Nanowire Addressing with Randomized-Contact Decoders

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Talk Outline

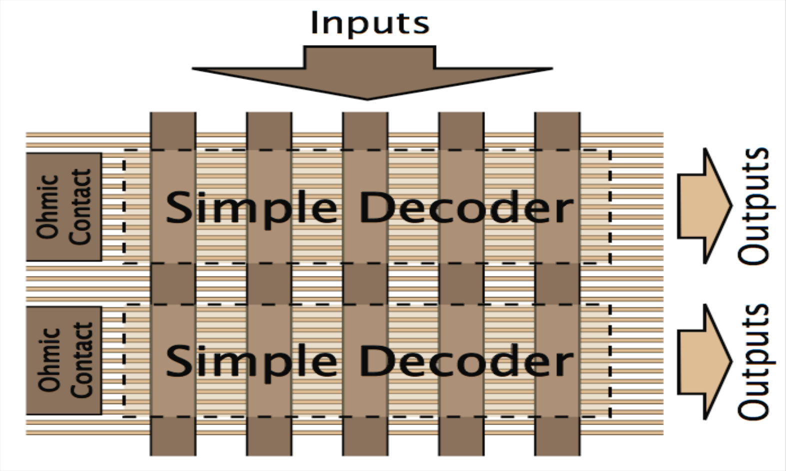
I. **Nanowire (NW) decoders and their applications.**
II. Decoding technologies.
III. NW addressability.
IV. The randomized-contact decoder (RCD).
V. Performance of RCD.
VI. Conclusions

What is a Nanowire (NW) Decoder?

- A **decoder** uses a small number of inputs wires to activates one of several output wires.
  - Used in crossbars, PLAs, etc…
- A **nanowire decoder**, mesowire (MW) inputs control NW outputs.
- **<u>Goal</u>**: Efficiently go from mesoscale (~100nm pitch) to nanoscale (~10nm pitch).

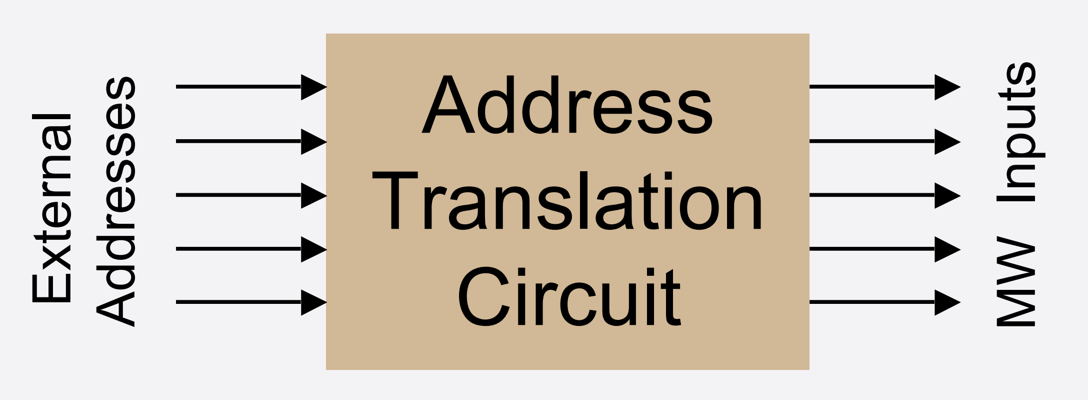
The Nanowire Decoder

- Efficient bridging between scales is best done by assembling multiple simple decoders.
- Decoders share MWs.
- Each decoder has:
  - Mesoscale contact(s) for each group of NWs.
  - MWs address individual NWs within each group.



