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Large-margin Structured Learning for Link Ranking

Stephen H. Bach[1], Bert Huang[1], and Lise Getoor[2]
[1] University of Maryland, College Park
[2] University of California, Santa Cruz

Introduction

- **Link prediction** is the task of predicting which nodes are linked in a network
- We introduce a **large-margin method** for learning to **rank in structured domains**, where ranking positions are interdependent
- By training a structured predictor to rank, we **improve performance** on link prediction as measured by a ranking loss

Large-margin Structured Learning

With observations $\mathbf{X}$ and training output $\mathbf{Y}$ in structured output space $\mathcal{Y}$, find parameters λ of a function f_λ such that the following condition holds:

$$f_\lambda(\mathbf{Y}, \mathbf{X}) \geq f_\lambda(\tilde{\mathbf{Y}}, \mathbf{X}) + L(\mathbf{Y}, \tilde{\mathbf{Y}}), \quad \forall \tilde{\mathbf{Y}} \in \mathcal{Y}$$

which leads to this learning objective:

$$\min_{\lambda} \quad \frac{1}{2}\|\lambda\|^2 + C \max_{\tilde{\mathbf{Y}} \in \mathcal{Y}} \left(f_\lambda(\tilde{\mathbf{Y}}, \mathbf{X}) - f_\lambda(\mathbf{Y}, \mathbf{X}) + L(\mathbf{Y}, \tilde{\mathbf{Y}}) \right)$$

Perform subgradient descent, taking steps via

$$\nabla_\lambda = \lambda + C\left(\phi(\mathbf{Y}^\star, \mathbf{X}) - \phi(\mathbf{Y}, \mathbf{X})\right)$$

where

$$\mathbf{Y}^\star = \arg\max_{\tilde{\mathbf{Y}}} f_\lambda(\tilde{\mathbf{Y}}, \mathbf{X}) + L(\mathbf{Y}, \tilde{\mathbf{Y}})$$

Computational challenge is finding $\mathbf{Y}^\star$ efficiently, which depends on forms of both f_λ and L

Structured Ranking

Train a structured predictor to rank via large-margin learning, i.e., use ranking loss for $L(\mathbf{Y}, \tilde{\mathbf{Y}})$

ROC loss (one minus area under ROC curve):

$$L^{\mathrm{ROC}}(\mathbf{Y}, \tilde{\mathbf{Y}}) \equiv \frac{1}{|\mathcal{P}||\mathcal{N}|} \sum_{(i,j)|Y_i > Y_j} \mathbb{I}\left[\tilde{Y}_j > \tilde{Y}_i\right]$$

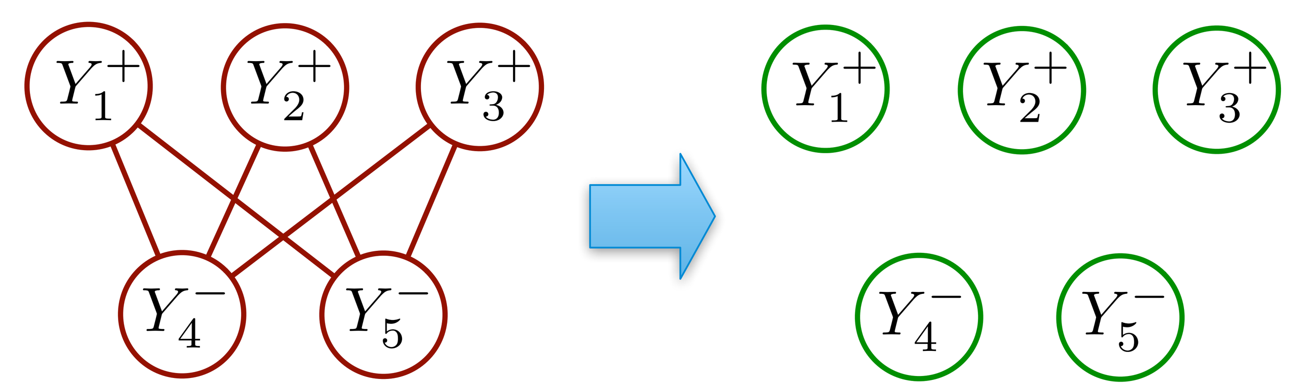
Pseudo ROC loss (convex lower bound):

$$L^{\mathrm{pROC}}(\mathbf{Y}, \tilde{\mathbf{Y}}) \equiv \frac{1}{|\mathcal{P}||\mathcal{N}|} \sum_{(i,j)|Y_i > Y_j} \max\left\{0, \tilde{Y}_j - \tilde{Y}_i\right\}$$

Gradient Computation

Finding $\mathbf{Y}^\star$ with L^{pROC} is a non-convex objective. We iteratively solve a linear approximation to find a local optimum and reduce problem size.

