

# Ideal and Resistive Nanowire Decoders

## General Models for Nanowire Addressing

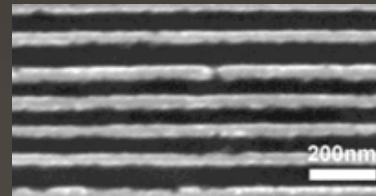
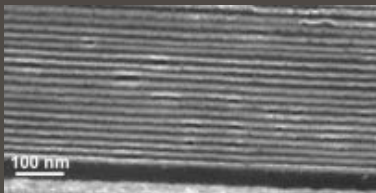
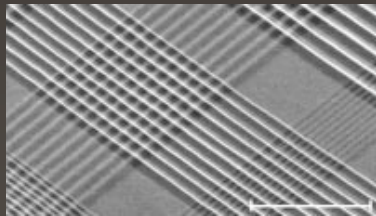
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# The Nanowire

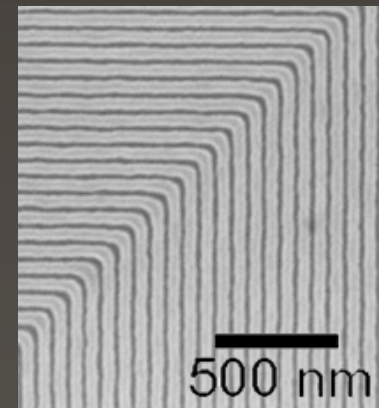
- Nanoscale computing requires nanoscale wires (NWs) and nanoscale devices.
- Sets of parallel NWs have been produced.
- Devices will reside at NW intersections.
- To control these devices, we must gain control over individual NWs.

# NW Technologies

SNAP NWs  
(Heath, Caltech)



Copolymer  
Directed Growth  
(Stoykovich, UW)



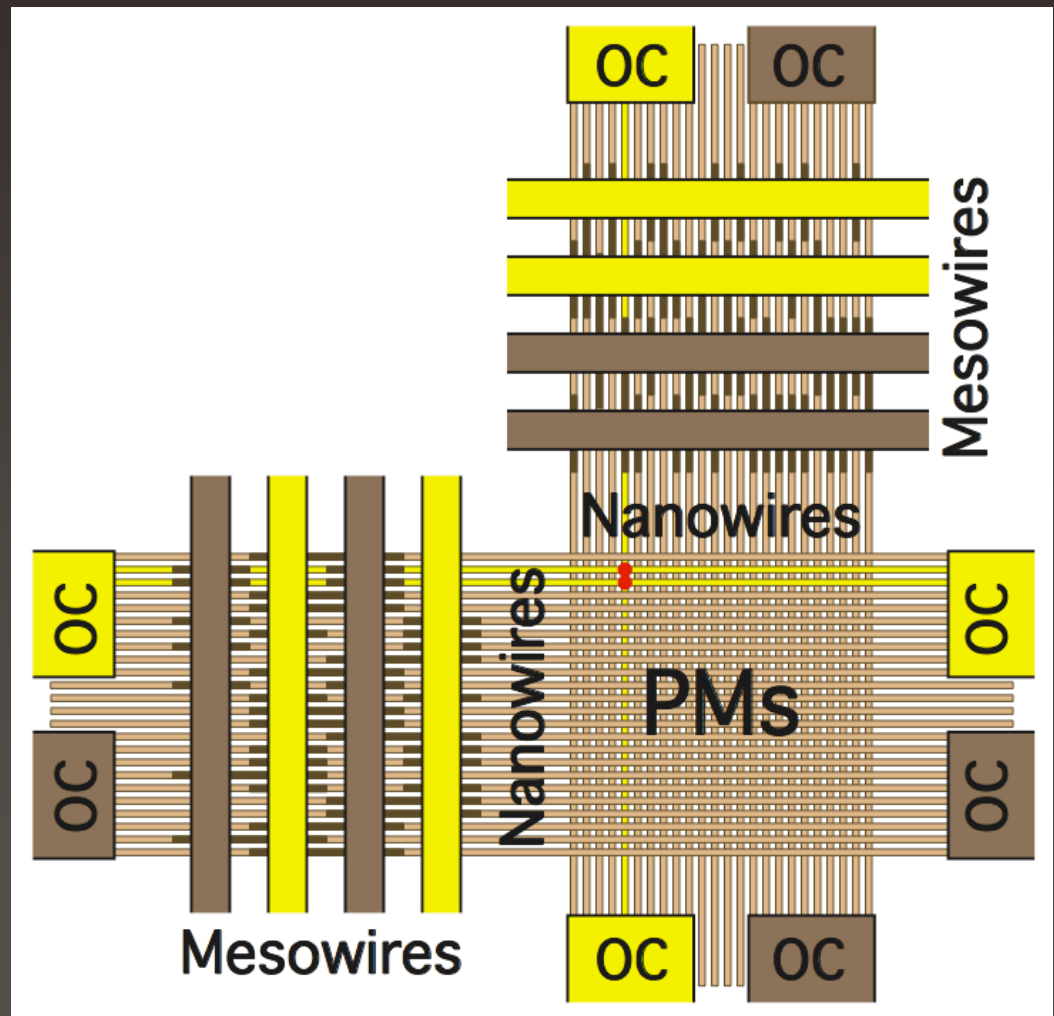
CVD NWs  
(Lieber, Harvard)

