TAC 2000: Bid Determination in Simultaneous Auctions

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eBay Auctions

Complements

- $u(A\bar{B}) + u(\bar{A}B) \leq u(AB)$
- camera, flash, and tripod

Substitutes

- $u(A\bar{B}) + u(\bar{A}B) \geq u(AB)$
- Canon AE-1 and Canon A-1

BD Problems

Allocation

- given only the set of goods I already hold, what is the maximum utility I can attain?

Acquisition

- given the set of goods I already hold, and given ask prices in all open auctions, on what set of additional goods should I bid to maximize utility less costs?

Completion

- given the set of goods I already hold, and given ask and *bid* prices in all open auctions, on what additional set of goods should I place bids or *asks* to maximize my utility plus profits less costs?

TAC Market Game

Agent's Score = Utility − Costs + Profits

Supply

- Flights: Inbound and Outbound
- Hotels: Grand Hotel and Le FleaBag Inn
- Entertainment: Red Sox, Symphony, Phantom

Auctions

- Flights: infinite supply, prices follow random walk, clear continuously, no resale permitted
- Hotels: ascending, multi-unit, 16th price auctions, transactions clear at auction close (early closings after random period of inactivity), no resale
- Entertainment: continuous double auctions, initial endowment, resale is permitted

TAC Market Game

Demand

Client	IAD	IDD	HV	BRS	SY	PH
1	1	3	99	134	118	65
2	1	4	131	170	47	49
3	1	2	147	13	55	49
4	3	4	145	130	60	85
5	1	4	82	136	68	87
6	2	4	53	94	51	105
7	1	3	54	156	126	71
8	1	5	113	119	187	143

Feasible Packages

- arrival date prior to departure date
- same hotel on all intermediate nights
- at most one entertainment event per night
- at most one of each type of entertainment

TAC Market Game

Utility = 1000 - travelPenalty + hotelBonus + funBonus

$$\text{travelPenalty} = 100(|\text{IAD} - \text{AD}| + |\text{IDD} - \text{DD}|)$$

$$\text{hotelBonus} = \begin{cases} \text{HV} & \text{if H = G} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{funBonus} = \text{entertainment values}$$

Allocation

Client	AD	DD	H	Ticket	Utility
1	1	3	G	SY1, BRS2	1351
2	1	3	G	BRS1	1201
3	1	2	G	—	1147
4	3	4	G	BRS3	1275
5	1	3	F	BRS1, PH2	1123
6	3	4	G	PH3	1058
7	1	3	F	SY1, BRS2	1282
8	1	5	G	PH1, SY3, BRS4	1562

TAC Agent Architecture

(A) While some auctions remain open, do

1. Update current prices and holdings
2. Estimate future prices, supply, and demand
3. Run completer to determine buy/sell quantities
4. Place bid/ask prices strategically

(B) After all auctions close, run allocator

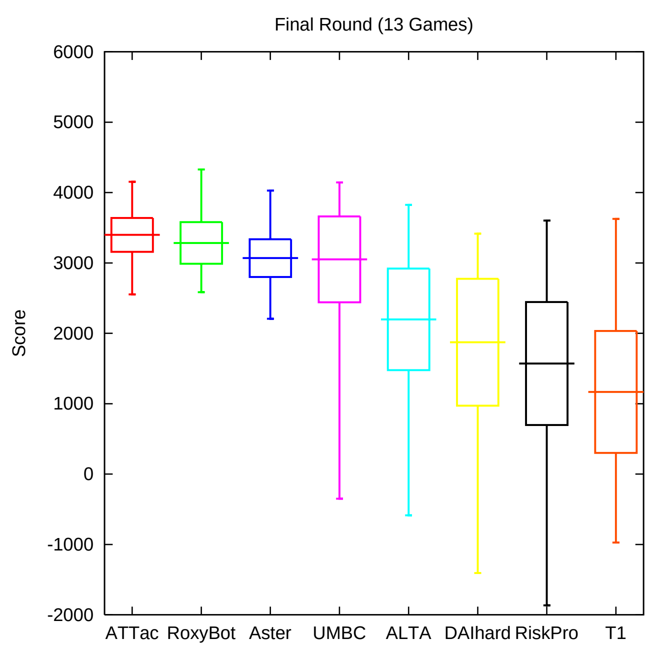
Overview

Theoretical Observations

- BD in double auctions can be reduced to BD in single-sided auctions
- BD in simultaneous auctions are isomorphic to WD in combinatorial auctions