Challenges for NW Decoders

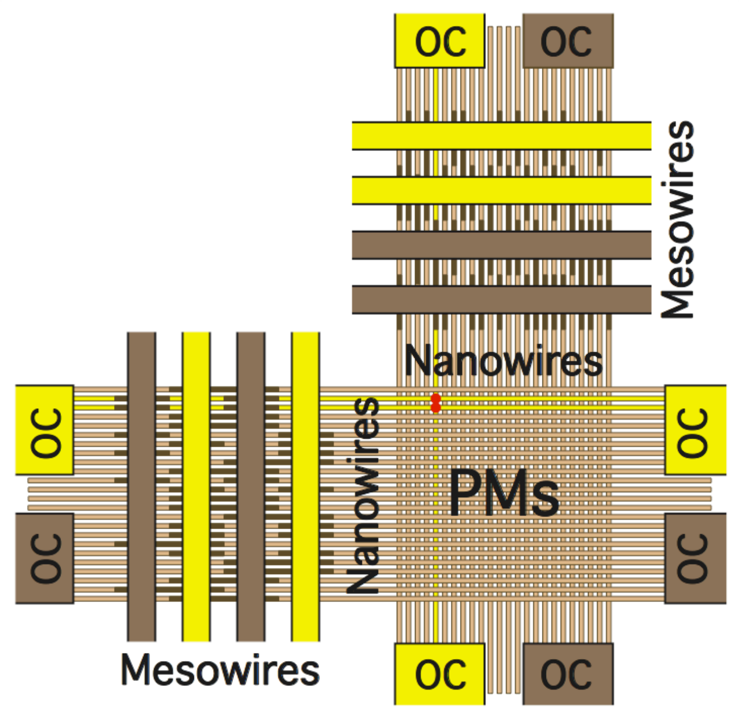
- NW addresses assigned stochastically.
  - Some NWs may not be addressable.
  - All addresses must be discovered!



- Address translation circuit (ATC) is required to map external to internal addresses.

Decoder Applications

- Crossbar memories and PLAs
  - MWs activate one NW in each dimension of crossbar.
  - NWs provide control over programmable crosspoints.
- Biological sampling
  - Antibodies are attached to NWs.
  - Charge-carrying proteins lock into antibodies.
  - NW resistance increases as proteins attach.



Our Results

- Model randomized-contact decoder (RCD) as well as faults in decoder assembly.
- Obtain analytical bounds on the probability that most, or all, NWs are addressable
- Compare RCD address translation strategies.
- Compare RCD with other types of decoders.

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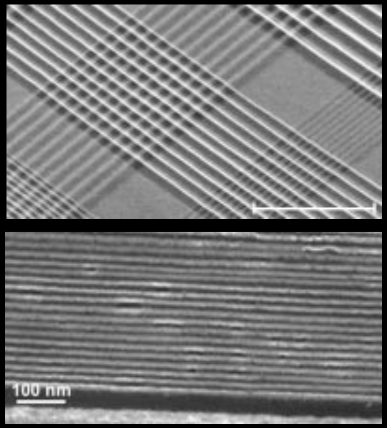
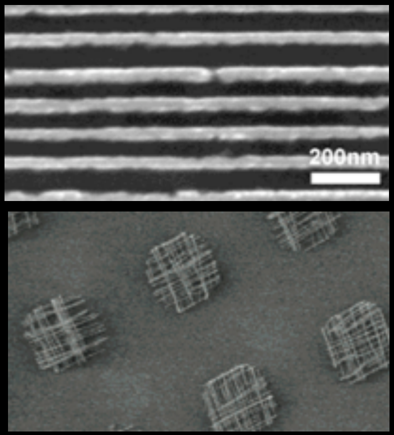
Uniform Nanowires

- Created using nanolithography or stamping.