# The Crossbar

The crossbar is currently the most feasible nano-scale architecture.

By addressing individual NWs, we can control programmable molecules at NW crosspoints.

Crossbars are a basis for memories and circuits.



# Nanowire Control

- Mesoscale contacts apply a potential along the lengths of NWs.
- Mesoscale wires (MWs) apply fields to across NWs, some of which form FETs.
- NW/MW junctions can form FETs using a variety of technologies:
  - ⇒ Modulation-doping
  - ⇒ Random Particle deposition
  - ⇒ Masking NWs with dielectric material

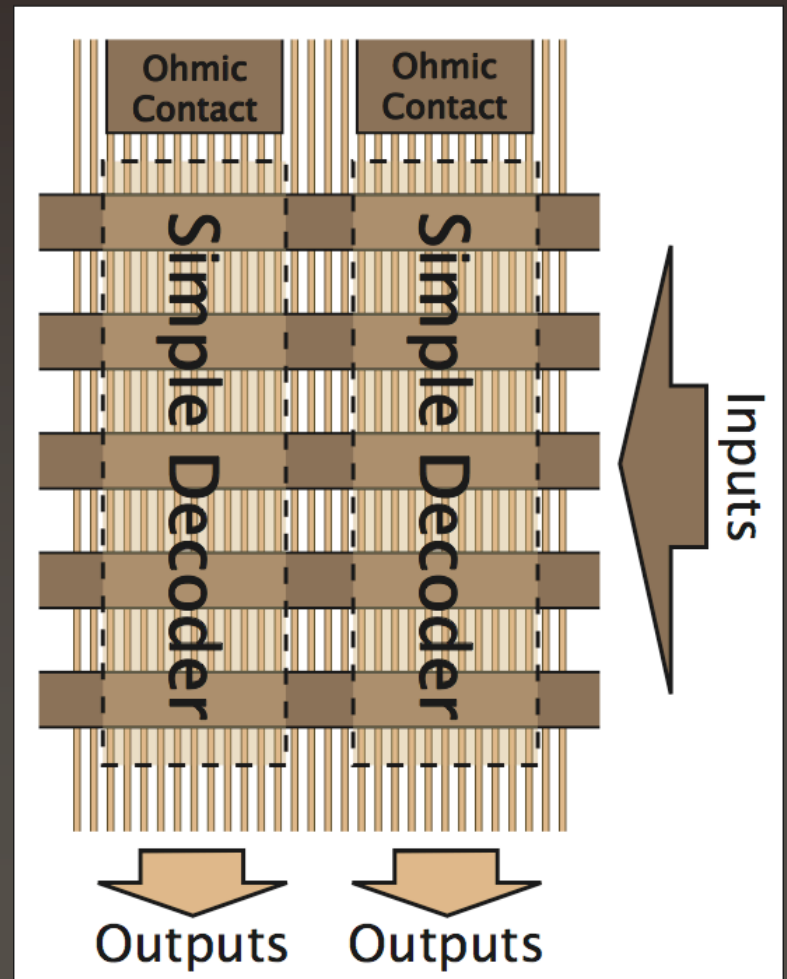
# Simple NW Decoders

- A potential is applied along the NWs.
- $M$  MW inputs control  $N$  NW outputs. Each MW controls a subset of NWs.
- When a MW produces a field, the current in each NW it controls is greatly reduced.
- Each MW “subtracts” out subsets of NWs. This permits  $M \ll N$ .
- Decoders are assembled stochastically and become difficult to produce as  $N$  is large.



