

# Super-resolution with Structured Motion

Gabby Litterio, David Lizarazo-Ferro,  
Pedro Felzenszwalb, Rashid Zia

Brown University

## Overview



- **Super-resolution**: reconstruct high-resolution image from one or more low-resolution images.

## Overview



- **Super-resolution**: reconstruct high-resolution image from one or more low-resolution images.
- Focus on imaging constraints, low-level priors and sensor/camera motion.
- Connection between super-resolution and **box filters**.
- We recover high-resolution images using motion blur.

Motion + Low-resolution  $\rightarrow$  High-resolution

# Motion + Low-resolution $\rightarrow$ High-resolution

## Multiple sub-pixel translations



1 of 64 images



Interlaced data



Deconvolution

# Motion + Low-resolution $\rightarrow$ High-resolution

## Multiple sub-pixel translations



1 of 64 images



Interlaced data



Deconvolution

Motion blur

# Motion + Low-resolution $\rightarrow$ High-resolution

## Multiple sub-pixel translations



1 of 64 images

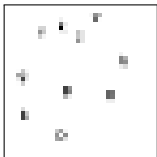


Interlaced data



Deconvolution

## Motion blur



# Motion + Low-resolution $\rightarrow$ High-resolution

## Multiple sub-pixel translations



1 of 64 images

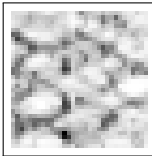
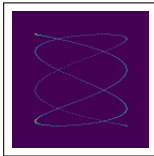
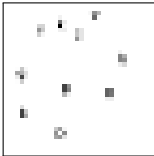


Interlaced data



Deconvolution

## Motion blur





# Motion + Low-resolution $\rightarrow$ High-resolution

## Multiple sub-pixel translations



1 of 64 images

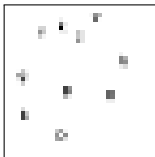


Interlaced data

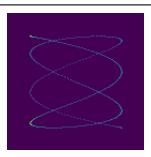


Deconvolution

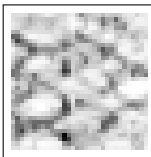
## Motion blur



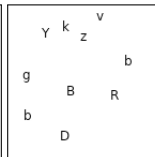
Static Image



Trajectory



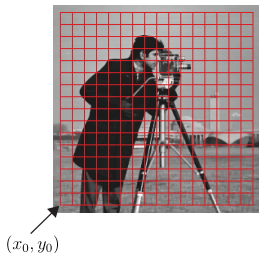
Image



Reconstruction

## Imaging Model

Camera with a translating sensor (or equivalent).



Sensor integrates brightness  $g(x, y)$  over square pixels.

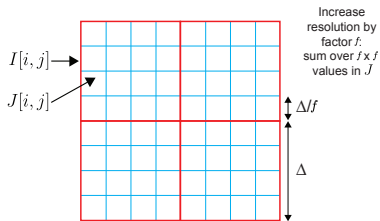
Static exposure

$$I[i, j] = \int_0^{\Delta} \int_0^{\Delta} g(x_0 + i\Delta + x, y_0 + j\Delta + y) dx dy.$$

Moving exposure

$$I[i, j] = \int_0^T \int_0^{\Delta} \int_0^{\Delta} g(x_0(t) + i\Delta + x, y_0(t) + j\Delta + y) dx dy dt.$$

# Box Convolution



$$I = (J \otimes B) \downarrow_f$$

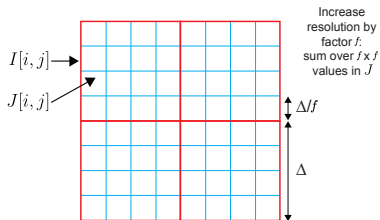
$f$ : resolution increase factor

$J$ : high-resolution image

$I$ : low-resolution image

$B$ :  $f \times f$  box filter

# Box Convolution



$$I = (J \otimes B) \downarrow_f$$

$f$ : resolution increase factor

$J$ : high-resolution image

$I$ : low-resolution image

$B$ :  $f \times f$  box filter

## Capturing a super-resolution image

1)  $f \times f$  subpixel shifts  $\rightarrow$  collection of images  $\{I_{k,l}\}$ .

2) Interlacing  $\{I_{k,l}\} \rightarrow H$ .

