$O(1)$	$O(1)$	PTAS

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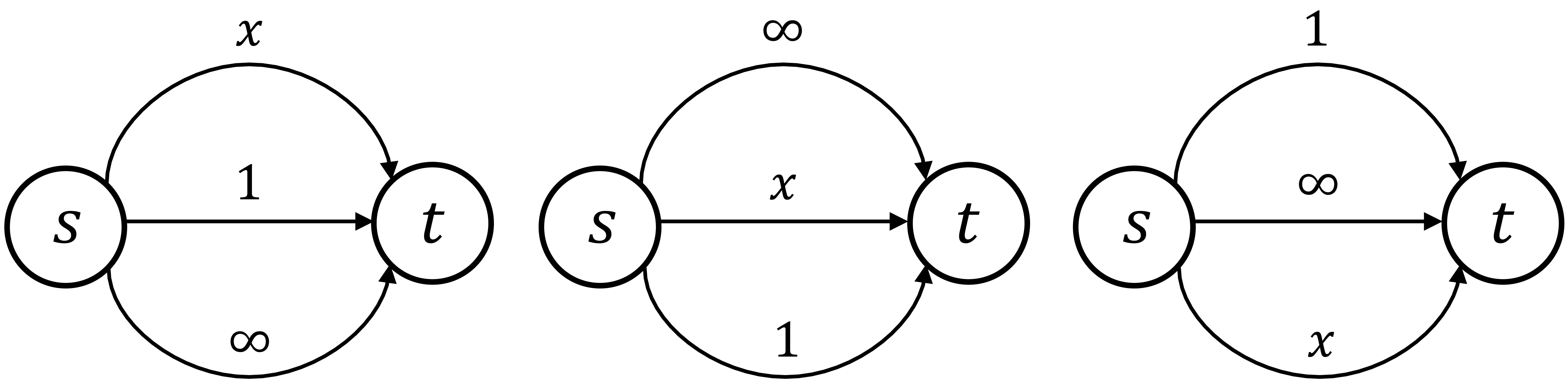
Information Structure Design

- Public vs. private signaling schemes.
- Co-designing the information structure and the mechanism.
- Collecting and trading information.
- Real-world applications.

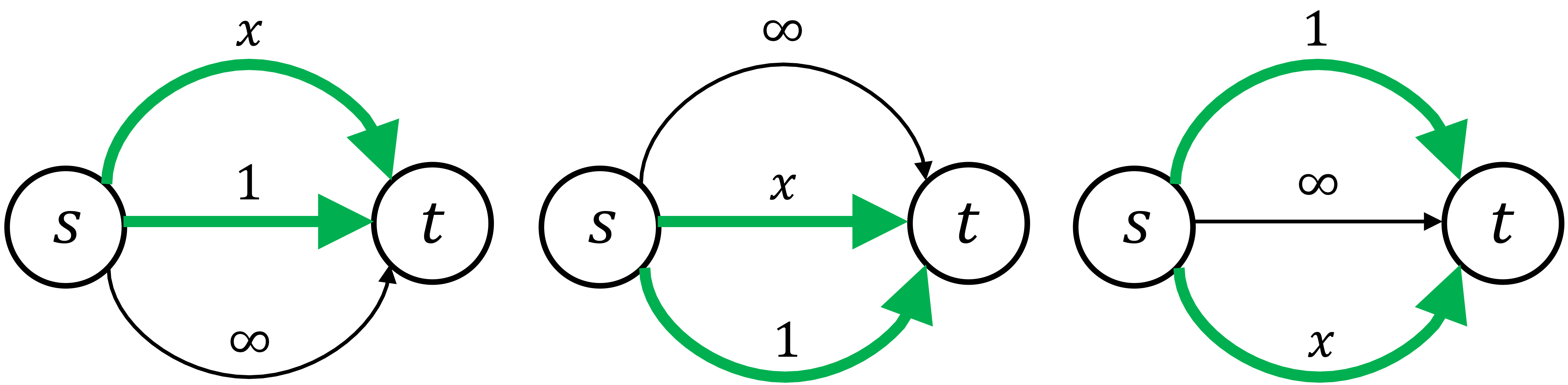
Private vs. Public

- What if the principal can send different signals to different agents?
- The principal can do better (in routing games)! [CDX '17]

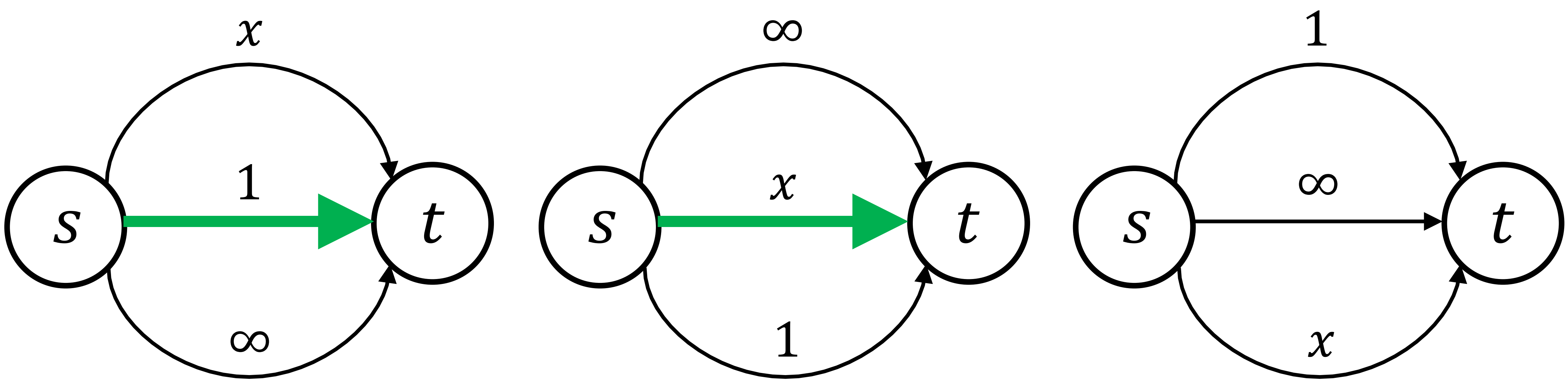
Public vs. Private: Pigou's Example



Public vs. Private: Pigou's Example



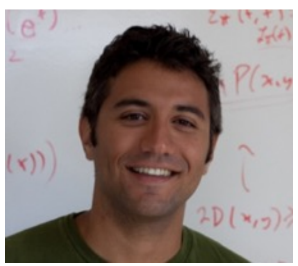
Public vs. Private: Pigou's Example



Collaborators

Umang Bhaskar

Ho Yee Cheung



Shaddin Dughmi

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Young Kun Ko

Chaitanya Swamy

Shang-Hua Teng

Haifeng Xu

My Thesis

- Network Routing Games
- Normal Form Games
- Mixture Selection Framework
- Future Work