MATH 1040 Capstone: Conway's Game of Life on the Einstein Hat Tiling

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Abstract

The goal of this project was to see if a glider could be found for Conway's Game of Life on the Einstein Hat Tiling. Conway's Game of Life is a cellular automaton devised by the British mathematician John Horton Conway in 1970. One plays the Game of Life by creating an initial configuration and observing how it evolves under the given rules. It is Turing complete, meaning it can be used to create a Turing machine/computer. My code for this project can be found in the GitHub repository: https://github.com/henrypdonahue/EinstiensHat-GoL

1. Conway's Game of Life

Conway's Game of Life is a cellular automaton that is played on a 2D square grid. Each "cell" (a square regularly) on the grid can be either populated or unpopulated (alive or dead). Once begun, time ticks in single steps, with each time step the following rules are enforced for each cell:

- 1. (Underpopulation) Any populated/alive cell with fewer than two live neighbours dies
- 2. (Overpopulation) Any populated/alive cell with more than three live neighbours dies
- 3. (No change) Any live cell with two or three populated/alive neighbours remains unchanged/lives to the next generation
- 4. (Populate) Any unpopulated or dead cell with exactly three populated/alive neighbours comes to life

The novelty of Conway's Game of Life is that this set of rules is Turing Complete. A Turing Complete system is a system in which a program can be written that will find an answer to a given input. In the Game of Life it is possible to create Digital Logic Gates. It is therefore possible to create an entire chip or even computer. It is possible to create the game of life within the game of life. Referenced is a paper about the creation of a binary adder in Conway's Game of life. [1]

There are three objects that one is able to create in Conway's Game of life that makes this possible: The Glider, the Glider Gun and the Eater.

- 1. (Glider) Moves in one direction forever, osculates shape as it moves
- 2. (Glider Gun) Create gliders moving in the same direction periodically
- 3. (Eater) Any live cell with two or three populated/alive neighbours remains unchanged/lives to the next generation

If these objects exist in an alternative Game of Life tiling, the tiling would also be Turing complete.

Figure 1: Glider



Figure 2: Glider Gun



Figure 3: Eater

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Figure 4: Hat tiling I generated



Figure 5: Hat tiling showing flipped tiles

2. Generating the Einstein Hat Tiling

The first part of my project was generating the Einstein Hat Tiling, so I could then implement the Game of Life on the tiling. This tiling was discovered in 2022 by hobbyist David Smith. Smith recruited help from mathematicians Craig S. Kaplan, Joseph Samuel Myers, and Chaim Goodman-Strauss, and in 2023 the group released their paper proving that the hat, when considered with its mirror image, forms an aperiodic tiling of the plane. Their proof awaits peer-review and formal publication. The existence of an Einstein tiling such that the tiling does not involve mirror images is still an unsolved problem. I used the public code cited in this paper and the following source as a jumping off point for my tiling. The most difficult part of generating this tiling was to reconcile how the flipped tile fits in. The file on my GitHub repository called "EinsteinTiling.js". [2] [3]

3. Results

After successfully implementing the Game of Life on the Einstein tiling, I found that the aperiodic nature of the tiling prevents the creation of a glider, and the rest of the needed objects for the tiling to be Turing Complete. This is where my project has almost endless continuation. I am working on implementing more and more complex algorithms to try and find these objects. I have tried using an SMT solver, SAT solvers and traditional algorithms I developed in attempts to find these objects. One note is that the idea of a glider would have to be



Figure 6: Initial populated cells



Figure 7: After one time step (Paremeters: 4, 2, 1)

redefined. As there is no symmetry, the idea of a shape moving continuously while maintaining it's shape would need to be reinterpreted. While deliberating about this, I realized that tuning the "hyper-parameters" or values to overpopulate, populate, etc, was also particular interesting and useful. I then implemented the ability for the front end user to change the hyperparameters to experiment with the tiling. I think that changing the exact rules slightly is needed for the tiling to be Turing Complete.

4. References

References

- Nicholas Carlini. Digital logic gates on conway's game of life part 1, April 2020. https://nicholas.carlini.com/writing/2020/ digital-logic-game-of-life.html.
- [2] Arthur O'Dwyer. Escheresque parquet deformations of an aperiodic monotile, Mar 2023. https://quuxplusone.github.io/blog/2023/03/30/hat-parquet/.
- [3] David Smith, Joseph Samuel Myers, Craig S. Kaplan, and Chaim Goodman-Strauss. An aperiodic monotile, April 2023. https://cs.uwaterloo.ca/~csk/hat/.