Empirical Tests: TAC-2000

- Heuristic search vs. Integer linear programming



Pricelines

Buying Priceline

$\vec{p}_g = \langle 0, 0, 0, 0, 20, 30 \rangle$

Given a set of buying pricelines $P = \{\vec{p}_g \mid g \in G\}$ and a set of packages S, we define the utility and cost of S:

$$\begin{aligned}
\text{Util}(S) &= \sum_{\vec{q} \in S} u(\vec{q}) \\
\forall g, \quad \text{Used}(S, g) &= \sum_{\vec{q} \in S} q_g \\
\forall g, \quad \text{Cost}_g(S, P) &= \sum_{n=1}^{\text{Used}(S,g)} p_{gn} \\
\text{Cost}(S, P) &= \sum_{g \in G} \text{Cost}_g(S, P)
\end{aligned}$$

Pricelines

Selling Priceline

$\vec{\pi}_g = \langle 10, 5, 0, 0 \rangle$

Given a set of selling pricelines $\Pi = \{\vec{\pi}_g \mid g \in G\}$ and a set of packages S, we define profit analogously to cost:

$$
\begin{aligned}
\forall g, \quad \text{Unused}(S, g, \Pi) &= \max\{|\vec{\pi}_g| - \text{Used}(S, g), 0\} \\
\forall g, \quad \text{Profit}_g(S, \Pi) &= \sum_{n=1}^{\text{Unused}(S,g,\Pi)} \pi_{gn} \\
\text{Profit}(S, \Pi) &= \sum_{g \in G} \text{Profit}_g(S, \Pi)
\end{aligned}
$$

Formalization

Acquisition

Inputs: set of packages Q

set of buying pricelines P

utility function $u : Q \to \mathbb{R}^+$

Output: $S^* \in \arg\max_{S \subseteq Q} (\text{Util}(S) - \text{Cost}(S, P))$

Completion

Inputs: set of packages Q

set of buying pricelines P

set of selling pricelines Π

utility function $u : Q \to \mathbb{R}^+$

Output: $S^* \in \arg\max_{S \subseteq Q} (\text{Util}(S) - \text{Cost}(S, P) + \text{Profit}(S, \Pi))$

Theoretical Observation 1

BD in double auctions reduces to BD in single-sided auctions: *i.e.*, completion $\mapsto$ acquisition

Buying Priceline
$\vec{p}_g = \langle 0, 0, 0, 0, 20, 30 \rangle$

Selling Priceline
$\vec{\pi}_g = \langle 10, 5, 0, 0 \rangle$

1st Reduction

- extend package input set with single-item packages, one for each copy of each item in selling pricelines; assign selling prices as utilities for these packages: $\vec{\pi}_g \mapsto$ 4 new packages with utilities 10, 5, 0, 0

2nd Reduction

- add reversed selling pricelines to buying pricelines: $\vec{p}_g + \text{reverse}(\vec{\pi}_g) = \langle 0, 0, 5, 10, 20, 30, \infty, \infty, \ldots \rangle$

1st Reduction

Theorem For all P, Π, Q, and u,

$$\text{Completion}(P, \Pi, Q, u) = \text{Acquisition}(P, Q \cup Q', u \cup u') \cap Q$$

where $Q' = \{\vec{e}_{gn} \mid g \in G, n = 1 \ldots |\vec{\pi}_g|\}$
and $u' = \{\vec{e}_{gn} \mapsto \pi_{gn} \mid g \in G, n = 1 \ldots |\vec{\pi}_g|\}$

Proof

A solution to the completion problem is a subset of Q that maximizes the function

$$f(X) = \text{Util}(X) - \text{Cost}(X, P) + \text{Profit}(X, \Pi)$$

A solution to the acquisition problem posed in the statement of the theorem is a subset of $Q \cup Q'$ that maximizes

$$f'(X) = \text{Util}(X) - \text{Cost}(X, P)$$

We show that for all $S \subseteq Q$ and $S' \subseteq Q'$, $f(S) = f'(S \cup S')$

1st Reduction (Continued)