SNAP NWs
(Heath, Caltech)



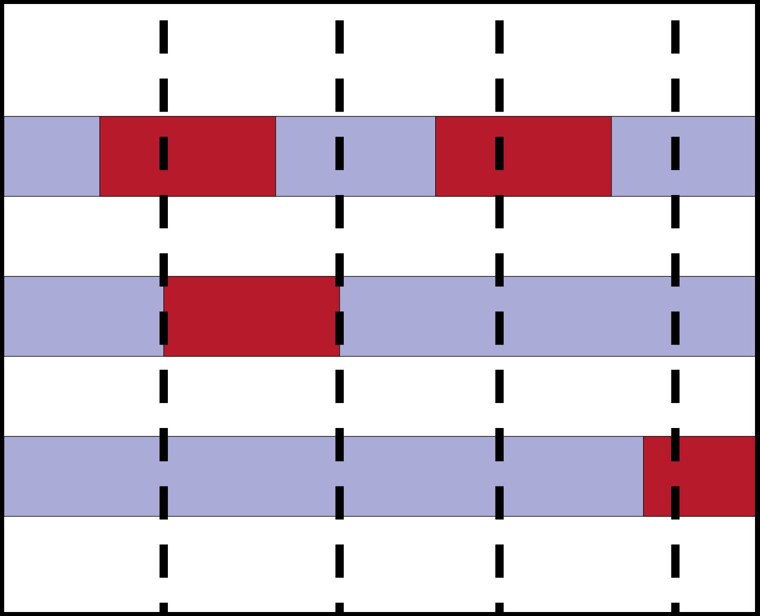
CVD NWs
(Lieber, Harvard)

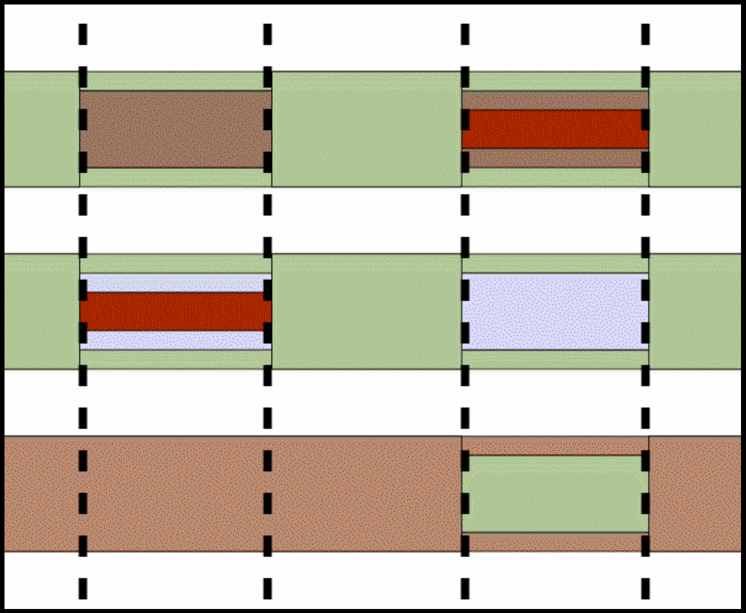
- Must be differentiated after assembly.

NWs Differentiated During Manufacture

Modulation-doped

Core-Shell





Misaligned NWs

Aligned NWs

MW Control Over NWs

- MWs fields control NW resistances.
  - NWs can have lightly and heavily doped sections.
  - Lightly-doped NWs can be shielded in sections.
  - Fields can be intensified by high-K dielectrics.

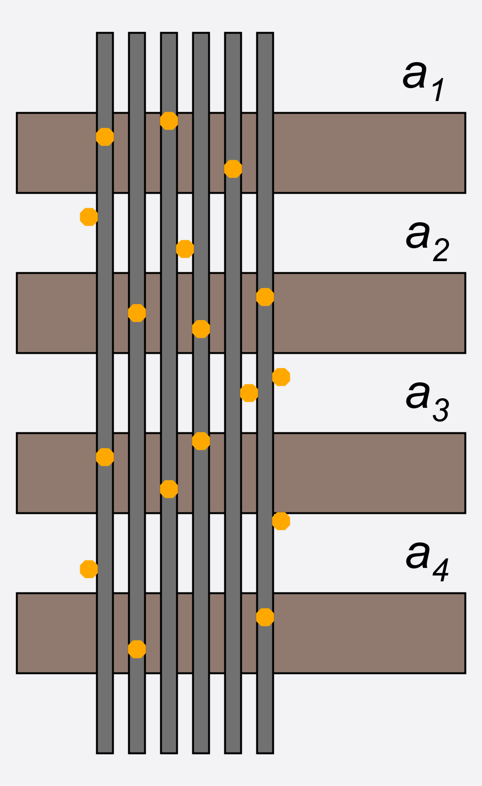
- Binary versus modulated fields
  - In most decoders, a MW is either “on” or “off.”
  - An IBM device combines multiple fields, selecting NWs based on their spatial location (IEDM’05).