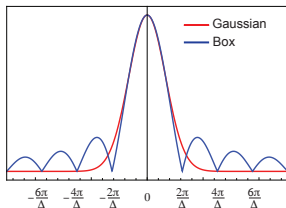
$$H = J \otimes B$$

3) Recovering  $J$  involves a deconvolution.

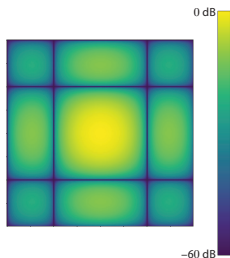
## Box Filter

$$H = J \otimes B$$

- Luckily, a box filter is a **bad** low-pass filter.
- $H$  determines **most** of the Fourier coefficients of  $J$ .
- In this setting sparse signals can be recovered with convex optimization [Donoho 89] [Candes, Romberg, Tao 06] [F 25].



FT of 1D Gaussian vs box



DFT of a 4x4 box

## Revisiting the Limits of Super-Resolution [Baker and Kanade]

[Baker and Kanade]

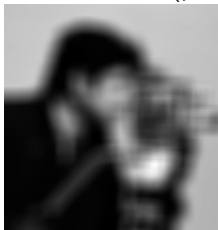
The Limits of Super-Resolution and How to Break them, 2002.

## Revisiting the Limits of Super-Resolution [Baker and Kanade]

[Baker and Kanade]

The Limits of Super-Resolution and How to Break them, 2002.

Two (particular) solutions  $J$  with  $H = (J \otimes B)$



$H$



Quadratic prior [BK]



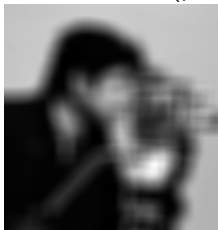
TV prior

## Revisiting the Limits of Super-Resolution [Baker and Kanade]

[Baker and Kanade]

The Limits of Super-Resolution and How to Break them, 2002.

Two (particular) solutions  $J$  with  $H = (J \otimes B)$



$H$



Quadratic prior [BK]



TV prior

Sparse images can be recovered via convex optimization:

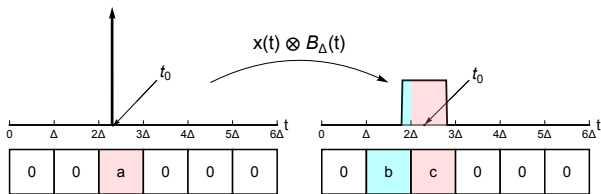
$$J^* = \operatorname{argmin}_J \|H - (J \otimes B)\|_2^2 + \lambda TV(J)$$

$$TV(J) = \sum_{i,j} |J[i,j] - J[i+1,j]| + |J[i,j] - J[i,j+1]|$$



# Motion Blur Can Help

Localizing a point source in one dimension

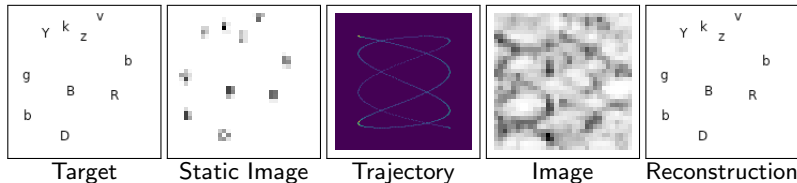


$$a = b + c,$$

$$t_0 = (k + 0.5)\Delta + c/(b + c).$$

## Motion Blur for Super-Resolution

We can use *large* motions to superimpose multiple low-res images.



$$I = (J \otimes Q \otimes B) \downarrow_f$$

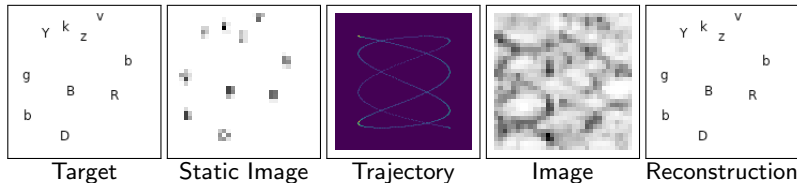
$Q$ : occupancy mask (blur kernel)

Reconstruction:

$$J^* = \operatorname{argmin}_J \|I - (J \otimes Q \otimes B) \downarrow_f\|_2^2 + \lambda \|J\|_1$$