$f'(S \cup S') = \text{Util}(S) + \text{Util}(S') - (\text{Cost}(S, P) + \text{Cost}(S', P))$

since S and S' are disjoint and the functions Util and Cost are summations. $\text{Cost}(S', P) = 0$ because, by the definition of Q', all goods in S' are owned by the agent. A short calculation also shows $\text{Profit}(S, \Pi) = \text{Util}(S')$:

$$\begin{aligned} \text{Profit}(S, \Pi) &= \sum_{g \in G} \sum_{n=1}^{\text{Unused}(S,g,\Pi)} \pi_{gn} \\ &= \sum_{\vec{q} \in S'} u'(\vec{q}) \\ &= \text{Util}(S') \end{aligned}$$

Therefore,

$$\begin{aligned} f'(S \cup S') &= \text{Util}(S) + \text{Util}(S') - (\text{Cost}(S, P) + \text{Cost}(S', P)) \\ &= \text{Util}(S) + \text{Profit}(S, \Pi) - \text{Cost}(S, P) \\ &= f(S) \end{aligned}$$

Finally, for S, S' that maximize g, S also maximizes f. $\square$

2nd Reduction

Theorem For all P, Π, Q, and u,

$$\text{Completion}(P, \Pi, Q, u) = \text{Acquisition}(P', Q, u)$$

where $P' = \{\vec{p}_g + \text{reverse}(\vec{\pi}_g) \mid g \in G\}$

Proof

A solution to the completion problem is a subset of Q that maximizes the function

$$f(S) = \text{Util}(S) - \text{Cost}(S, P) + \text{Profit}(S, \Pi)$$

A solution to the acquisition problem posed in the statement of the theorem is a subset of Q that maximizes

$$f'(S) = \text{Util}(S) - \text{Cost}(S, P')$$

Thus, it suffices to show that there exists some constant C *s.t.* for all $S \subseteq Q$, $f(S) = f'(S) + C$.

2nd Reduction (Continued)

Let C_g represent the total profits the agent could earn if it were to sell all its copies of good g:

$$C_g = \sum_{n=1}^{|\vec{\pi}_g|} \pi_{gn}$$

We show that for all goods g,

$$\text{Profit}_g(S, \Pi) - \text{Cost}_g(S, P) = C_g - \text{Cost}_g(S, P')$$

Therefore

$$\begin{aligned} f(S) &= \text{Util}(S) - \text{Cost}(S, P) + \text{Profit}(S, \Pi) \\ &= \text{Util}(S) - \sum_g \text{Cost}_g(S, P) + \sum_g \text{Profit}_g(S, \Pi) \\ &= \text{Util}(S) - \sum_g \text{Cost}_g(S, P') + \sum_g C_g \\ &= \text{Util}(S) - \text{Cost}(S, P') + C \\ &= f'(S) + C \end{aligned}$$

2nd Reduction (Continued)

Two cases arise. In the first case, $|\vec{\pi}_g| \geq \text{Used}(S, g)$, which implies that $\text{Unused}(S, g, \Pi) \geq 0$, $\text{Cost}_g(S, P) = 0$. The agent does not use all the goods it owns; it earns profit on unused goods and incurs no additional costs:

$$
\begin{aligned}
C_g - \text{Cost}_g(S, P') &= \sum_{n=1}^{|\vec{\pi}_g|} \pi_{gn} - \sum_{n=1}^{\text{Used}(S,g)} p'_{gn} \\
&= \sum_{n=1}^{|\vec{\pi}_g|} \pi_{gn} - \sum_{n=1}^{\text{Used}(S,g)} \pi_{gn} \\
&= \sum_{n=1}^{|\vec{\pi}_g|} \pi_{gn} - \left(\sum_{n=1}^{|\vec{\pi}_g|} \pi_{gn} - \sum_{n=1}^{\text{Unused}(S,g,\Pi)} \pi_{gn} \right) \\
&= \sum_{n=1}^{\text{Unused}(S,g,\Pi)} \pi_{gn} \\
&= \text{Profit}_g(S, \Pi) - \text{Cost}_g(S, P)
\end{aligned}
$$

2nd Reduction (Continued)