Prior Work

- RCDs - **studied here**
  - (Hogg *et al* IEEE Nano ’06)
- Randomized mask-based decoders
  - (Beckman *et al* Science ’05, Rachlin *et al* ISVLSI ‘06)
- IBM modulated field strength decoder
  - (Gopalakrishnan, IEDM ’05)
- Grid-based decoders (such as CMOL)
  - (Likharev FPGA’06, DiSpigna IEEE Nano ‘06)
- Modulation-doped NW decoders
  - (DeHon *et al* IEEE Nano’03, Gojman *et al* ACM JETCS ’05)
- Core-shell NW decoder
  - (Savage *et al* ACM JETCS ’06)

Randomized-Contact Decoder

- Contacts made at random between NWs and MWs.
- When contact is made, the MW controls the NW.
  - The NW's resistance is high when the MW is on.
- Control may be incomplete, creating a possible error.



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Modeling an Ideal and Non-Ideal NW Decoders

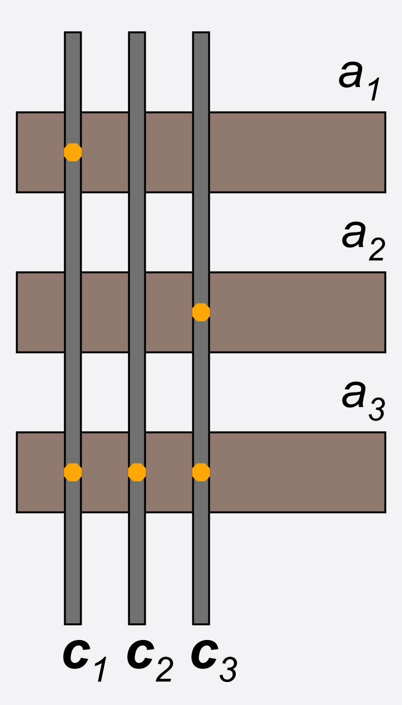
- Each NW is assigned a codeword $\mathbf{c} = (c_1, c_2, \ldots, c_M)$.
  - $c_j = 1$ if the NW is controlled (turned off) by j^{th} MW.
  - $c_j = 0$ if the NW is unaffected by j^{th} MW.
  - $c_j = e$ if the NW is only partially controlled by j^{th} MW.

- Ideal (non-ideal) resistive model:
  - NW's resistance is low when all MWs are off.
  - $c_j = 1$ resistance increase $= \infty$ ($> r_{high}$) when j^{th} MW active.
  - $c_j = 0$ if resistance increase $= 0$ ($< r_{low}$) when j^{th} MW active.
  - $c_j = e$ otherwise.

NW Addressability

- A NW is “addressed” if its resistance is low and the combined resistance of all other NWs is high.

- Example
  - $\boldsymbol{c}_1 = (1, 0, 1)$, $\boldsymbol{c}_2 = (0,0,1)$, $\boldsymbol{c}_3 = (0,1,1)$
  - 1s of $\boldsymbol{c}_1$ and $\boldsymbol{c}_3$ contain 1s of $\boldsymbol{c}_2$.
  - $\boldsymbol{c}_2$ off $\Rightarrow$ $\boldsymbol{c}_1$ off.
  - $\boldsymbol{c}_2$ off $\Rightarrow$ $\boldsymbol{c}_3$ off.
  - Can only individually address a codeword if it is not implied by another codeword.



