Ideal and Resistive Nanowire Decoders

General Models for Nanowire Addressing

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The Nanowire

- Nanoscale computing requires nanoscale wires (NWs) and nanoscale devices.
- Sets of parallel NWs have been produced.
- Devices will reside at NW intersections.
- To control these devices, we must gain control over individual NWs.
NW Technologies

SNAP NWs
(Heath, Caltech)

CVD NWs
(Lieber, Harvard)

Copolymer Directed Growth
(Stoykovich, UW)
The crossbar is currently the most feasible nano-scale architecture.

By addressing individual NWs, we can control programmable molecules at NW crosspoints.

Crossbars are a basis for memories and circuits.
Nanowire Control

• Mesoscale contacts apply a potential along the lengths of NWs.

• Mesoscale wires (MWs) apply fields to across NWs, some of which form FETs.

• NW/MW junctions can form FETs using a variety of technologies:
  ⇒ Modulation-doping
  ⇒ Random Particle deposition
  ⇒ Masking NWs with dielectric material
Simple NW Decoders

- A potential is applied along the NWs.
- $M$ MW inputs control $N$ NW outputs. Each MW controls a subset of NWs.
- When a MW produces a field, the current in each NW it controls is greatly reduced.
- Each MW “subtracts” out subsets of NWs. This permits $M << N$.
- Decoders are assembled stochastically and become difficult to produce as $N$ is large.
Composite Decoders

- A composite decoder uses multiple simple decoders to control many NWs.
- The simple decoders share MW inputs.
- This space savings allows for mesoscale inputs.
Ideal Decoders

• To analyze a decoder, we must model how MWs control NWs.

• In an ideal decoder, a MW’s electric field completely turns off the NWs it controls. Other NWs are unaffected.

• This model is accurate if the FETs formed from MW/NW junctions have high on/off ratios.
Binary Codewords

• In an ideal decoder, we associate an $M$-bit codeword, $c_i$, with each NW, $n_i$.

• The $j^{th}$ MW controls the $i^{th}$ NW if and only if the $j^{th}$ bit of $c_i$, $c_{ij}$, is $1$.

• We also represent the decoder’s input as an $M$-bit binary vector, $A$.

• $n_i$ carries a current if and only if $A \cdot c_i = 0$. 
Codeword Assignment

• Decoders are assembled stochastically.

• Codewords are assigned to NWs according to a probability distribution.

• This distribution is a way of comparing decoding technologies.

⇒ With no misalignment, modulation-doping is at least as good as particle deposition.
Codeword Interaction

- If $c_{bj} = 1$ where $c_{aj} = 1$, $c_a$ implies $c_b$. Inputs that turn off $n_a$ turn off $n_b$.
- A set of codewords, $S$, is **addressable** if some input turns off all NWs not in $S$.
- $S = \{c_i\}$ is addressable if and only if no codeword implies $c_i$. $S$ is addressed with input $A = \overline{c_i}$.
Decoders for Memories

• A $B$-bit memory maps $B$ addresses to $B$ disjoint sets of storage devices.

• A $D$-address memory decoder addresses $D$ disjoint subsets of NWs.

• Equivalently, the decoder contains $D$ addressable codewords.
Resistive Decoders

- Decoders that rely on FETs are not ideal.
- MWs carrying a field increase each NW’s resistance by some amount.
- In a resistive decoder, codewords are real-valued. In real-valued codeword $r_i$, $r_{ij}$ is the resistance induced in $n_i$ by the $j^{th}$ MW.
- On input $A$, $n_i$’s resistance is $r_{base} + A \cdot r_i$. 
Ideal vs. Resistive

- In a resistive memory decoder the addressed NWs must output more current than the other NWs.

- Consider 1-hot codewords:
  - The addressed wire has resistance $< r_{base} + M r_{low}$
  - Remaining wires have resistance $> (r_{base} + r_{high})/N$

- We require that $r_{high} >> M N r_{low}$ and $N r_{base}$
  - If $r_{ij} \leq r_{low}$, $c_{ij} = 0$.
  - If $r_{ij} \geq r_{high}$, $c_{ij} = 1$.
  - If $r_{low} < r_{ij} < r_{high}$, $c_{ij}$ is an error.
Ideal Decoders with Errors

• To apply the ideal model to resistive decoders, consider binary codewords with random errors.

• If \( c_{ij} = e \), the \( j^{th} \) MW increases \( n_i \)‘s resistance by an unknown amount.

• Consider input \( A \) such that the \( j^{th} \) MW carries a field. \( A \) functions reliably if a MW for which \( c_{ik} = 1 \) carries a field.
Balanced Hamming Distance

Consider two error-free codewords, $c_a$ and $c_b$. Let $|c_a - c_b|$ denote the number of inputs for which $c_{aj} = 1$ and $c_{bj} = 0$.

The balanced Hamming distance (BHD) between $c_a$ and $c_b$ is $2 \cdot \min(|c_a - c_b|, |c_b - c_a|)$.

If $c_a$ and $c_b$ have a BHD of $2d + 2$ they can collectively tolerate up to $d$ errors.
Fault-Tolerant Random Particle Decoders

• In a particle deposition decoder, $c_{ij} = 1$ with some fixed probability, $p$.

• If each pair of codeword has a BHD of at least $2d + 2$, the decoder can tolerate $d$ errors per pair.

• This holds with probability $> 1 - f$ when

$$M > \frac{(d + (d^2 + 4 \ln(N^2/f))^{1/2})^2}{4p(1 - p)}$$
Codeword Discovery

- Random codewords must be discovered to map memory addresses to decoder inputs.
- Input $A'$ contains $A$ if $A'_j = 1$ where $A_j = 1$.
- If $c_i$ is addressable, $A = \overline{c_i}$ produces a current, but inputs containing $A$ do not.
- By testing if inputs produce currents, the codewords in an error-free decoder are discovered without nanoscale measurement.
Codeword Discovery with Errors

- If errors are present, we cannot just test for the presence or absence of current.
- If inputs $A$ and $B$ both produce sufficiently large currents, we can be certain that both address some NW.
- If their union produces a small current, the inputs address distinct codewords.
Conclusion

• Stochastically assembled decoders can reliably control NWs even if errors occur.
• Our decoder model applies to many viable technologies. It provides conditions that a decoder must be meet.
• Discovery algorithms verify that a decoder functions properly without requiring nanoscale measurements.