In the second case, $|\vec{\pi}_g| \leq \text{Used}(S,g)$, which implies that $\text{Unused}(S,g,\Pi) = 0$, $\text{Profit}_g(S,\Pi) = 0$. The agent uses all the goods it owns; it earns no profits and perhaps incurs additional costs buying further copies of goods:

$$
\begin{aligned}
C_g - \text{Cost}_g(S,P') &= \sum_{n=1}^{|\vec{\pi}_g|} \pi_{gn} - \sum_{n=1}^{\text{Used}(S,g)} p'_{gn} \\
&= \sum_{n=1}^{|\vec{\pi}_g|} \pi_{gn} - \left(\sum_{n=1}^{|\vec{\pi}_g|} \pi_{gn} + \sum_{n=|\vec{\pi}_g|+1}^{\text{Used}(S,g)} p_{gn} \right) \\
&= - \sum_{n=|\vec{\pi}_g|+1}^{\text{Used}(S,g)} p_{gn} \\
&= - \sum_{n=1}^{\text{Used}(S,g)} p_{gn} \\
&= \text{Profit}_g(S,\Pi) - \text{Cost}_g(S,P) \; \square
\end{aligned}
$$

Theoretical Observation 2

BD in simultaneous auctions are isomorphic to WD in combinatorial auctions

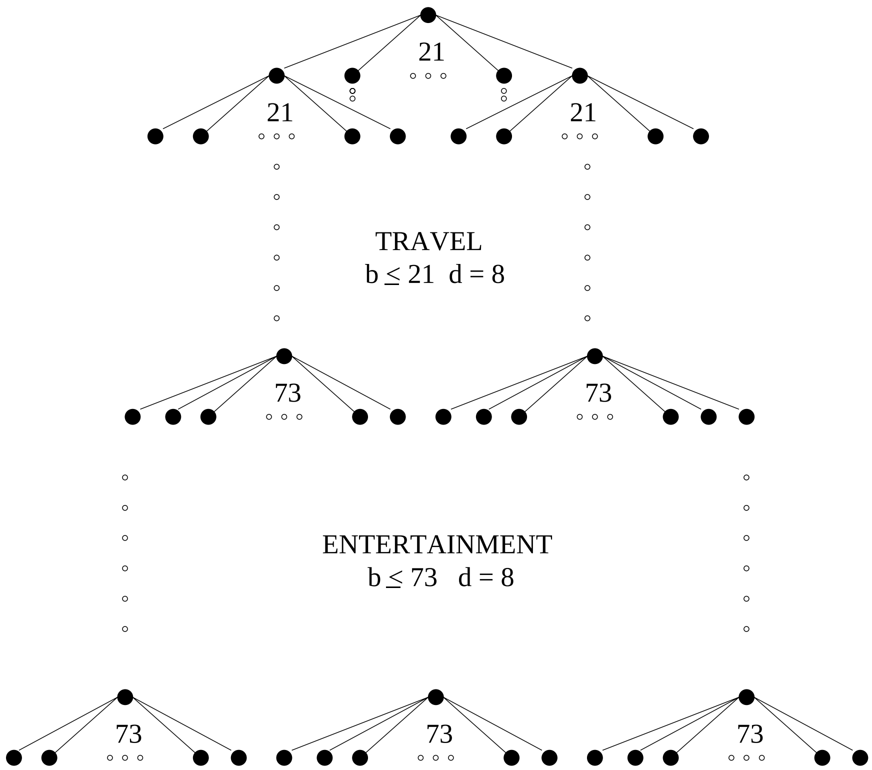
WD $\cong$ Allocation

- WD: auctioneer seeks the set of combinatorial bids maximizes profits, given feasibility constraints

WDR $\cong$ Acquisition $\cong$ Completion

- WDR (WD with reserve prices): auctioneer seeks the set of combinatorial bids that maximizes the difference between profits and reserve prices

Heuristic Search Solutions



A^* Search (provably optimal)

- intricate set of admissible heuristics [Greenwald and Boyan, 2001]

Beam Search (approximately optimal)

- "rollout" heuristic: at each node x, the heuristic value is that of a greedy assignment initiated at x

ILP Solution: Allocation

Index	Description	Number	TAC Values
i	clients	I	8
j	days	J	4
k	event types	K	3
l	hotel types	L	2
m	flight types	M	2
t	travel packages	T	20
s	entertainment tickets	S	12