# Composite Decoders

- A composite decoder uses multiple simple decoders to control many NWs.
- The simple decoders share MW inputs.
- This space savings allows for mesoscale inputs.



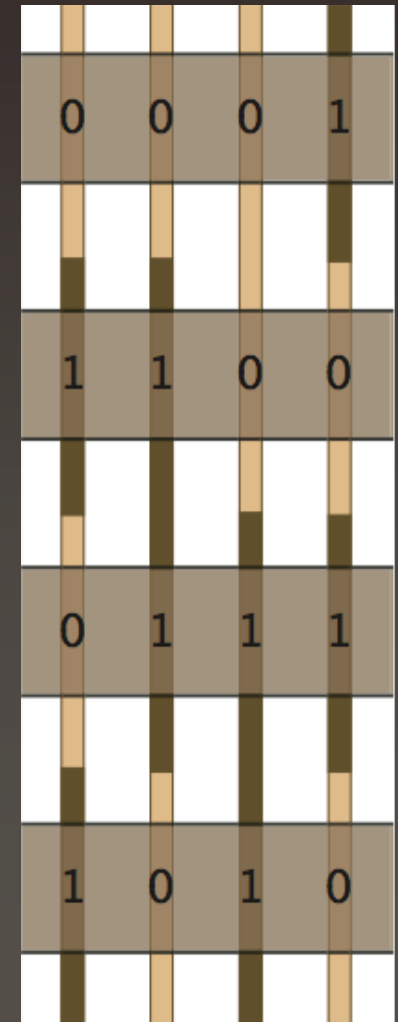
# Ideal Decoders

- To analyze a decoder, we must model how MWs control NWs.
- In an **ideal decoder**, a MW's electric field completely turns off the NWs it controls. Other NWs are unaffected.
- This model is accurate if the FETs formed from MW/NW junctions have high on/off ratios.



# Binary Codewords

- In an ideal decoder, we associate an  $M$ -bit codeword,  $c_i$ , with each NW,  $n_i$ .
- The  $j^{\text{th}}$  MW controls the  $i^{\text{th}}$  NW if and only if the  $j^{\text{th}}$  bit of  $c_i$ ,  $c_{ij}$ , is 1.
- We also represent the decoder's input as an  $M$ -bit binary vector,  $A$ .
- $n_i$  carries a current if and only if  $A \cdot c_i = 0$ .

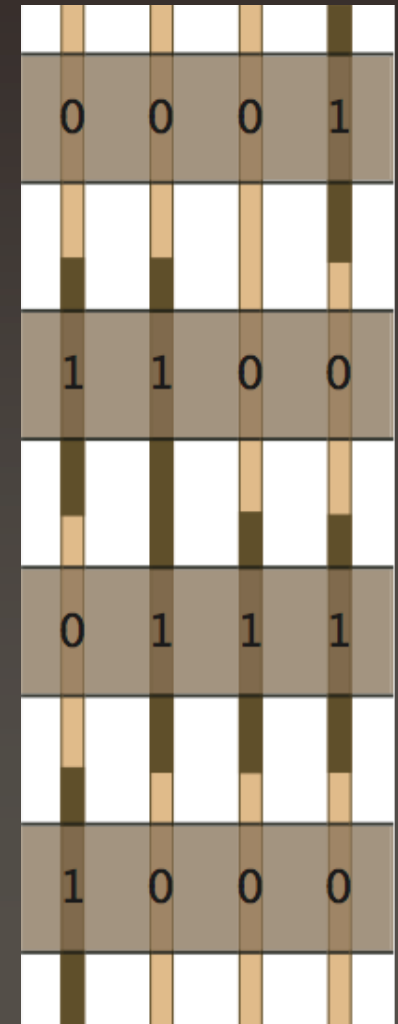


# Codeword Assignment

- Decoders are assembled stochastically.
- Codewords are assigned to NWs according to a probability distribution.
- This distribution is a way of comparing decoding technologies.
  - ⇒ With no misalignment, modulation-doping is at least as good as particle deposition.