Individually Addressable NWs in Ideal Decoders

- A NW n_i is **individually addressable (i.a.)** if a MW input $\boldsymbol{a} = (a_1, a_2, ..., a_M)$ exists such that n_i is addressed.

- (1, 0, 1,0), (0,1,0,1) and (0,1,1,0) are all i.a. assuming no other codewords are present.

- In ideal model, an i.a. NW with codeword $\boldsymbol{c}$ is addressed by $\boldsymbol{a} = \overline{\boldsymbol{c}}$ (Boolean complement).

Best Case Addressability

- A ***k*-hot code** contains all codewords with exactly k 1s in their M positions.
- Given M MWs, the set of $\lceil M/2 \rceil$-hot NWs has the largest number of i.a. NWs (Rachlin *et al*, ISVLSI ’06).
- Independent random contacts prevent the use of k-hot codes.

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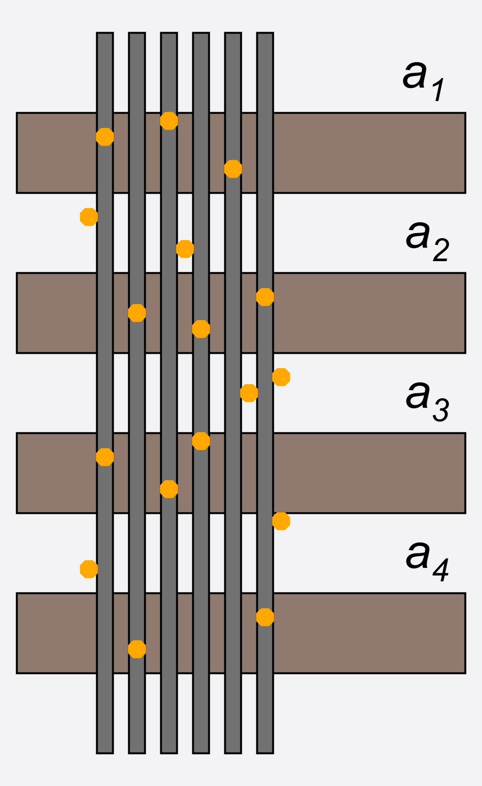
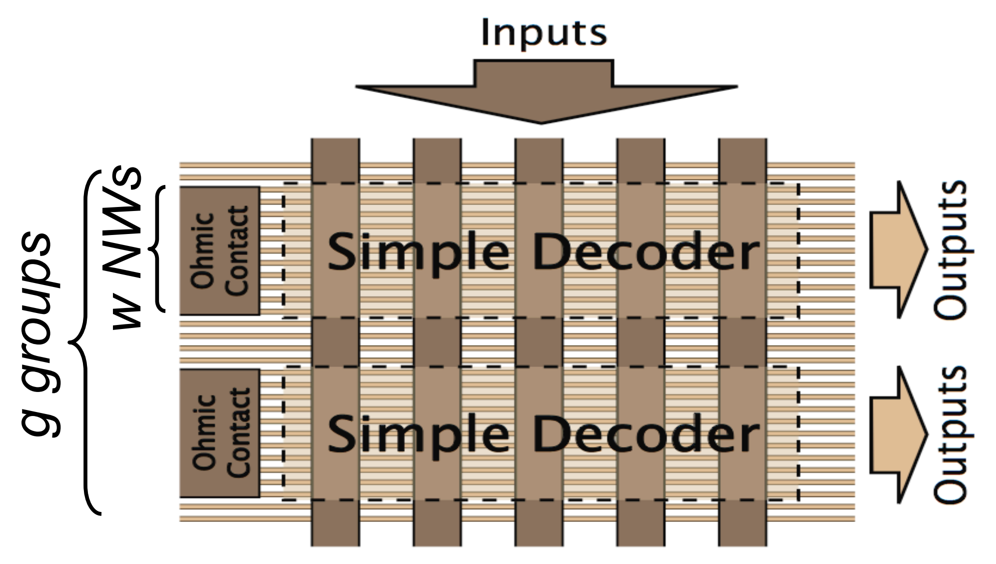
Background on Randomized-Contact Decoder (RCD)