## Motion Blur for Super-Resolution

We can use *large* motions to superimpose multiple low-res images.



$$I = (J \otimes Q \otimes B) \downarrow_f$$

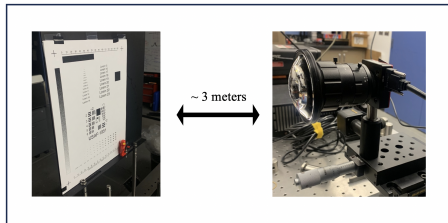
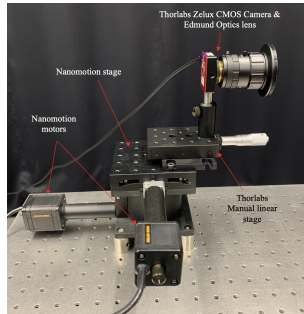
$Q$ : occupancy mask (blur kernel)

Reconstruction:

$$J^* = \operatorname{argmin}_J \|I - (J \otimes Q \otimes B) \downarrow_f\|_2^2 + \lambda \|J\|_1$$

Blind deconvolution (solving for  $Q/J$  simultaneously) can increase resolution

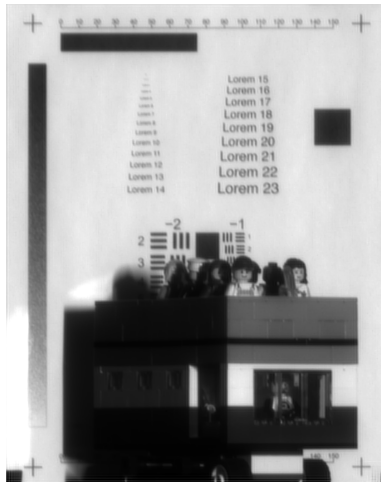
# Experimental Setup



## Multiple Shifts



Static image (1 of 64)

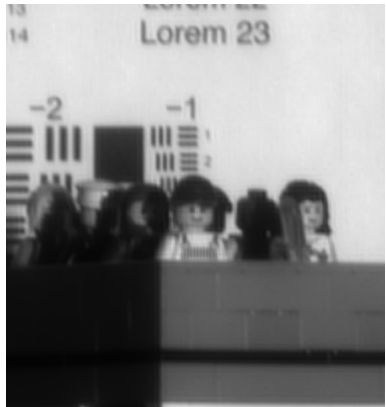


TV reconstruction

## Multiple Shifts

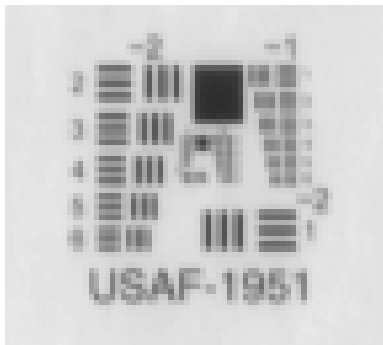


Static image (1 of 64)

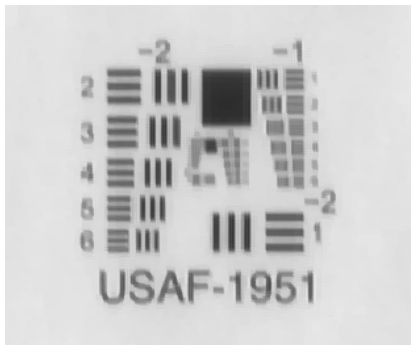


TV reconstruction

# Scanning Motion



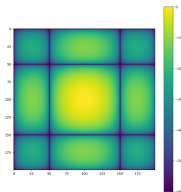
Static image



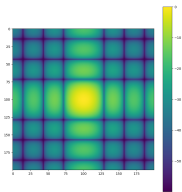
TV reconstruction

## Summary and Future Work

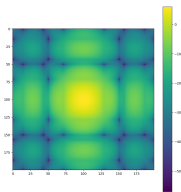
- Sparse image priors resolve the ambiguity of super-resolution under translation.
- We can recover high-resolution images *using* motion blur.
- Other implementations: pixel-shift camera, nano-positioning stage, satellite/drone, etc.
- Multiple pixel sizes (magnifications) can remove all ambiguities.



$|\text{DFT}(B_4)|$



$|\text{DFT}(B_7)|$



$|\text{DFT}(B_4)| + |\text{DFT}(B_7)|$