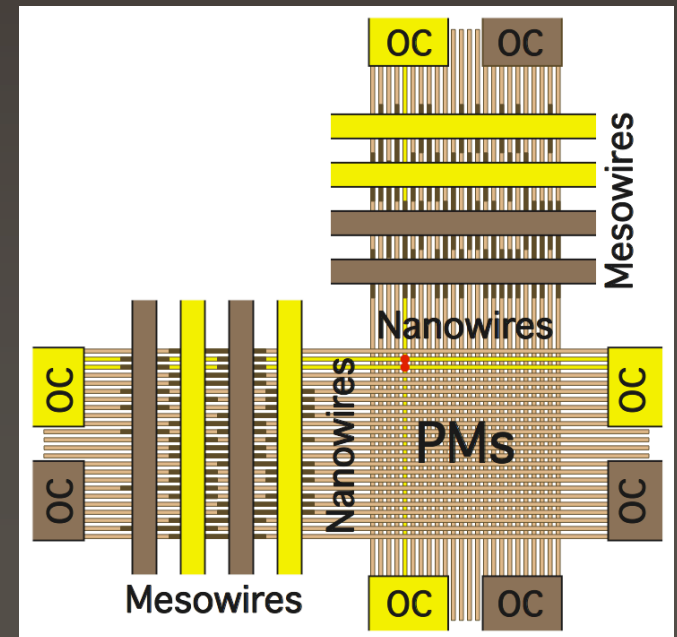
# Codeword Interaction

- If  $c_{bj} = 1$  where  $c_{aj} = 1$ ,  $c_a$  **implies**  $c_b$ .  
Inputs that turn on  $n_a$  turn off  $n_b$ .
- A set of codewords,  $S$ , is **addressable** if some input turns off all NWs not in  $S$ .
- $S = \{c_i\}$  is addressable if and only if no codeword implies  $c_i$ .  $S$  is addressed with input  $A = \bar{c}_i$ .



# Decoders for Memories

- A  $B$ -bit memory maps  $B$  addresses to  $B$  disjoint sets of storage devices.
- A  **$D$ -address memory decoder** addresses  $D$  disjoint subsets of NWs.
- Equivalently, the decoder contains  $D$  addressable codewords.

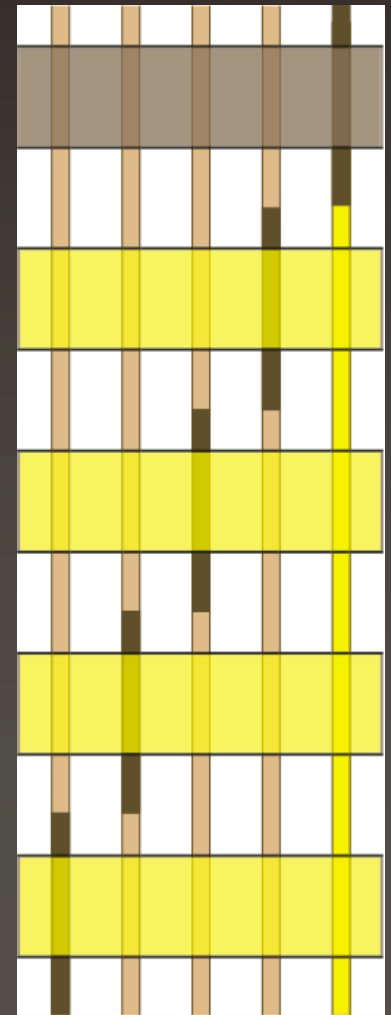


# Resistive Decoders

- Decoders that rely on FETs are not ideal.
- MWs carrying a field increase each NW's resistance by some amount.
- In a **resistive decoder**, codewords are real-valued. In real-valued codeword  $r_i$ ,  $r_{ij}$  is the resistance induced in  $n_i$  by the  $j^{\text{th}}$  MW.
- On input  $A$ ,  $n_i$ 's resistance is  $r_{base} + A \cdot r_i$ .

