Verifiable Order Queries on a List in Zero-Knowledge

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Overview

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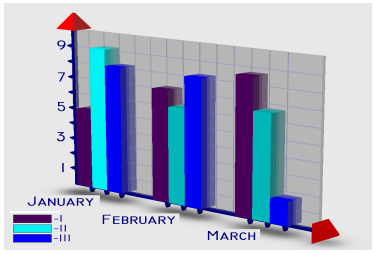
Outsourced Data

Sales Reports

Cloud

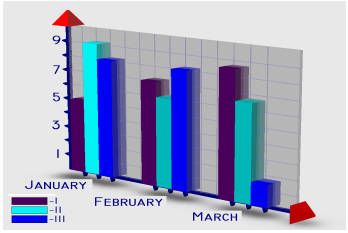
Revealing anything more?

Sales Report



Authentic?

Sales Report



Company

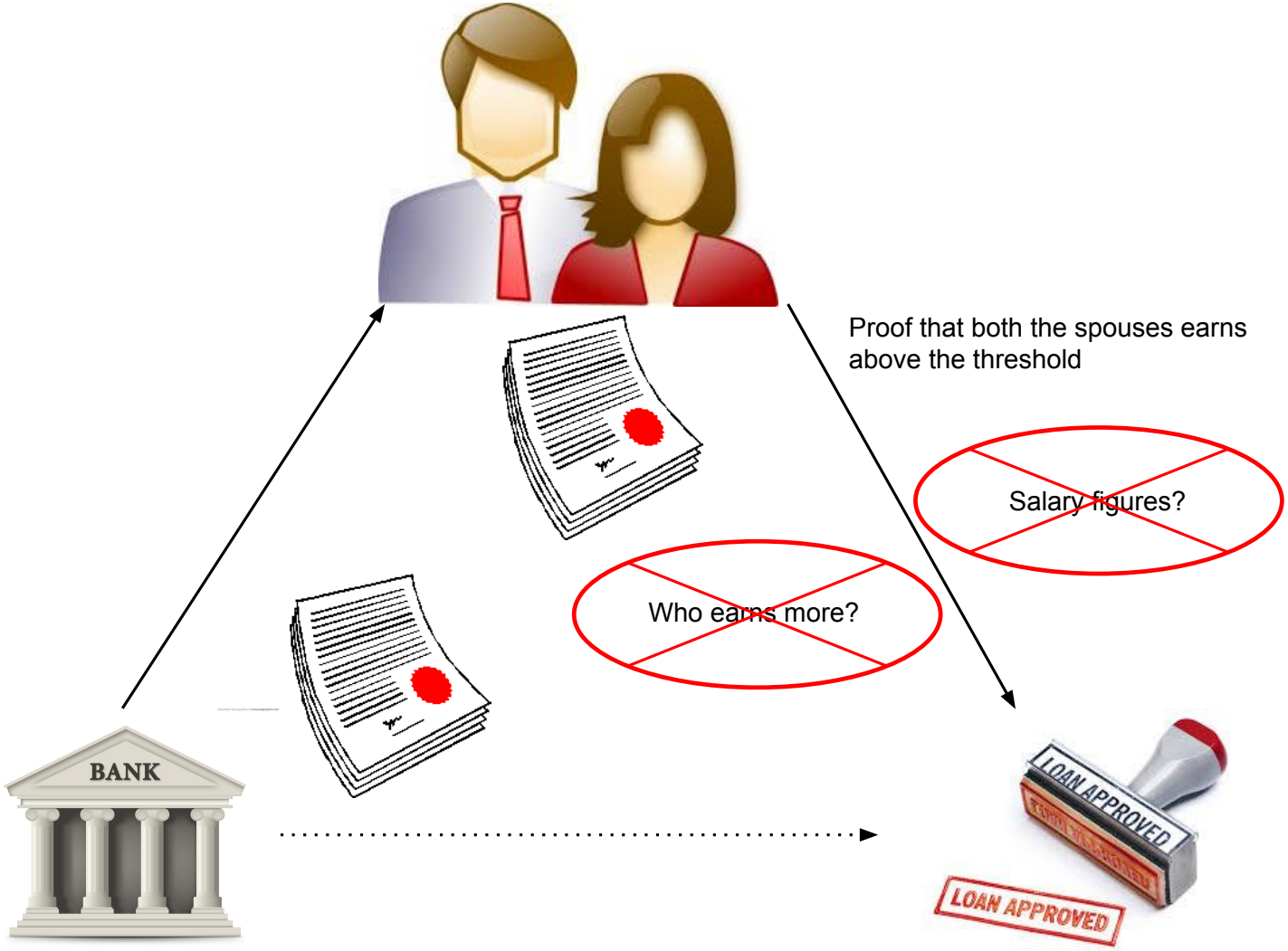
Product I sold more than product II in March

Client

Health Records

Vaccine	Date given (mm/dd/yyyy)	Administered by
Hepatitis B		
Diptheria, Tetanus		
Haemophilus influenzae type b		
Polio		
Rotavirus		
Measles, Mumps and Rubella		
Varicella		
Hepatitis A		
Meningococcal		
Human papillomavirus		

Proving values above a thresohold



Other Statistical Queries - Max and Min

Let $\mathcal{S}$ be a collection of elements of a totally ordered set, $\mathcal{L}$.

Max($\mathcal{S}$) and Min($\mathcal{S}$) return the highest ranked and the lowest ranked element of $\mathcal{S}$ with respect to the list ordering.

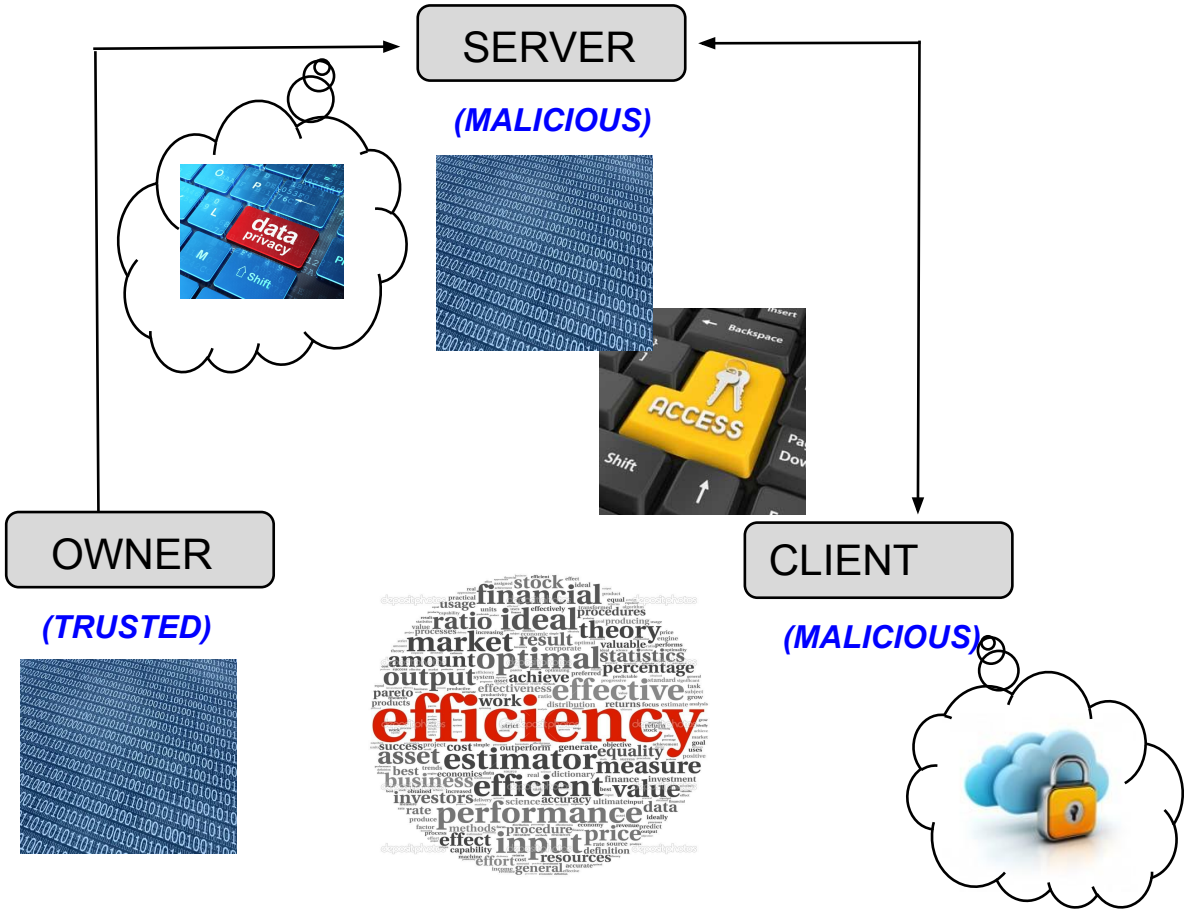
The proof should reveal nothing beyond the answer, i.e., every element in $\mathcal{S}$ is ranked lower than Max($\mathcal{S}$).

Other Statistical Queries - top-t

The query $\text{Top}(t, \mathcal{S})$ returns the top t ranked elements along with a proof of the answer.