- Kuekes and Williams 2001 patent.
- Hoggs, *et al* (IEEE Trans. Nano, March 2006)
  - Focuses on simulation and empirical analysis
- Our contributions
  - Tight probabilistic analysis of RCD.
  - Bounds on the effect of errors.
  - Comparison of addressing strategies.



RCD Model



- *g* contact groups, *w* NWs per group
- $N = gw$
- N_a = number of individually addressable NWs

RCD Model (continued)

- M number of MWs.

- p = probability $c_j = 1$.
- q = probability $c_j = 0$.
- $r = 1-p-q$ = probability $c_j = e$.
  - When errors occur, MW control is uncertain.

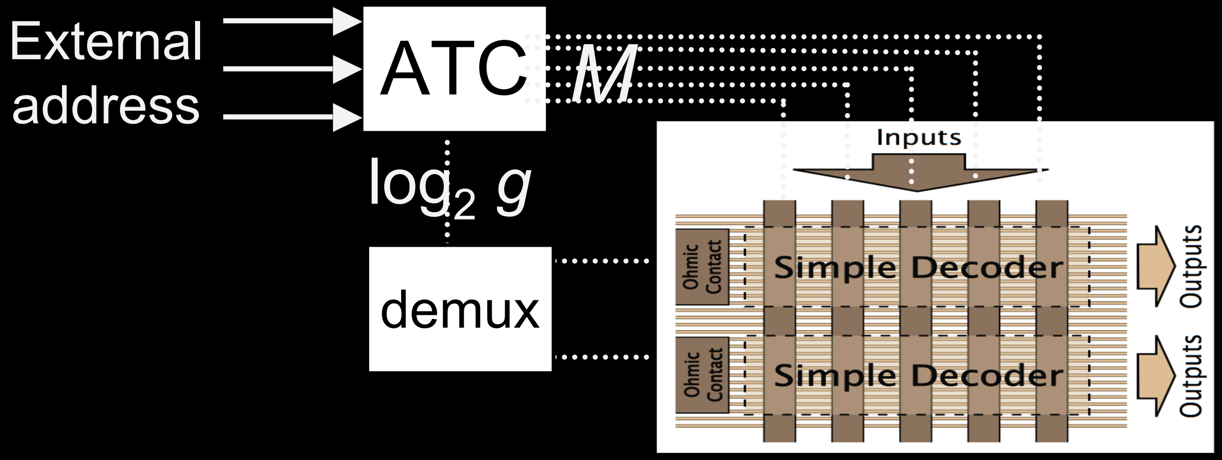
- **Goal:** Given “addressing strategy”, p and q, find M *so* N_a NWs are i.a. w/ high probability.

Three Decoder Addressing Strategies

- **All Wires Addressable (AWA)**
  - In every contact group all wires are i.a.
- **All Wires Almost Always Addressable (AWA^3)**
  - Only use contact groups in which all wires are i.a.
- **Take What You Get (TWYG)**
  - Use all i.a. NWs in all contact groups.
- Given N_a, w, g and M we can estimate the area of the ATC, crossbar and MWs for each strategy.

Address Translation

- **Address translation circuit (ATC)** maps fixed external addresses to random internal ones.



N_a words in ATC, each of $(M + \sigma)$ bits

- $\sigma = 0$ for AWA
- $\sigma \approx 0$ for AWA^3
- $\sigma = \log_2 g$ for TWYG

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Challenges

- Our goal is to determine the number of i.a. NWs, N_a, given M, p, q, g *and* w.
- Whether or not a NW is i.a. depends on the other randomly assigned codewords in the NW's contact group.
- We want to bound the number of i.a. NWs with high probability, not just find the mean.