Constant	Description
U_{it}	utility to client i of travel package t
U_{is}	utility to client i of entertainment ticket s
X_{jk}	number of entertainment tickets of type k for day j
Y_{jl}	number of hotel reservations of type l for day j
Z_{jm}	number of flights of type m for day j

Variable	Description	Number	TAC Values
v_{it}	is client i assigned package t?	$I \times T$	160
w_{is}	is client i entertainment ticket s?	$I \times S$	96

ILP Solution: Allocation (Continued)

Objective Function

$$\max \left(\sum_{i,t} U_{it} v_{it} + \sum_{i,s} U_{is} w_{is} \right)$$

Constraints

0. all variables are non-negative integers: $\forall i, s, t, v_{it}, w_{is} \in \mathbb{Z}^+$

1. I (8) constraints: cannot assign more than 1 travel package per client $\forall i, \sum_t v_{it} \leq 1$

2. cannot assign more tickets than number available
   - $J \times K$ (12) constraints: can assign at most X_{jk} entertainment tickets of type k for day j

     $$\forall j, k \sum_i \sum_{\{s | \text{DAY}(s)=j, \text{TYPE}(s)=k\}} w_{is} \leq X_{jk}$$

   - $J \times (L + M)$ (16) constraints: analogously, for hotels and flights

ILP Solution: Allocation (Continued)

3. $I \times K$ (24) constraints: can assign at most 1 entertainment ticket of type k per client

$$\forall i, k \sum_j \sum_{\{s | \text{DAY}(s)=j, \text{TYPE}(s)=k\}} w_{is} \leq 1$$

4. $I \times J$ (32) constraints: if client is in town on day j, can be assigned at most 1 ticket; if client is not in town on day j, cannot be assigned any tickets

$$\forall i, j \sum_k \sum_{\{s | \text{DAY}(s)=j, \text{TYPE}(s)=k\}} w_{is} \leq \text{INTOWN}_{ij}$$

where

$$\text{INTOWN}_{ij} = \sum_{\{t | \text{IN}(t) \leq j \leq \text{OUT}(t)\}} v_{it}$$

ILP Solution: Completion

Constant	Description
C_{jg}	denote the supply of good g on day j
D_{jg}	denote the demand of good g on day j
P_{jgn}	price of buying the nth copy of good g on day j
Π_{jgn}	price of selling the nth copy of good g on day j

Variable	Description
q_{jgn}	is the quantity of good g bought on day $j \geq n$?
σ_{jgn}	is the quantity of good g sold on day $j \geq n$?

Objective Function

$$\max \left(\sum_{i,t} U_{it} v_{it} + \sum_{i,s} U_{is} w_{is} + \sum_{j,k,n} \Pi_{jkn} \sigma_{jkn} - \sum_{j,k,n} P_{jkn} q_{jkn} \right)$$

Abbreviations

$$\text{BUY}_{jg} = \sum_{n=1}^{C_{jg}} q_{jgn} \qquad \text{SELL}_{jg} = \sum_{n=1}^{D_{jg}} \sigma_{jgn}$$

ILP Solution: Completion (Continued)

Constraints

2'. cannot assign more goods than the number owned plus what is bought

- $J \times K$ (12) constraints: can assign at most X_{jl} entertainment tickets of type k on day j plus the number bought minus the number sold

$$\forall j, k, \sum_i \sum_{\{s | \text{DAY}(s)=j, \text{TYPE}(s)=k\}} w_{is} \leq X_{jk} + \text{BUY}_{jk} - \text{SELL}_{jk}$$

- $J \times (L + M)$ (16) constraints: analogously, for hotels and flights

5. constrained by market supply and demand

- $J \times (K + L + M)$ (28) constraints: cannot buy more goods than market supply

$$\forall j, g, \text{BUY}_{jg} \leq C_{jg}$$

- $J \times (K + L + M)$ (28) constraints: cannot sell more goods than market demand

$$\forall j, g, \text{SELL}_{jg} \leq D_{jg}$$

ILP Solution: Completion (Revisited)

Constant	Description
C_{jg}	denote the supply of good g on day j
P_{jgn}	price of buying the nth copy of good g on day j

Variable	Description
q_{jgn}	is the quantity of good g bought on day $j \geq n$?