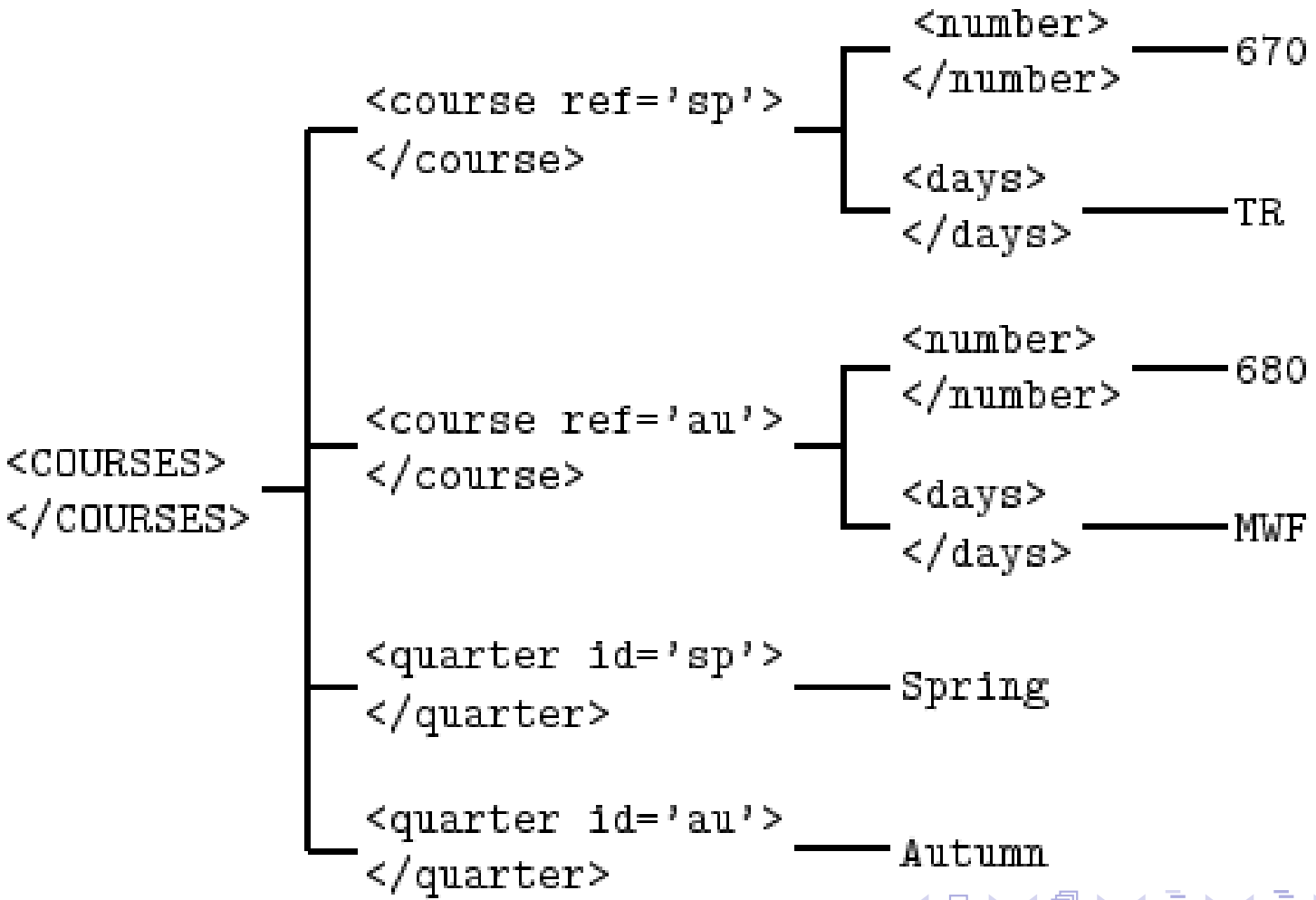
The proof should not even reveal the order among the returned elements.

Model


SERVER
(MALICIOUS)
data
privacy
ACCESS
OWNER
(TRUSTED)
CLIENT
(MALICIOUS)
efficiency
financial
ratio
ideal
theory
market
optimal
statistics
output
effective
estimator
measure
efficient
value
performance
input
price

Structured Data

Data is stored in datastructures like ordered sets, ordered trees.



Lists

A list is an ordered set of distinct elements.

$\mathcal{L} = \{y_1, y_2, \ldots, y_n\}$.

Example: Student id's in a course, sorted in the non-decreasing order of their test score.

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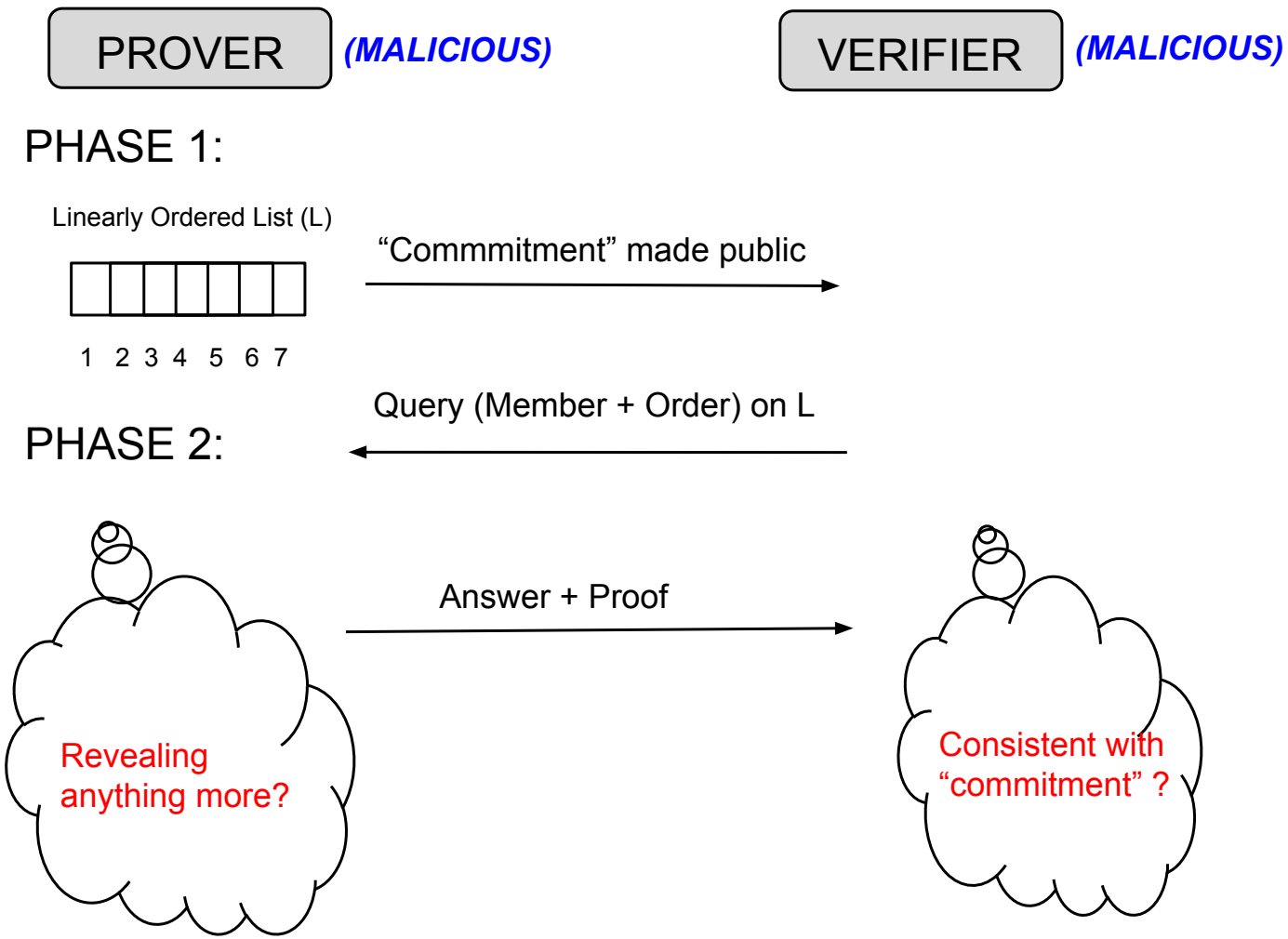
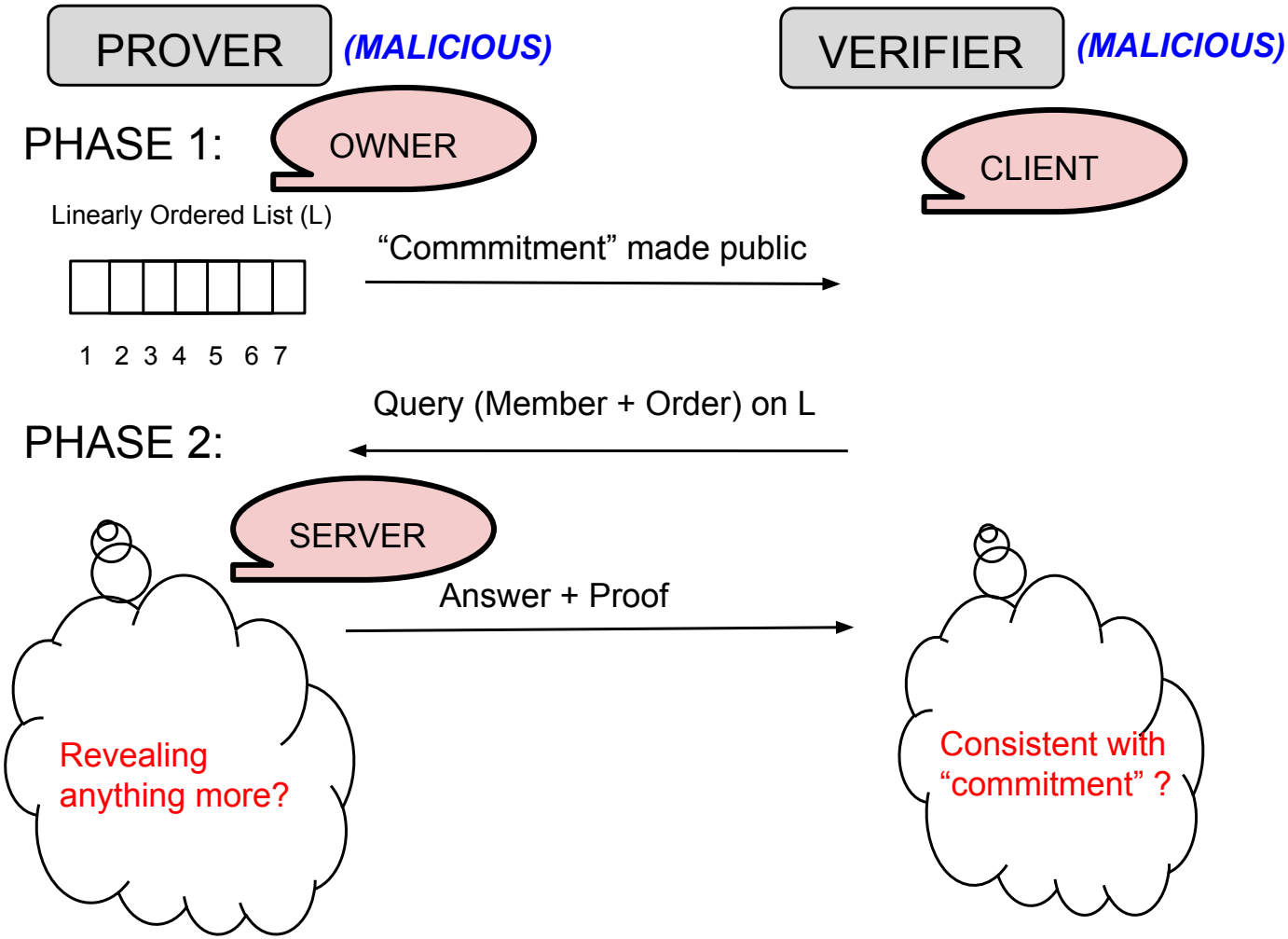
Zero-Knowledge List (ZKL)