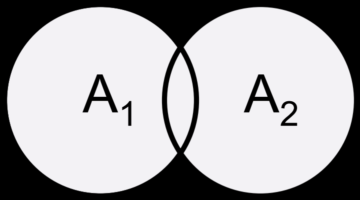
Methods for Bounding N_a

- Chebyshev's inequality:

$$P(|x - E[x]| \geq k \sqrt{Var[x]}) \leq 1/k^2$$

  - Uses mean and variance in number of i.a. NWs in each contact group.
  - Used for **Take What You Get**.

- Principle of Inclusion/Exclusion
  - $P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$
  - Used for **AWA** and **AWA³**.



Bounds on Addressable NWs Using Chebyshev's Inequality

Theorem Let $\alpha = 16\varepsilon^{-1}/g$. With probability at least $1-\varepsilon$, RCD with $N = gw$ NWs has at least $N_a = 3/4\ N(\alpha+1)/(\alpha+2)$ i.a. NWs if

$$M \geq \ln(\ N(2 + \alpha)\ )/(\ -g \ln(1 - pq)\ )$$

- $\alpha = 8$ is a reasonable value.

- Bound on M reflects errors if $r = 1-(p+q) > 0$.

Bounds on Addressable Wires Using Exclusion/Inclusion

Theorem In a simple RCD minimum value of *M* such that all NWs are addressable with probability 1-ε satisfies following lower bound

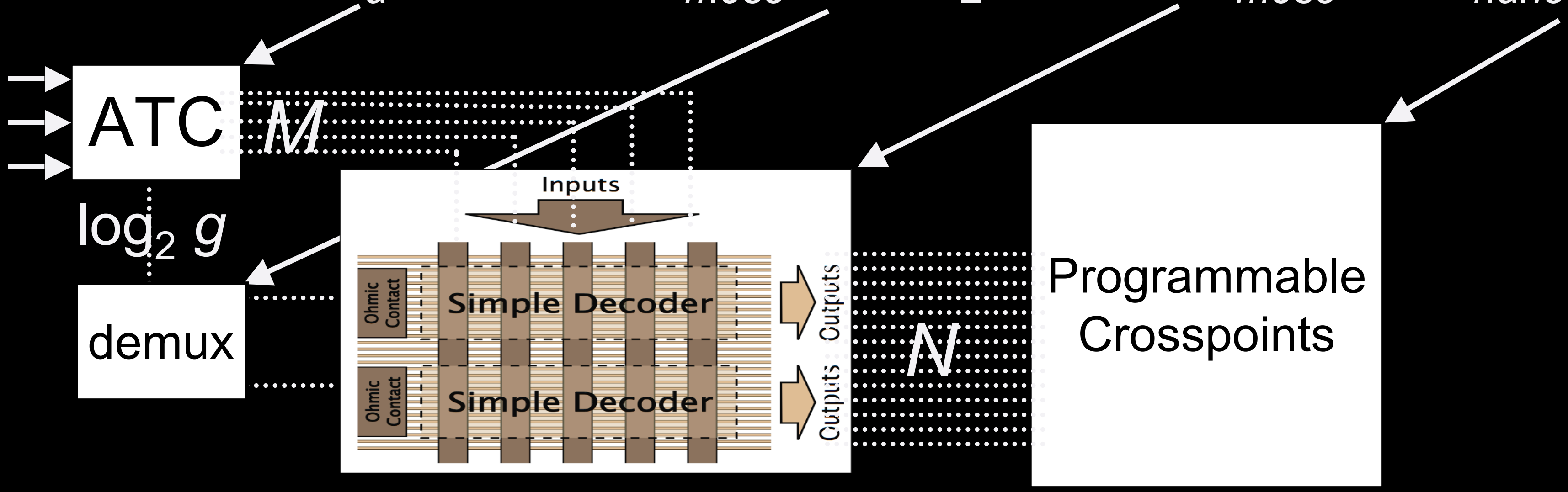
$$\frac{\ln(N(N-1)/2\epsilon)}{-\ln(1-pq)} \leq M \leq \frac{\ln(N(N-1)/\epsilon)}{-\ln(1-pq)}$$

when $M \geq \max\left(pq\min(p,q)^{-1}, \frac{\ln(10N(N-1))}{-\ln(1-pq)}\right)$

- Again, bound reflects errors if (p + q < 1).