Objective Function

$$\max \left(\sum_{i,t} U_{it} v_{it} + \sum_{i,s} U_{is} w_{is} - \sum_{j,k,n} P_{jkn} q_{jkn} \right)$$

Abbreviation

$$\text{BUY}_{jg} = \sum_{n=1}^{C_{jg}} q_{jgn}$$

ILP Solution: Completion (Continued)

Constraints

2'. cannot assign more goods than the number bought

- $J \times K$ (12) constraints: can assign at most the number of entertainment tickets bought of each type k on each day j

$$\forall j, k, \sum_{i} \sum_{\{s | \text{DAY}(s)=j, \text{TYPE}(s)=k\}} w_{is} \leq \text{BUY}_{jk}$$

- $J \times (L + M)$ (16) constraints: analogously, for hotels and flights

5. constrained by market supply and demand

- $J \times (K + L + M)$ (28) constraints: cannot buy more goods than market supply

$$\forall j, g, \text{BUY}_{jg} \leq C_{jg}$$

Experimental Setup

ALLOCATION

Raw Data

- 16 games of the TAC finals
- 128 agents: 8 clients per agent

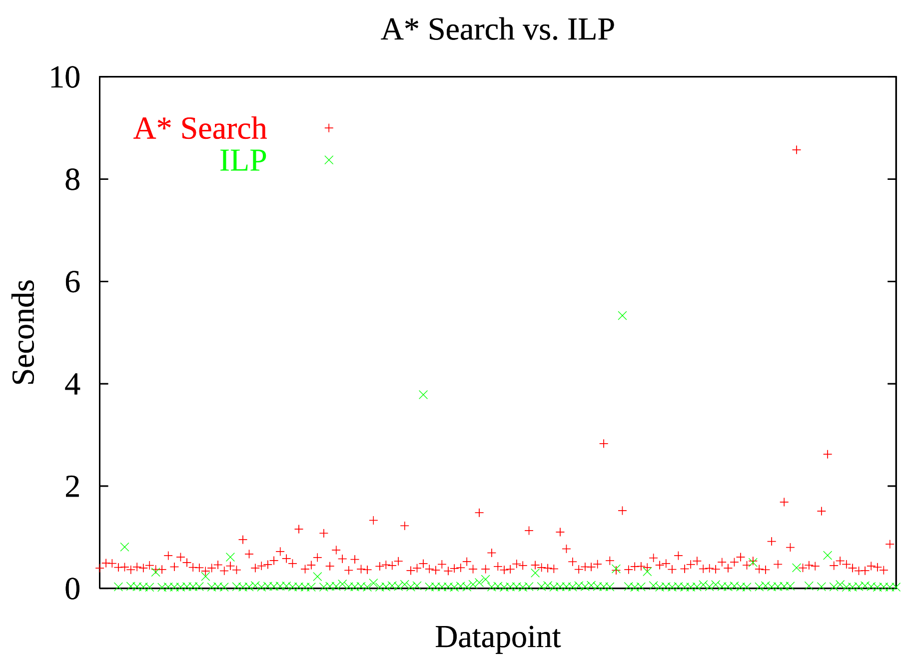
Compiled Data

- 128 agents: 8 clients per agent
- 64 agents: 16 clients per agent
- 32 agents: 32 clients per agent
- 16 agents: 64 clients per agent