Zero-Knowledge List (ZKL)



Query Operations

$\mathcal{L}$ = linearly ordered list of elements:

Membership Query

Client Query: Is element $y \in \mathcal{L}$?

Server Response: If $y \in \mathcal{L}$ return $\mathcal{L}(y) =$ true + proof of $y \in \mathcal{L}$; else return $\mathcal{L}(y) =$ false + proof of $y \notin \mathcal{L}$;

Order Query

Client Query: A pair of query elements (y, w)

Server Response: If $y, w \in \mathcal{L}$, (y, w) rearranged according to their order in $\mathcal{L}$ + membership proof + order proof. Proof of non-membership otherwise.

Security Properties

Completeness: Honestly generated proofs are always accepted by the client.

Soundness: Proofs for answers to queries, inconsistent with the initial commitment do not pass the verification.

Zero-Knowledge: Proofs do not reveal anything beyond the answers, i.e., the proofs are simulatable.

Zero-Knowledge Set

First let us consider an unordered finite set $\mathcal{S}$ of key value pairs.

$\mathcal{S}(x) = v$ denotes v is the value corresponding to the key x. For the keys that are not present in $\mathcal{S}$, $x \notin \mathcal{S}$, we write $\mathcal{S}(x) = \perp$.

Queries are of the form "is key x in $\mathcal{S}$?".

Return $\mathcal{S}(x)$ if $x \in \mathcal{S}$ + proof.

Return $\perp$ + proof if $x \notin \mathcal{S}$.

Zero-Knowledge protocol to prove non-negativity of an integer

The prover sends a commitment c to a non-negative value x to the verifier and proves, without opening c, that $x \geq 0$.

We concisely write this as : $\mathsf{P} \leftrightarrow \mathsf{V}(x, r : c = C(x; r) \wedge x \geq 0)$.

The protocol is constructed on two facts:

- a negative number cannot be a sum of squares
- every non-negative integer is a sum of four squared integers

ZKL Setup

Let $\mathcal{L} = \{y_1, \ldots, y_n\}$ and $\text{rank}(\mathcal{L}, y_j) = j$.

For every $y_j \in \mathcal{L}$, compute $\mathbb{H}(y_j)$ and $C(j)$.

Instantiate ZKS with $(\mathbb{H}(y_j), C(j))$ as (key,value) pair .

Order Query in ZKL

Let $\mathcal{L} = \{Alice, Bob, Charles\}$.

Proving $Alice < Charles \in \mathcal{L}$:

As a part of the proof of membership membership of Alice, the verifier receives $C(1)$ and (3) for Bob.

These commitments are never opened.

Additionally, the prover returns a fresh commitment to 1, $C^*(1)$, and its opening information ρ.

Prover constructs proof using $\mathsf{P} \leftrightarrow \mathsf{V}(x, r : c = C(x; r) \wedge x \geq 0)$ to convince the verifier that $C(3 - 1 - 1)$ is a commitment to value ≥ 0.

Verification

The verifier verifies the commitment $C^*(1)$ using ρ.

The verifier computes $C(3-1-1) := C(3)/(C(1) \times C^*(1))$ using the homomorphic property of the integer commitment scheme.

Then it verifies the proof of non-negativity of $C(3-1-1)$ following the steps of the NIZK protocol
$\mathsf{P} \leftrightarrow \mathsf{V}(x, r : c = C(x; r) \wedge x \geq 0)$.

Performance

- The prover executes the commitment phase in time and space proportional to ZKS commitment.
- In the query phase, the prover computes the proof of the answer in time and space proportional to ZKS prover.
- The verifier verifies the proof in time and space proportional to ZKS verifier.

Performance (Continued)

The construction has the following performance, where n is the list size, m is the query size, each element of the list is a k-bitstring and N is the number of all possible k-bit strings:

- The prover executes the commitment phase in $O(n \log N)$ time and space.
- In the query phase, the prover computes the proof of the answer in $O(m \log N)$ time.
- The verifier verifies the proof in $O(m \log N)$ time and space.

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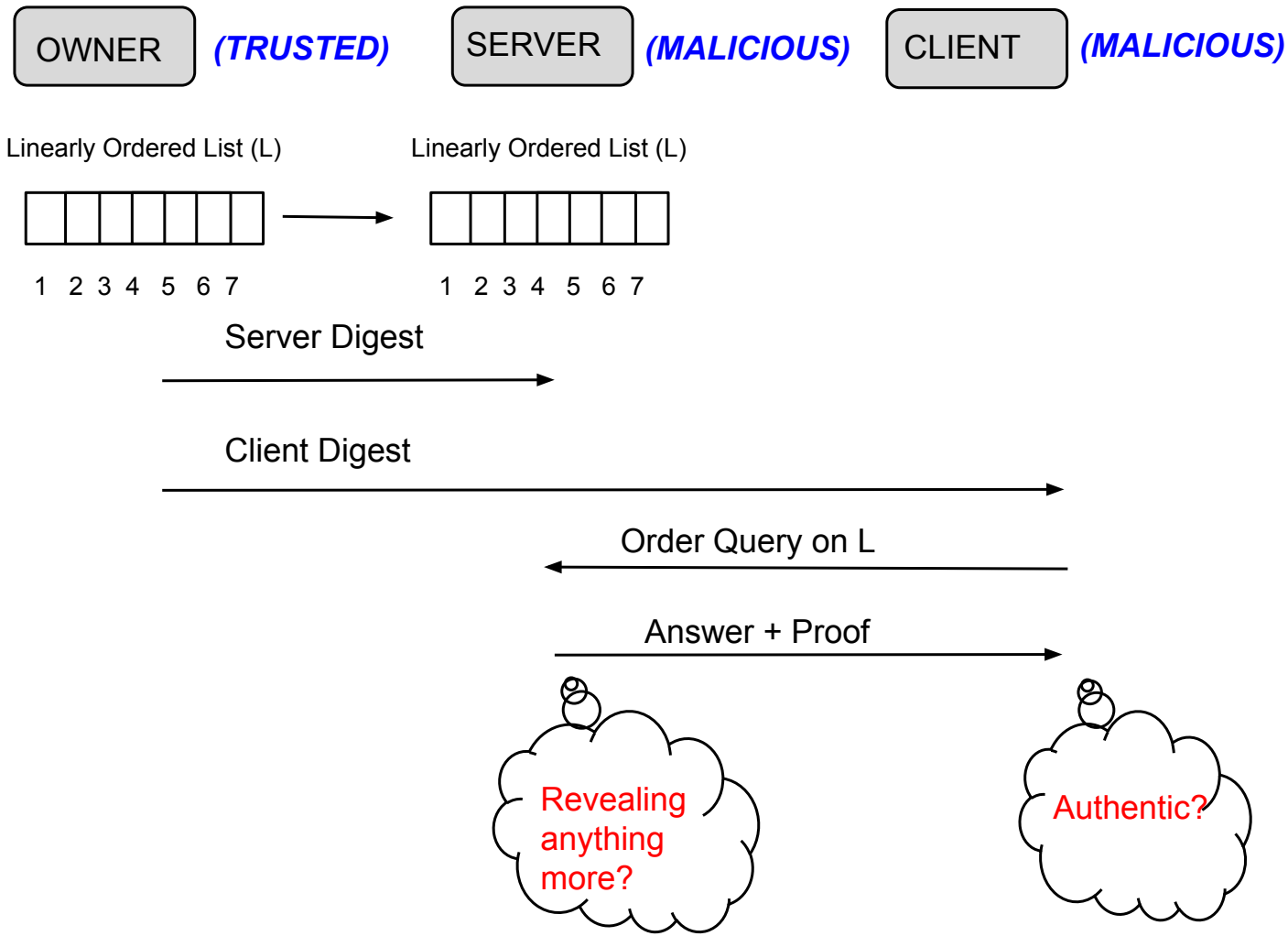
Model