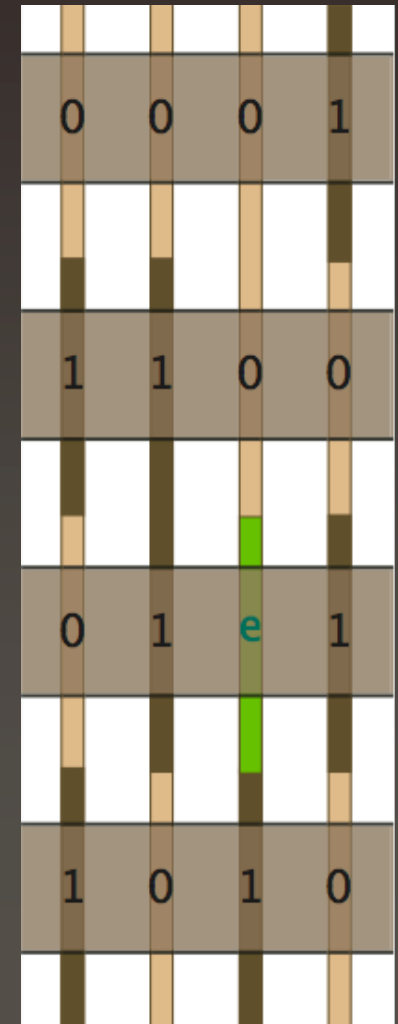
# Ideal vs. Resistive

- In a resistive memory decoder the addressed NWs must output more current than the other NWs.
- Consider 1-hot codewords:
  - ⇒ The addressed wire has resistance  $< r_{base} + Mr_{low}$
  - ⇒ Remaining wires have resistance  $> (r_{base} + r_{high})/N$
- We require that  $r_{high} \gg MNr_{low}$  and  $Nr_{base}$ 
  - If  $r_{ij} \leq r_{low}$ ,  $c_{ij} = 0$ .
  - If  $r_{ij} \geq r_{high}$ ,  $c_{ij} = 1$ .
  - If  $r_{low} < r_{ij} < r_{high}$ ,  $c_{ij}$  is an error.



# Ideal Decoders with Errors

- To apply the ideal model to resistive decoders, consider binary codewords with random **errors**.
- If  $c_{ij} = e$ , the  $j^{\text{th}}$  MW increases  $n_i$ 's resistance by an unknown amount.
- Consider input  $A$  such that the  $j^{\text{th}}$  MW carries a field. A functions reliably if a MW for which  $c_{ik} = 1$  carries a field.



# Balanced Hamming Distance

1	0	1	0	1	0	1	0	1	0	1	0
0	1	1	0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

- Consider two error-free codewords,  $c_a$  and  $c_b$ . Let  $|c_a - c_b|$  denote the number of inputs for which  $c_{aj} = 1$  and  $c_{bj} = 0$ .
- The balanced Hamming distance (BHD) between  $c_a$  and  $c_b$  is  $2 \cdot \min(|c_a - c_b|, |c_b - c_a|)$ .
- If  $c_a$  and  $c_b$  have a BHD of  $2d + 2$  they can collectively tolerate up to  $d$  errors.



# Fault-Tolerant Random Particle Decoders

- In a particle deposition decoder,  $c_{ij} = 1$  with some fixed probability,  $p$ .
- If each pair of codeword has a BHD of at least  $2d + 2$ , the decoder can tolerate  $d$  errors per pair.
- This holds with probability  $> 1 - f$  when

$$M > \frac{(d + (d^2 + 4 \ln(N^2/f))^{1/2})^2}{4p(1 - p)}$$

# Codeword Discovery

- Random codewords must be discovered to map memory addresses to decoder inputs.
- Input  $A'$  **contains**  $A$  if  $A'_j = 1$  where  $A_j = 1$ .
- If  $c_i$  is addressable,  $A = \bar{c}_i$  produces a current, but inputs containing  $A$  do not.
- By testing if inputs produce currents, the codewords in an error-free decoder are discovered without nanoscale measurement.

# Codeword Discovery with Errors

- If errors are present, we cannot just test for the presence or absence of current.
- If inputs  $A$  and  $B$  both produce sufficiently large currents, we can be certain that both address some NW.
- If their union produces a small current, the inputs address distinct codewords.

# Conclusion

- Stochastically assembled decoders can reliably control NWs even if errors occur.
- Our decoder model applies to many viable technologies. It provides conditions that a decoder must meet.
- Discovery algorithms verify that a decoder functions properly without requiring nanoscale measurements.