A^* Search vs. ILP: Raw Data

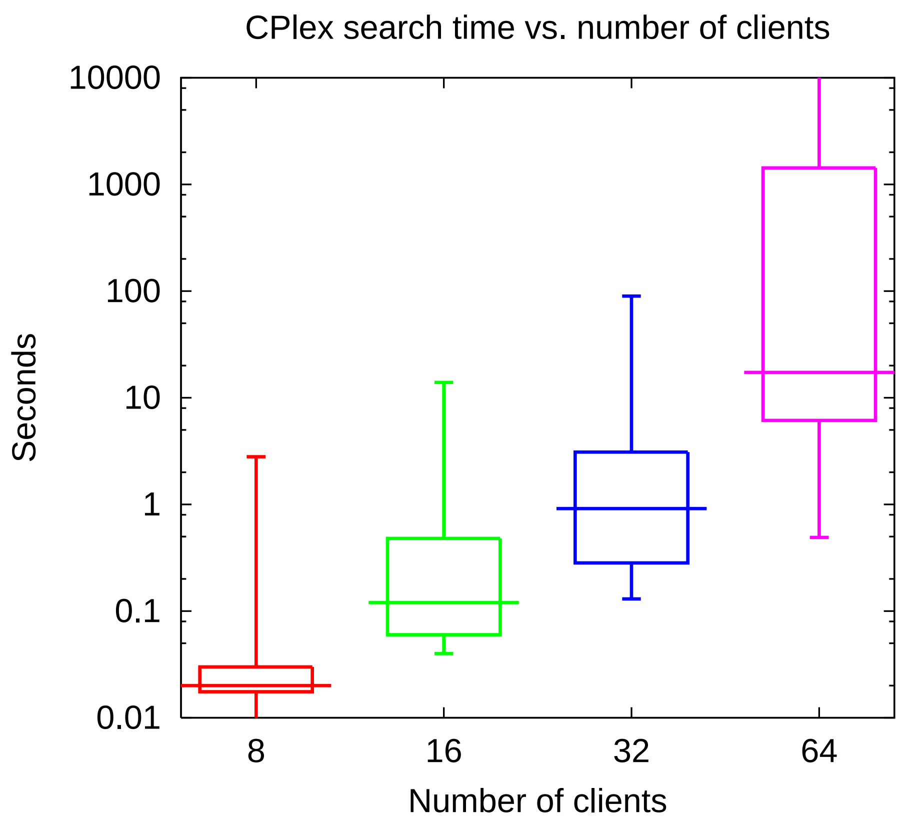
- A^*: median run time 0.59 sec on a 600 MHz PC; worst run time 8.6 sec

- ILP: median run time 0.02 sec using CPLEX 6.5.3 on a 400 MHz SPARCstation with 2Gb of RAM; worst run time 419.4 sec



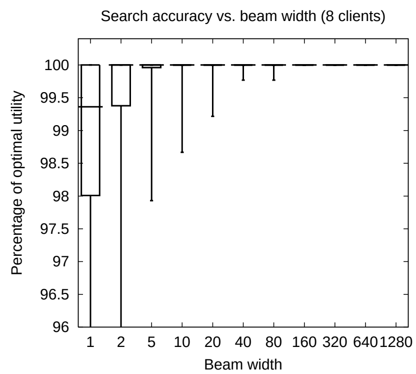
ILP: Compiled Data

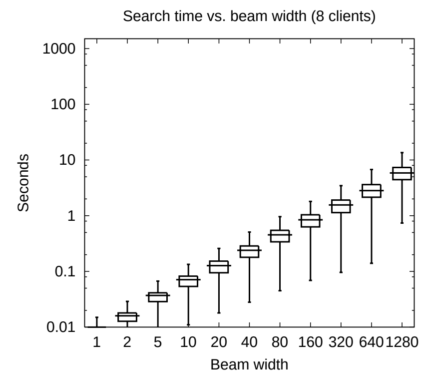
- ILP solved all but one of the 64 client cases
- ILP is fast on average, but its variance is high