OWNER
(TRUSTED)
SERVER
(MALICIOUS)
CLIENT
(MALICIOUS)
Linearly Ordered List (L)
Linearly Ordered List (L)
1 2 3 4 5 6 7
1 2 3 4 5 6 7
Server Digest
Client Digest
Order Query on L
Answer + Proof
Revealing anything more?
Authentic?

Query

$\mathcal{L}$ = linearly ordered list of elements:

Order Query

Client Query: A pair of query elements (x, y) of $\mathcal{L}$

Server Response: (x, y) rearranged according to their order in $\mathcal{L}$ + order proof

Adversarial Model

Both the server and the client can act as adversaries:

Server: May try to forge proofs for incorrect answers to membership or ordering queries

Client: Tries to learn from the proofs additional information about list $\mathcal{L}$ beyond what she has asked for

Security Properties

Completeness: Honestly generated proofs are always accepted by the client.

Soundness: Proofs for incorrect answers forged by the server do not pass the verification by the client.

Zero-Knowledge: Proofs do not reveal anything beyond the answers, i.e., the proofs are simulatable.

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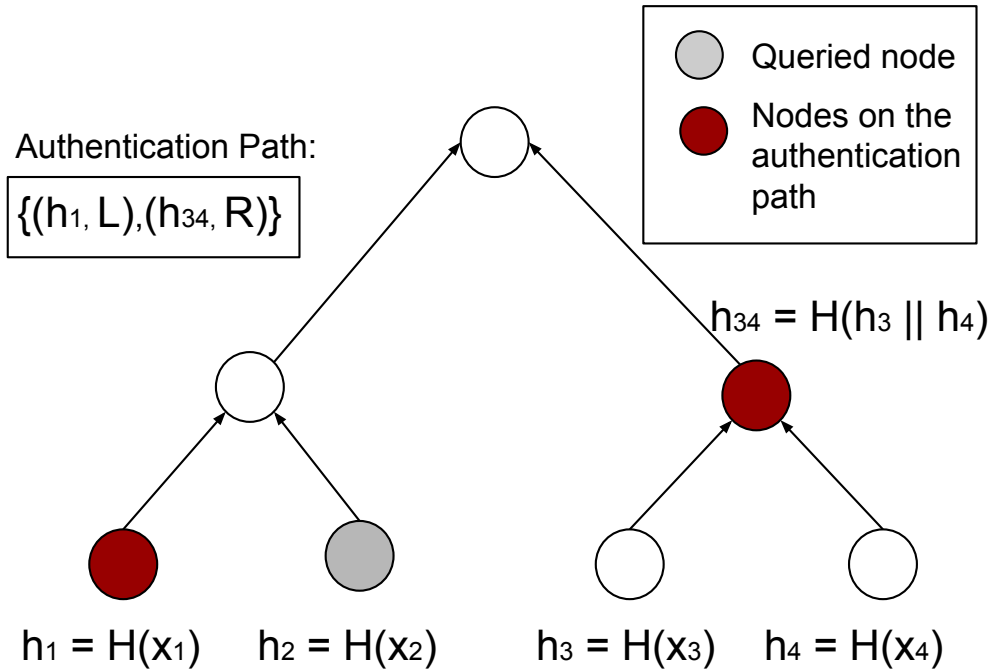
Merkle's Hash Tree



Merkle's Hash Tree where the elements are stored at the leaves in sorted order reveals the size of the source list and the rank of an element

Authenticated skip list

The proof of membership/non-membership is a sequence of nodes visited when searching for the element.

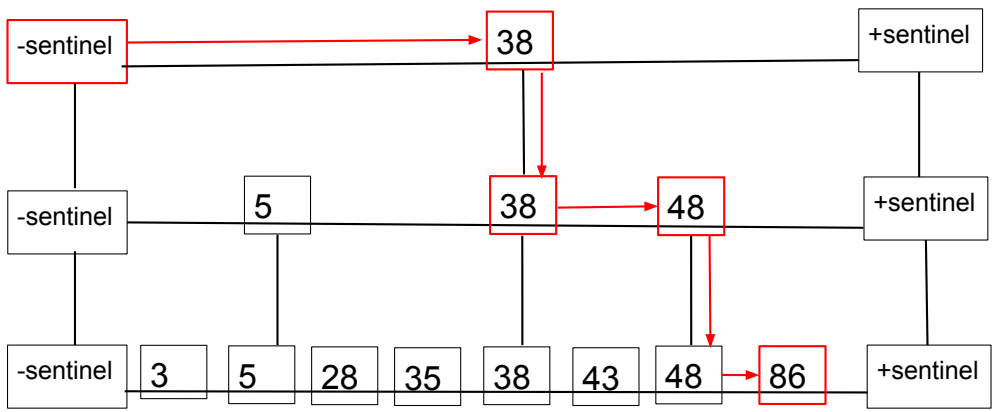


Figure: Elements searched for the value 86 are shown in red.

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Efficiency Goals

Let n = number of data elements in the source list $\mathcal{L}$ and let m = number of elements in the queried sublist

- **Server storage:** $O(n)$, irrespective of the number of queries answered
- **Proof size:** proportional to $O(m)$
- **Setup time:** proportional to $O(n)$
- **Query time:** proportional to $O(m)$
- **Verification time:** proportional to $O(m)$

Related Work

	[SBZ01]	[JMSW02]	[CLX09]	[BBD+10]	[SPB+12]	[PSPDM12]	[KAB12]	This Work
Obliviousness				✓	✓	✓		✓
Setup time	$n \log n$	n	n	n^2	n^2	n	n	n
Space	n	n	n	n^2	n^2	n	n^2	n
Query time	m	$n \log n$	n	mn	m	n	n	$\min(m \log n, n)$
Verification time	$m \log n \log m$	$m \log n$	n^2	m^2	m^2	m	m	m
Integrity proof	m	$m \log n$	n	m^2	m	m	m	m
Order proof	m	$m \log n$	n	m^2	m^2	m	n	m
Assumption	RSA	RSA	SRSA Division	EUCMA	nEAE	AnAHF	ROH, RSA	nBDHI

Table: Comparison with previous works where n = size of the list, m = size of the queried sublist

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Privacy Preserving Authenticated List (PPAL)

$\mathcal{L}$ = ordered list of distinct elements.

- $(\text{digest}_C, \text{digest}_S) \leftarrow \text{Setup}(1^k, \mathcal{L})$(Owner)
  digest_S= Digest for the server
  digest_C= Digest for the client
- $(\text{order}, \text{proof}) \leftarrow \text{Query}(\text{digest}_S, \mathcal{L}, \delta)$(Server)
  δ = sublist of a list $\mathcal{L}$ is defined as: Elements(δ) $\subseteq$ Elements($\mathcal{L}$).
  order = $\pi_{\mathcal{L}}(\delta)$
  proof = proof of order.

Privacy Preserving Authenticated List (PPAL) (Cont.)

- $b \leftarrow \text{Verify}(\text{digest}_C, \delta, \text{order}, \text{proof})$ (Client)
  $b = \text{ACCEPT}$ iff $\text{Elements}(\delta) \subseteq \text{Elements}(\mathcal{L})$ and $\text{order} = \pi_{\mathcal{L}}(\delta)$.
  Otherwise, $b = \text{REJECT}$.

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Aggregate Signature

Given signatures $\sigma_1, \ldots, \sigma_n$ on distinct messages $M_1, \ldots, M_n$ from n distinct users $u_1, \ldots, u_n$, an aggregate signature aggregates these signatures into a single short signature σ.

