MATH 1040 Capstone:
Conway's Game of Life on the Einstein Hat Tiling

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Abstract

The goal of this project was to see if a glider could be found for Conway's Game of Life on the Einstein Hat Tiling. Conway's Game of Life is a cellular automaton devised by the British mathematician John Horton Conway in 1970. One plays the Game of Life by creating an initial configuration and observing how it evolves under the given rules. It is Turing complete, meaning it can be used to create a Turing machine/computer. My code for this project can be found in the GitHub repository: https://github.com/henrypdonahue/EinstiensHat-GoL

1. Conway's Game of Life

Conway's Game of Life is a cellular automaton that is played on a 2D square grid. Each "cell" (a square regularly) on the grid can be either populated or unpopulated (alive or dead). Once begun, time ticks in single steps, with each time step the following rules are enforced for each cell:

1. (Underpopulation) Any populated/alive cell with fewer than two live neighbours dies
2. (Overpopulation) Any populated/alive cell with more than three live neighbours dies
3. (No change) Any live cell with two or three populated/alive neighbours remains unchanged/lives to the next generation
4. (Populate) Any unpopulated or dead cell with exactly three populated/alive neighbours comes to life

The novelty of Conway's Game of Life is that this set of rules is Turing Complete. A Turing Complete system is a system in which a program can be written that will find an answer to a given input. In the Game of Life it is possible to create Digital Logic Gates. It is therefore possible to create an entire chip or even computer. It is possible to create the game of life within the game of life. Referenced is a paper about the creation of a binary adder in Conway's Game of life. [1]

There are three objects that one is able to create in Conway's Game of life that makes this possible: The Glider, the Glider Gun and the Eater.

1. (Glider) Moves in one direction forever, osculates shape as it moves
2. (Glider Gun) Create gliders moving in the same direction periodically
3. (Eater) Any live cell with two or three populated/alive neighbours remains unchanged/lives to the next generation

If these objects exist in an alternative Game of Life tiling, the tiling would also be Turing complete.

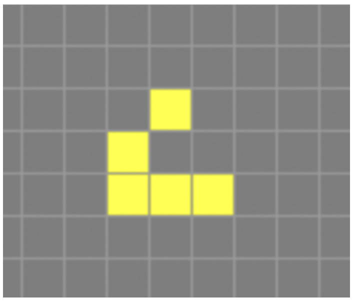

Figure 1: Glider

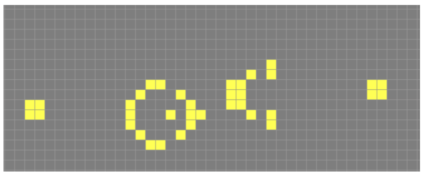

Figure 2: Glider Gun

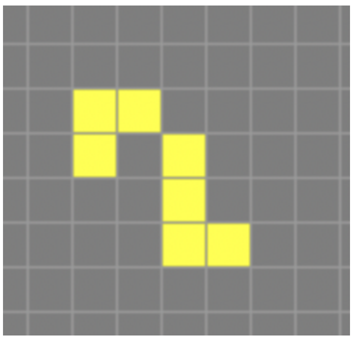

Figure 3: Eater