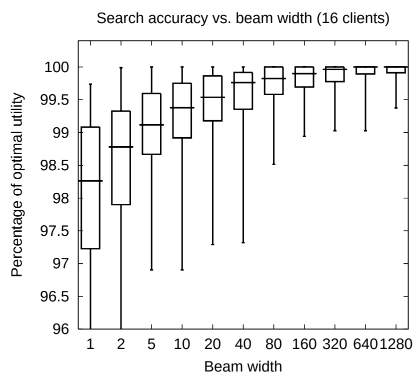


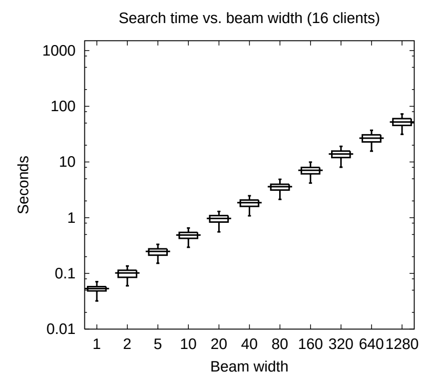
Beam Search: Compiled Data

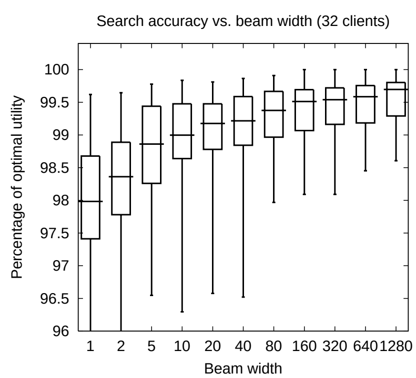
- Beam width of 1 (best-first search) yielded median accuracy of 99.4% for 8 clients with run times less than 0.01 sec
- Beam width of 1 (best-first search) yielded median accuracy of 97.9% for 64 clients in roughly 1 sec
- Beam width of 1280 yielded median accuracy of 99.4% for 64 clients, but run time was near 22 min
- Run times have low variance, and accuracy is always above 96% for all but the smallest of beam widths

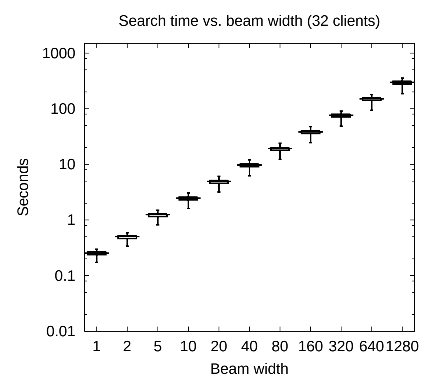

Search accuracy vs. beam width (8 clients)
Percentage of optimal utility
100
99.5
99
98.5
98
97.5
97
96.5
96
1 2 5 10 20 40 80 160 320 640 1280
Beam width

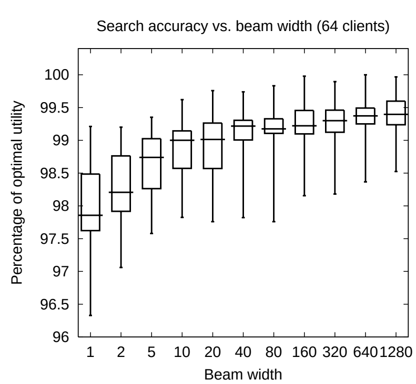

Search time vs. beam width (8 clients)
Seconds
1000
100
10
1
0.1
0.01
1 2 5 10 20 40 80 160 320 640 1280
Beam width

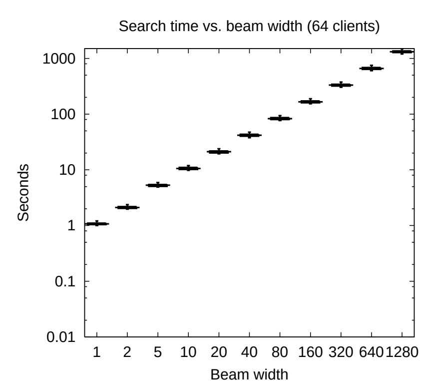

Search accuracy vs. beam width (16 clients)
Percentage of optimal utility
100
99.5
99
98.5
98
97.5
97
96.5
96
1 2 5 10 20 40 80 160 320 640 1280
Beam width


Search time vs. beam width (16 clients)
Seconds
1000
100
10
1
0.1
0.01
1 2 5 10 20 40 80 160 320 640 1280
Beam width


Search accuracy vs. beam width (32 clients)
Percentage of optimal utility
100
99.5
99
98.5
98
97.5
97
96.5
96
1 2 5 10 20 40 80 160 320 640 1280
Beam width


Search time vs. beam width (32 clients)
Seconds
1000
100
10
1
0.1
0.01
1 2 5 10 20 40 80 160 320 640 1280
Beam width


Search accuracy vs. beam width (64 clients)
Percentage of optimal utility
100
99.5
99
98.5
98
97.5
97
96.5
96
1 2 5 10 20 40 80 160 320 640 1280
Beam width


Search time vs. beam width (64 clients)
Seconds
1000
100
10
1
0.1
0.01
1 2 5 10 20 40 80 160 320 640 1280
Beam width

Summary

Theoretical Observations

- BD in double auctions can be reduced to BD in single-sided auctions
- BD in simultaneous auctions are isomorphic to WD in combinatorial auctions

Empirical Observations

- for TAC's dimensions, BD problems are tractable
- A^* scales poorly
  ILP fares better on average, but its variance is high
- heuristic approximation scales well: it produced near-optimal solutions with predictable time and space requirements