σ and the n messages convince the verifier that the n users indeed signed the n original messages (i.e., user i signed message M_i).

We use the special case where a single user signs n distinct messages $M_1, \ldots, M_n$.

Security: the aggregate signature σ is valid if and only if the aggregator used all σ_i's to construct it.

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Intuition

Every element of the list is associated with a member witness.

The rank of the element in encoded the member witness and then the rank information "blinded" with randomness.

Every pair of (element, member witness) is signed by the owner.

The signatures are aggregated to compute the list digest signature.

Intuition (Continued)

The list digest signature and some information is sent to the server.

The list digest signature is sent to the client.

The server can compute a valid digest signature for any sublist of the source list by exploiting the homomorphic nature of aggregate signatures, that is without owner's involvement.

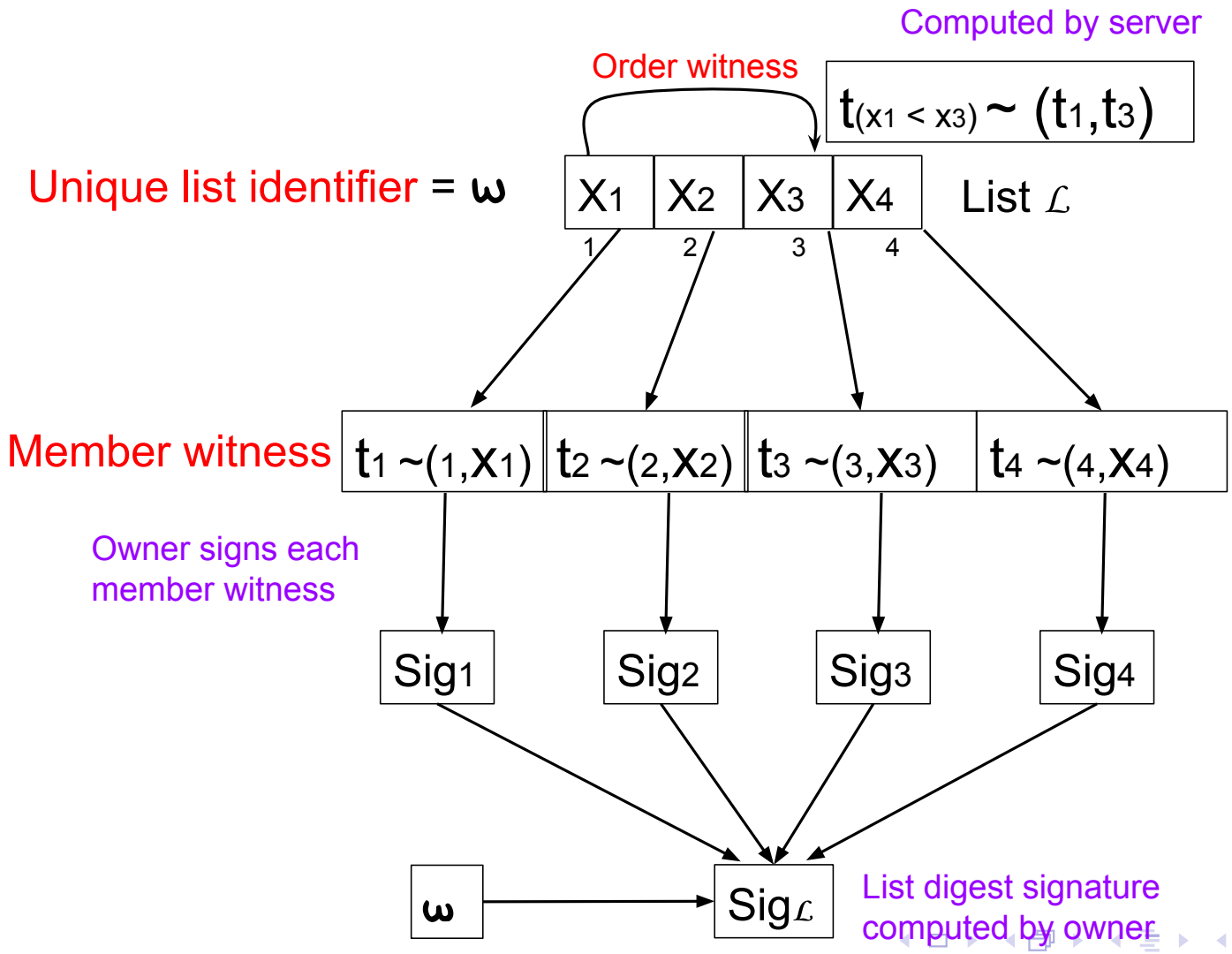
The client can verify the individual signatures in a single invocation to aggregate signature verification.

Intuition (Continued)

The owner sends to the server the random elements used in computing the member witnesses.

The server computes the order witnesses without the owner's involvement, using these random elements.

Schematically



Some Notations

- $k \in \mathbb{N}$ is the security parameter of the scheme
- G, G_1 multiplicative cyclic groups of prime order p
- p is a large k-bit prime
- g is a random generator of G
- $e : G \times G \rightarrow G_1$ is computable bilinear nondegenerate map
- $\mathcal{H} : \{0,1\}^* \rightarrow G$: full domain hash function
- System parameters: $(p, G, G_1, e, g, \mathcal{H})$
- We assume all the efficiently computable group operations and hashing can be performed in $O(1)$ time (abstracting the $O(\text{poly(k)})$ time and $O(\text{poly(k)})$ space required for representation of group elements)

Setup

$$(\text{digest}_C, \text{digest}_S) \leftarrow \text{Setup}(1^k, \mathcal{L})$$

Input: The security parameter 1^k and the list, $\mathcal{L} = \{x_1, \ldots, x_n\}$

Algorithm:

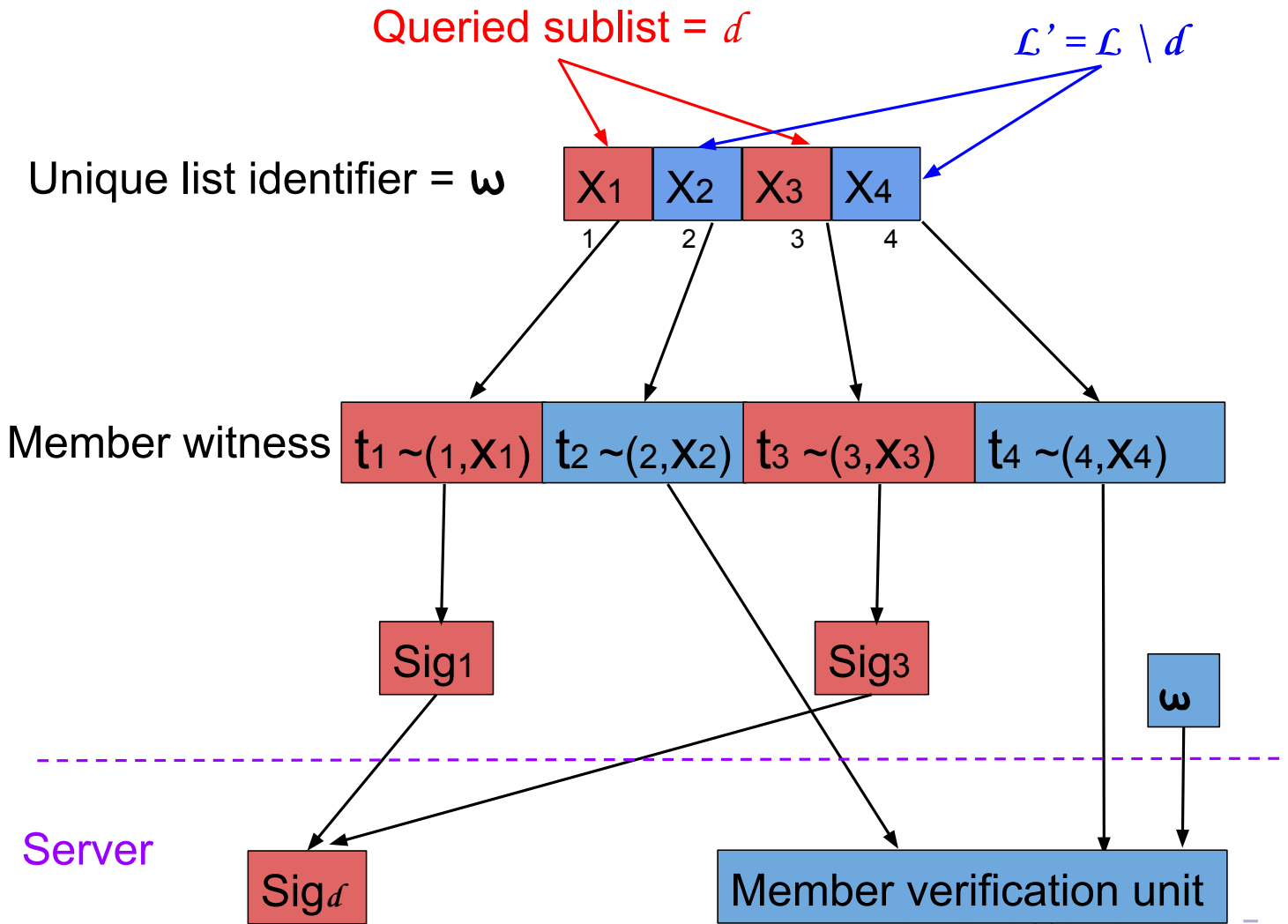
- The secret key $\text{sk}_o = \langle s, v \rangle$, $s \xleftarrow{\$} \mathbb{Z}_p^*$ and $v \xleftarrow{\$} \mathbb{Z}_p^*$ $O(1)$
- $g \xleftarrow{\$} G$ $O(1)$ For every element x_i in $\mathcal{L} = \{x_1, \ldots, x_n\}$
  - $r_i \xleftarrow{\$} \mathbb{Z}_p^*$ $\forall i : O(n)$
  - Set $\Omega_{\mathcal{L}} := \langle r_1, r_2, \ldots, r_n \rangle$, $r_i \neq r_j$ for $i \neq j$
  - Member witness, $t_{x_i \in \mathcal{L}} \leftarrow (g^{s^i})^{r_i}$ $\forall i : O(n)$