Memory Area Estimates

- Area = $\rho N_a\,(M+\sigma) + \lambda_{meso}^2\, g \log_2 g + (M\,\lambda_{meso} + N\,\lambda_{nano})^2$



- λ_{meso} = meso feature size, λ_{nano} = nano feature size
- ρ = area of CMOS bit
- Let $\lambda_{meso} = 10\,\lambda_{nano}$, $\rho = 100\,\lambda_{nano}^2$

Comparison of Addressing Strategies

- **Objective:** about N_a = 1,000 addressable NWs.
- Assuming error-free comparisons (p + q = 1, r = 0), strategies when w = 8:
  - AWA: $N_a = 1{,}024$, $M = 47$, $N = 1{,}204$, $\sigma = 0$
  - AWA^3: $N_a = 1{,}024$, $M = 30$, $N = 1{,}064$, $\sigma \approx 0$
  - TWYG: $N_a = 1{,}080$, $M = 16$, $N = 1{,}600$, $\sigma = 8$
- AWA^3 clearly dominates AWA.

Area Comparisons Between AWA^3, TWYG

- Compare using Area_{ATC} + $\text{Area}_{\text{XBar}}$, ignoring the smaller $\text{Area}_{\text{std dcdr}}$ term.
  - $\text{Area}_{\text{ATC}} = 100N_a\,(M + \sigma)\,\lambda_{nano}^2$
  - $\text{Area}_{\text{XBar}} = (10M + N)^2\,\lambda_{nano}^2$
- Parameters
  - AWA^3: $N_a = 1{,}024$ for $M = 30$, $g = 133$, $\sigma \approx 0$
  - TWYG: $N_a = 1{,}080$ for $M = 16$, $g = 200$, $\sigma = 8$
- Both methods use about the same area but AWA^3 is somewhat better than TWYG.

Tighter Bounds for TWYG

- TWYG analysis is less precise than other two
  - Distribution of N_a is close to Gaussian, which is not captured by Chebyshev's inequality.
  - Fewer than 10 standard deviations would suffice.

- Simulation shows that $M \approx 10$ suffices!
  - At $M = 10$, TWYG is best strategy.

TWYG: Random Contact vs. Differentiated NW Decoders

- RCD
  - $N_a = 1{,}080$ for $M = 16$, $g = 200$, $w = 8$.
- Differentiated NW Decoder
  - $M/2$-hot NWs (with .8 penalty for misalignment)
    - $N_a = 1{,}033$ for $M = 8$, $g = 180$, $w = 8$.
  - Core-shell NWs (no misalignment, but larger NWs)
    - $N_a = 1{,}013$ for $M = 12$, $g = 190$, $w = 8$.
- RCD competitive (*M* is reasonable).

The Effect of Partial Contacts

- Effect of partial contacts measured by $r = 1-(p+q)$
- Number of MWs for **Take What You Get:**

$$M \geq \ln(N(2 + \alpha))/(-g \ln(1 - pq))$$

- The effect of errors is to change *M* by factor of

$$ß = \ln(3/4) / \ln(1 - (1 - r)^2/4) \text{ when } p = q.$$

r	ß
.1	1.27
.2	1.65
.4	3.05

Conclusions

- Of 3 strategies **TWYG** is superior.
  - Analysis shows M can be small.
  - AWA^3 is a close second, much better than AWA.
- RCD decoder can tolerate faults efficiently.
- RCD is competitive with other NW decoders.
  - It may be easier to implement than other methods.
- Codeword discovery is still an open problem!