

The role of arithmetic in fast parallel matrix inversion

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Abstract

In the last two decades several NC algorithms for solving basic linear algebraic problems have appeared in the literature. This interest was clearly motivated by the emergence of a parallel computing technology and by the wide applicability of matrix computations. The traditionally adopted computation model, however, ignores the arithmetic aspects of the applications, and no analysis is currently available demonstrating the concrete feasibility of many of the known fast methods. In this paper we give strong evidence to the contrary, on the sole basis of the issue of robustness, indicating that some theoretically brilliant solutions fail the severe test of the “Engineering of Algorithms”. We perform a comparative analysis of several well known numerical matrix inversion algorithms under both fixed- and variable-precision models of arithmetic. We show that, for most methods investigated, a typical input leads to poor numerical performance, and that in the exact-arithmetic setting no benefit derives from conditions usually deemed favorable in standard scientific computing. Under these circumstances, the only algorithm admitting sufficiently accurate NC implementations is Newton’s iterative method, and the word size required to guarantee worst-case correctness appears to be the critical complexity measure. Our analysis also accounts for the observed instability of the considered superfast methods when implemented with the same floating-point arithmetic that is perfectly adequate for the fixed-precision approach.