Setup (Continued)

Algorithm:

- Sign x_i as $\sigma_i \leftarrow \mathcal{H}(t_{x_i \in \mathcal{L}} || x_i)^v$ $\boxed{\forall i : O(n)}$
- Pick nonce $\omega \xleftarrow{\$} \{0,1\}^*$ and salt $\leftarrow \mathcal{H}(\omega)^v$ $\boxed{O(1)}$
- List digest signature: $\sigma_{\mathcal{L}} \leftarrow \text{salt} \times \prod_{1 \leq i \leq n} \sigma_i$ $\boxed{O(n)}$
- Set $\Sigma_{\mathcal{L}} := \langle \{t_{x_i \in \mathcal{L}}, \sigma_i\}_{1 \leq i \leq n}, \mathcal{H}(\omega) \rangle$
- Compute $\langle g, g^s, g^{s^2}, \ldots, g^{s^n} \rangle$ $\boxed{O(n)}$
- Set $\text{digest}_C := (g^v, \sigma_{\mathcal{L}})$
- Set $\text{digest}_S := (g^v, \sigma_{\mathcal{L}}, \langle g, g^s, g^{s^2}, \ldots, g^{s^n} \rangle, \Sigma_{\mathcal{L}}, \Omega_{\mathcal{L}})$
- Return $(\text{digest}_C, \text{digest}_S)$

Query


Queried sublist = $\mathcal{d}$
$\mathcal{L}' = \mathcal{L} \setminus \mathcal{d}$
Unique list identifier = ω
X1
X2
X3
X4
1
2
3
4
Member witness
t1 ~(1,X1)
t2 ~(2,X2)
t3 ~(3,X3)
t4 ~(4,X4)
Sig1
Sig3
ω
Server
Sig$\mathcal{d}$
Member verification unit

Query (Continued)

$$(\text{order}, \text{proof}) \leftarrow \text{Query}(\text{digest}_S, \mathcal{L}, \delta)$$

Input:

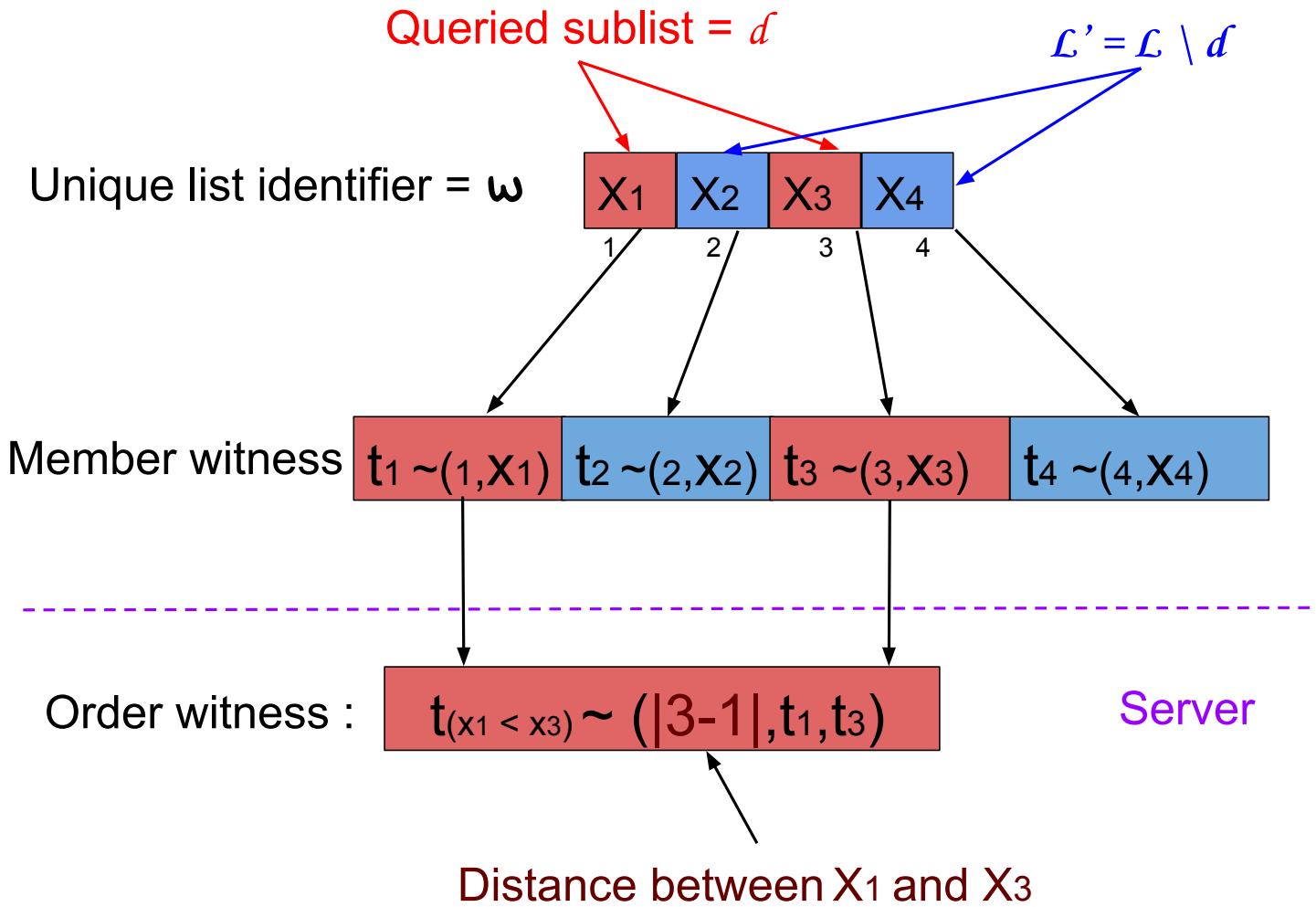
- The server digest, digest_S
- The list $\mathcal{L}$
- A sublist, $\delta = \{z_1, \ldots, z_m\}$, $z_i \in \mathcal{L}$, $1 \leq i \leq m$.

Algorithm:

- Let $\mathcal{L}' = \mathcal{L} \setminus \delta$
- order $\leftarrow \pi_{\mathcal{L}}(\delta) = \{y_1, \ldots, y_m\}$ $O(m)$
- $\Sigma_{\text{order}} = \langle \sigma_{\text{order}}, T, \lambda_{\mathcal{L}'} \rangle$
- $T = \{t_{y_1 \in \mathcal{L}}, \ldots, t_{y_m \in \mathcal{L}}\}$
- Digest signature for the sublist $\sigma_{\text{order}} \leftarrow \prod_{z_j \in \delta} \sigma_{\text{rank}(\mathcal{L}, z_j)}$ $O(m)$
- Member verification unit: $\lambda_{\mathcal{L}'} \leftarrow \mathcal{H}(\omega) \times \prod_{x \in \mathcal{L}'} \mathcal{H}(t_{x_{\text{rank}(\mathcal{L},x) \in \mathcal{L}}} || x)$

  $O(n-m)$, With preprocessing: $O(min\{m \log n, n\})$
- Set $\Sigma_{\text{order}} := (\sigma_{\text{order}}, T, \lambda_{\mathcal{L}'})$

Query (Continued)