Approximate $|\mathcal{P}||\mathcal{N}|$ terms with $|\mathcal{P}| + |\mathcal{N}|$ terms



where $\mathcal{P}$ is set of true links and $\mathcal{N}$ is false links

Algorithm 1 Gradient Computation for L^{pROC}

Input: model f_λ, input $\mathbf{X}$, training output $\mathbf{Y} = \mathcal{P} + \mathcal{N}$, initial guess $\tilde{\mathbf{Y}}$

Output: $\mathbf{Y}^\star = \arg\max_{\tilde{\mathbf{Y}}} f_\lambda(\tilde{\mathbf{Y}}, \mathbf{X}) + L^{\mathrm{pROC}}(\mathbf{Y}, \tilde{\mathbf{Y}})$

while not converged **do**
  $\tilde{\mathbf{Y}} \leftarrow \mathtt{Sort}(\tilde{\mathbf{Y}})$
  $\mathbf{c} \leftarrow \mathtt{UpdateCounts}(\tilde{\mathbf{Y}})$
  $\tilde{\mathbf{Y}} \leftarrow \arg\max_{\tilde{\mathbf{Y}}} f_\lambda(\tilde{\mathbf{Y}}, \mathbf{X}) + \frac{1}{|\mathcal{P}||\mathcal{N}|} \sum_{i=1}^{n} c_i \tilde{Y}_i$
end while
$\mathbf{Y}^\star \leftarrow \tilde{\mathbf{Y}}$

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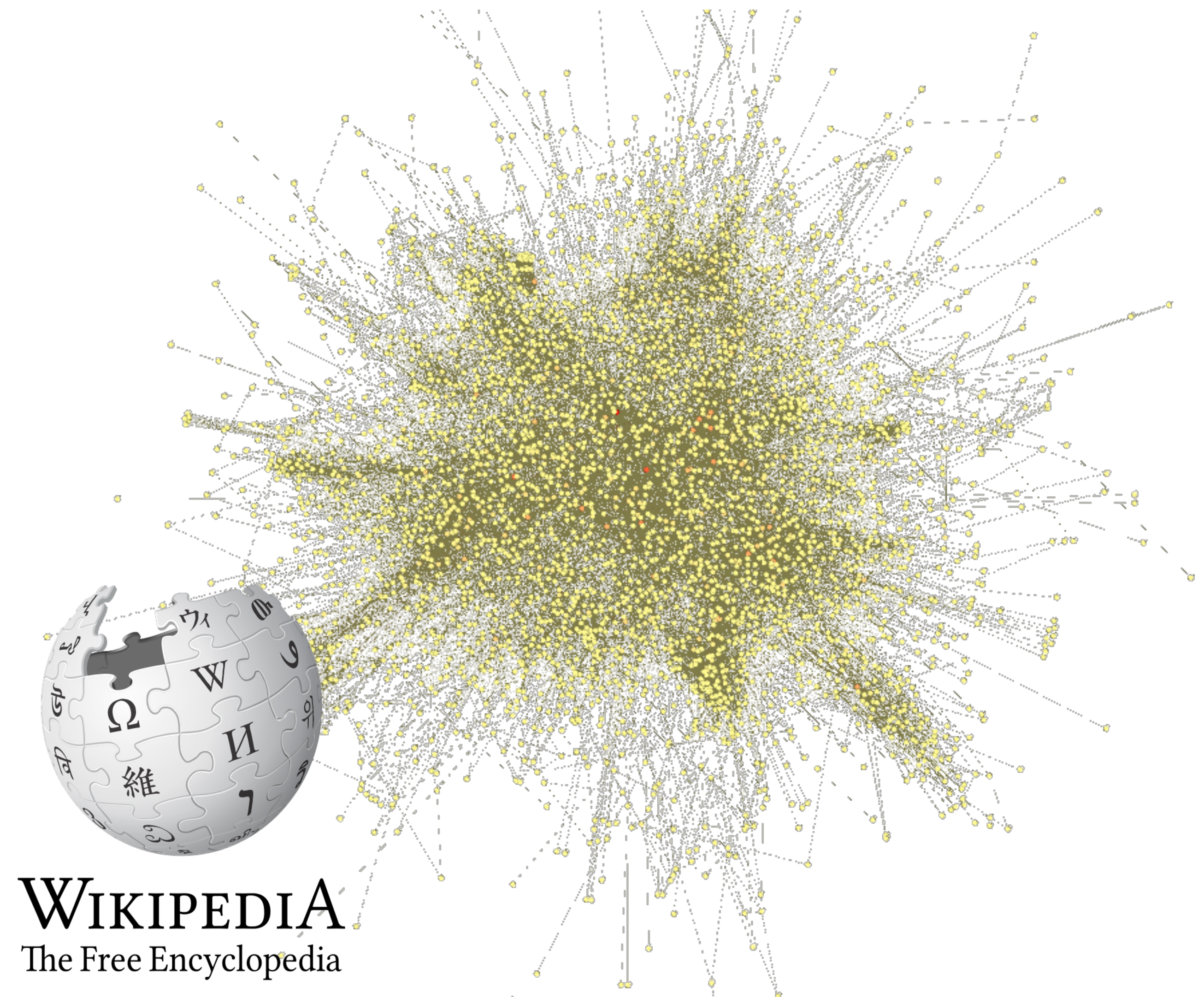
Preliminary Experimental Results

Task

Predict hyperlinks in set of featured Wikipedia articles using text of articles, category of articles, and structural dependencies among links.



Structured Predictor

Hinge-loss Markov random fields [Bach et al., UAI 2013] are **scalable** graphical models over **continuous** random variables

Most probable assignment to continuous variables can be used as a ranking of possible hyperlinks

Results

	ROC	P-R	Acc.
pROC	**0.869 (0.046)**	**0.601 (0.106)**	0.804 (0.112)
L1	0.843 (0.047)	0.556 (0.111)	**0.852 (0.086)**
Perceptron	0.842 (0.049)	0.524 (0.105)	0.667 (0.145)

Table 1: Average area under ROC and precision-recall curves and 0-1 accuracy with different loss functions during large-margin learning. Standard deviations are listed in parentheses. Scores statistically equivalent to the best scoring method by metric are typed in bold.

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