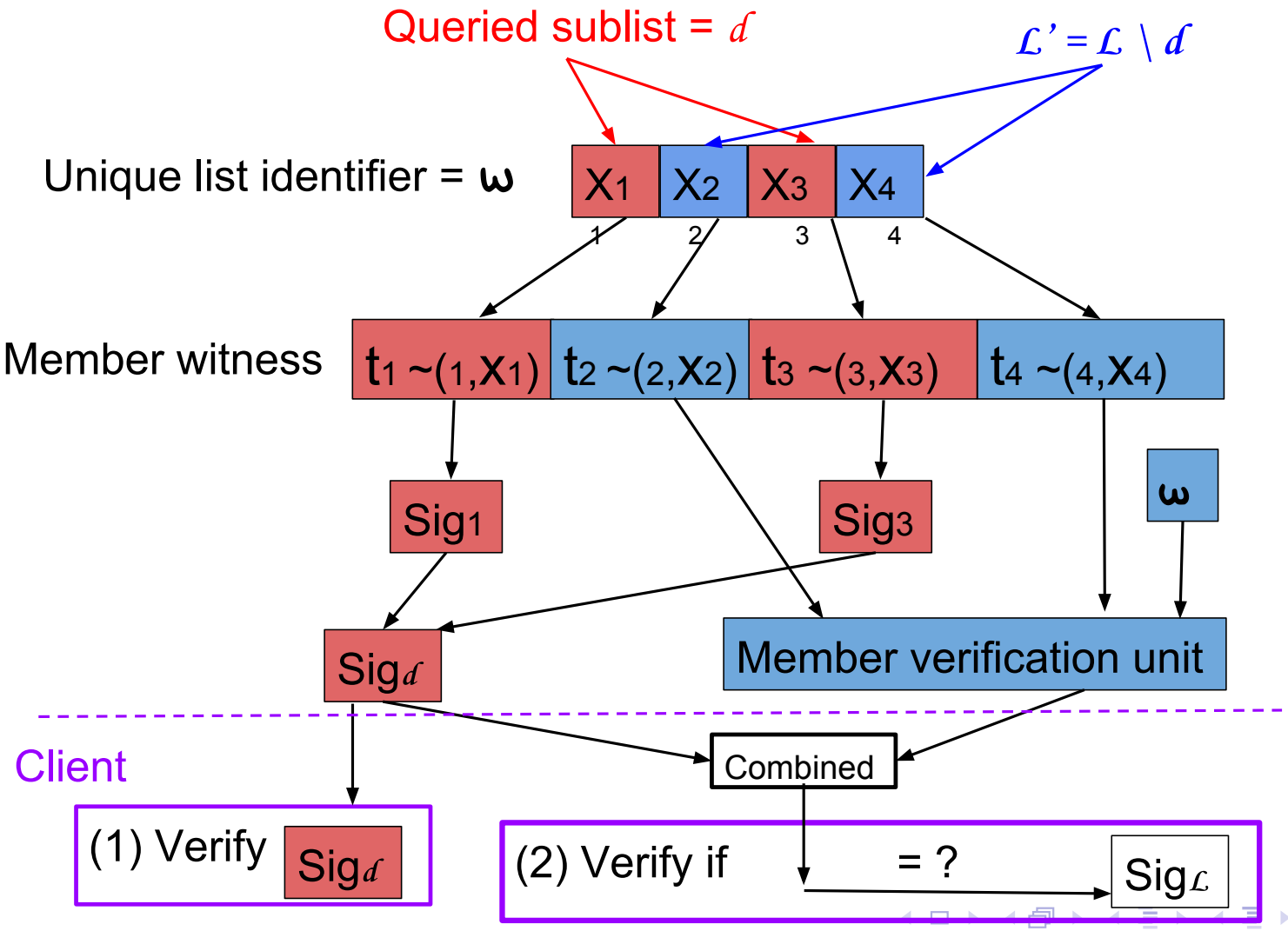


Query (Continued)

Algorithm:

- Let $i' = \text{rank}(\mathcal{L}, y_j), i'' = \text{rank}(\mathcal{L}, y_{j+1})$
- Let $d = |i' - i''|, r_1 = \Omega_{\mathcal{L}}[i']^{-1}$ and $r_2 = \Omega_{\mathcal{L}}[i'']$
- Order witness, $t_{y_j < y_{j+1}} = (g^{s^d})^{r_1 r_2}$ $\boxed{\forall i : O(m)}$
- Set $\Omega_{\text{order}} := \{t_{y_1 < y_2}, t_{y_2 < y_3}, \ldots, t_{y_{m-1} < y_m}\}$.
- Set proof $:= (\Sigma_{\text{order}}, \Omega_{\text{order}})$
- Return (order, proof)

Verify


Queried sublist = $\mathcal{d}$
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X1
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Member witness
t1 ~(1,X1)
t2 ~(2,X2)
t3 ~(3,X3)
t4 ~(4,X4)
Sig1
Sig3
ω
Sig$\mathcal{d}$
Member verification unit
Client
Combined
(1) Verify
Sig$\mathcal{d}$
(2) Verify if
= ?
Sig$\mathcal{L}$

Verify (Continued)

$b \leftarrow \text{Verify}(\text{digest}_C, \delta, \text{order}, \text{proof})$

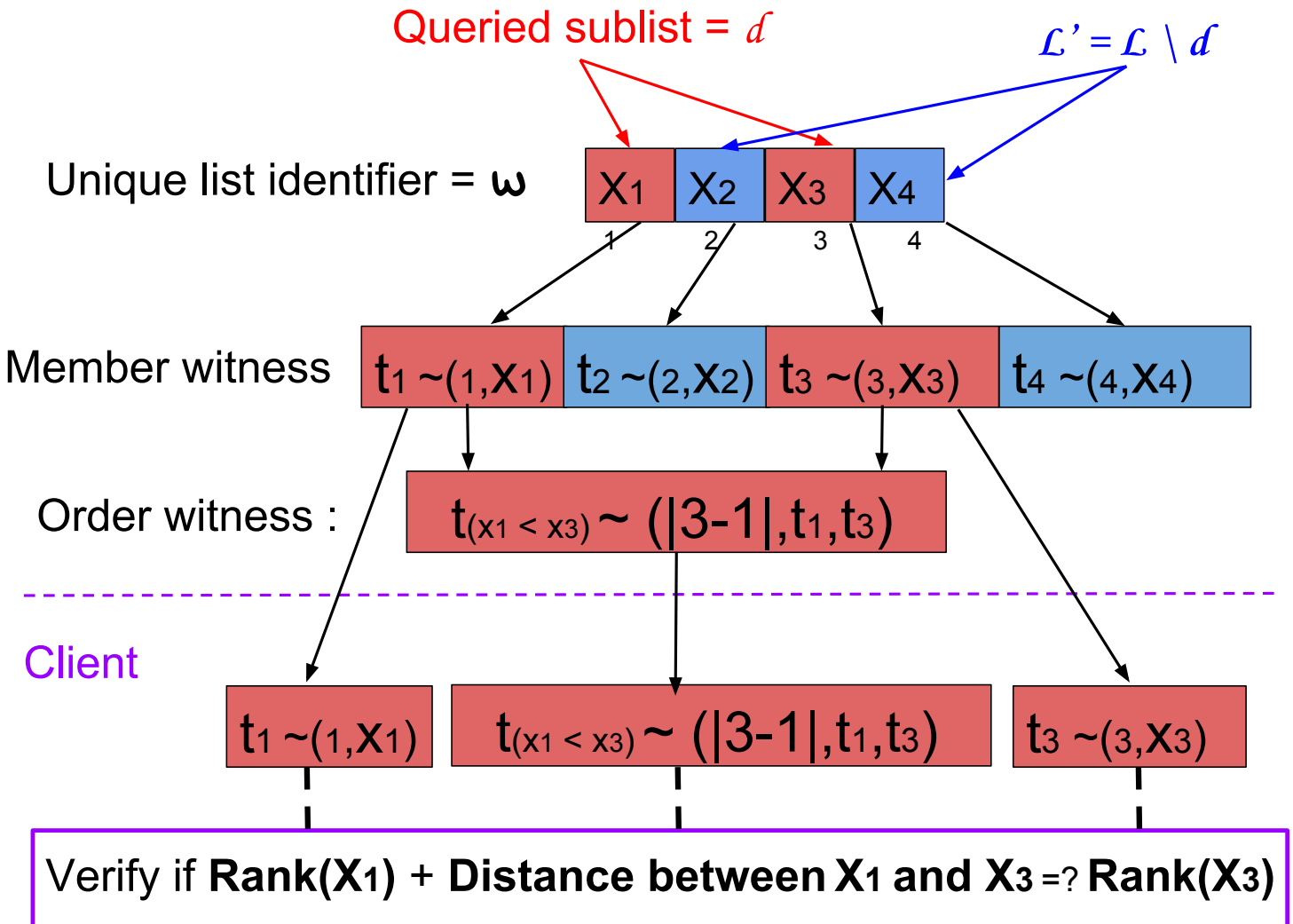
Input:

- The client digest digest_C
- A sublist $\delta = \{y_1, \ldots, y_m\}$
- A permutation on the elements of δ, order
- Proof of the order, proof

Algorithm:

- Check if $\text{Elements}(\text{order}) \stackrel{?}{=} \text{Elements}(\delta)$ $O(m)$
- Compute $\xi \leftarrow \prod_{y_j \in \delta} \mathcal{H}(t_{y_j \in \mathcal{L}} || y_j)$ $O(m)$
- Check if $e(\sigma_{\text{order}}, g) \stackrel{?}{=} e(\xi, g^v)$ and $e(\sigma_{\mathcal{L}}, g) \stackrel{?}{=} e(\sigma_{\text{order}}, g) \times e(\lambda_{\mathcal{L}'}, g^v)$ $O(1)$

Verify (Continued)



Verify (Continued)

Algorithm:

- For every $j \in [1, m-1]$, $e(t_{y_j \in \mathcal{L}}, t_{y_j < y_{j+1}}) \stackrel{?}{=} e(t_{y_{j+1} \in \mathcal{L}}, g)$. $\boxed{O(m)}$
- Return ACCEPT iff all the equalities verify, and REJECT, otherwise.

Preprocessing Step

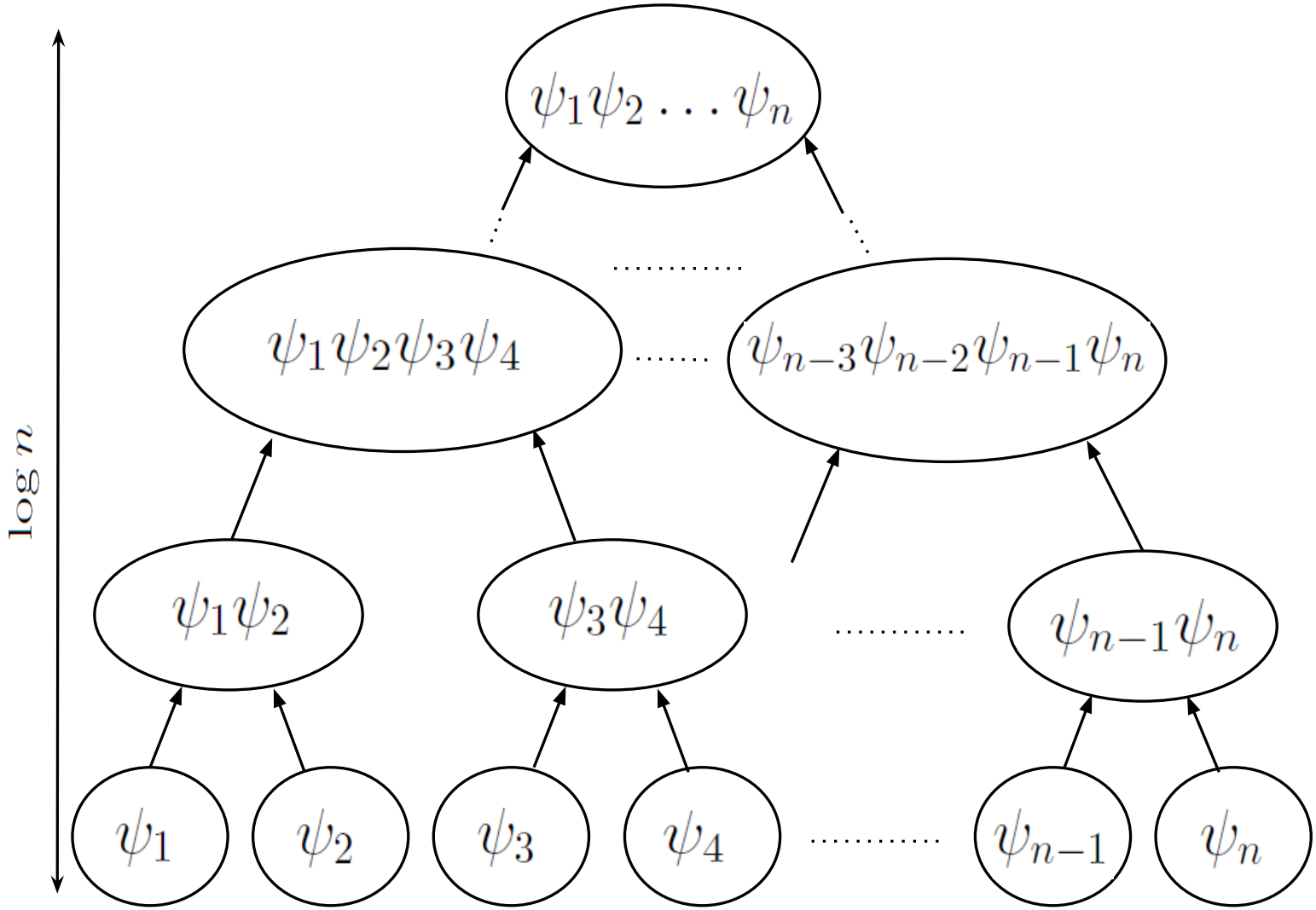
The server can precompute and store some products to reduce the overall running time of this algorithm to $O(m \log n)$ when $m \ll n$.

Let $\psi_i = \mathcal{H}(t_{x_i \in \mathcal{L}} || x_i)$ for every element in $\mathcal{L} = \{x_1, \ldots, x_n\}$.

A balanced binary tree is built over n leaves, where the ith leaf and stores ψ_i.

Each internal node of the tree stores the product of the values stored at its children.

Preprocessing Step (Continued)



Preprocessing Step (Continued)

Computing each internal node takes time $O(1)$ and the tree has $O(n)$ nodes

So, the precomputation takes time $O(n)$ and requires $O(n)$ storage.

Computing $\lambda_{\mathcal{L}'}$ will require computing the product of $m+1$ partial products.

Each partial product can be computed using $O(\log n)$ precomputed products.

The total time required to compute the product of $m+1$ partial products is $O((m+1)\log n) = O(m \log n)$.

Complexity Summary

Table: n = number of elements in the source list $\mathcal{L}$ and $m \leq n$, size of the queried sublist

	Time Complexity	Space Complexity
Owner	$O(n)$	$O(n)$
Server	Preprossessing: $O(n)$ $O(min\{m \log n, n\})$	$O(n)$
Client	$O(m)$	$O(m)$

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Hardness Assumption

Definition (n-Bilinear Diffie Hellman Inversion (n-BDHI) [BB04])

Let s be a random element of $\mathbb{Z}_p^*$ and n be a positive integer. Then, for every PPT adversary $\mathcal{A}$ there exists a negligible function $\nu(.)$ such that:

$$Pr[s \xleftarrow{\$} \mathbb{Z}_p^*; y \leftarrow \mathcal{A}(\langle g, g^s, g^{s^2}, \ldots, g^{s^n} \rangle) : y = e(g,g)^{\frac{1}{s}}] \leq \nu(k).$$

Dan Boneh and Xavier Boyen.
Efficient selective-id secure identity based encryption without random oracles.
In *Proceedings of Eurocrypt 2004, volume 3027 of LNCS*, pages 223–238. Springer-Verlag, 2004.

Soundness Proof Sketch

We assume that there exists a malicious server adv, which forges the order on a non-trivial sublist $\delta = \{x_1, \ldots, x_m\}$, where $m \geq 2$, for a list $\mathcal{L}$.

Then there exists at least one inversion pair (x_i, x_j) whose order is flipped in adv's forgery.

Wlog assume that $u < v$ where $u = \text{rank}(\mathcal{L}, x_i)$ and $v = \text{rank}(\mathcal{L}, x_j)$.

Soundness Proof Sketch (Continued)

Then adv must have forged the witness $t_{x_j<x_i} = \left(g^{s^{(u-v)}}\right)^{r_1 r_2^{-1}}$ that passes the verification, where $r_1, r_2 \in \mathbb{Z}_p^*$ are the randomnesses used in member witnesses of elements x_i and x_j, respectively.

By invoking adv and using its forged witness $t_{x_j<x_i}$, we can construct a PPT adversary that successfully breaks the n-BDHI hardness assumption by outputting
$e\left(t_{x_j<x_i}, \left(g^{s^{v-u-1}}\right)^{r_1{}^{-1} r_2}\right) = e(g,g)^{\frac{1}{s}}$

Zero-Knowledge Proof Sketch

We write a stateful simulator Sim that has access to the system parameters $(p, G, G_1, e, g, \mathcal{H})$.

Sim picks a random element $v \xleftarrow{\$} \mathbb{Z}_p^*$ and a random element $g_1 \xleftarrow{\$} G$ and publishes as $\text{digest}_C = (g^v, g_1^v)$ and keeps v as the secret key.

For a query on sublist $\delta = \{x_1, x_2, \ldots, x_m\}$, Sim makes an oracle access to list $\mathcal{L}$ to get the list order of the elements.

Sim fakes the proof objects as follows:

Zero-Knowledge Proof Sketch (Continued)

- Sim picks a random element $r_i \xleftarrow{\$} \mathbb{Z}_p^*$ sets the member authentication unit as $t_{y_i \in \mathcal{L}} := g^{r_i}$.
- Sim computes $\sigma_{y_i} \leftarrow \mathcal{H}(t_{y_i \in \mathcal{L}} || y_i)^v$.
- For every pair of elements y_i, y_{i+1} in order, Sim computes $t_{y_i < y_{i+1}} \leftarrow g^{r_{i+1}/r_i}$.

Thus Sim produces view identically distributed to real view of the adversary, given only oracle access to the list, using the fact that our PPAL construction uses witnesses blinded in their exponents.

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Summary

Theorem

The privacy-preserving authenticated list (PPAL) construction of satisfies the security properties of completeness, soundness and zero-knowledge in the random oracle model and under the n-BDHI assumption. Also, the construction has the following performance, where n denotes the list size and m denotes the query size.

- *The owner and the server use $O(n)$ space.*
- *The owner performs the setup phase in $O(n)$ time.*
- *The server performs the preprocessing phase in $O(n)$ time.*
- *The server computes the answer to a query and its proof in $O(\min\{m \log n, n\})$ time.*
- *The client verifies the proof in $O(m)$ time and space.*

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Thank you!