Dustin Wu
Brown University Computer Science Capstone
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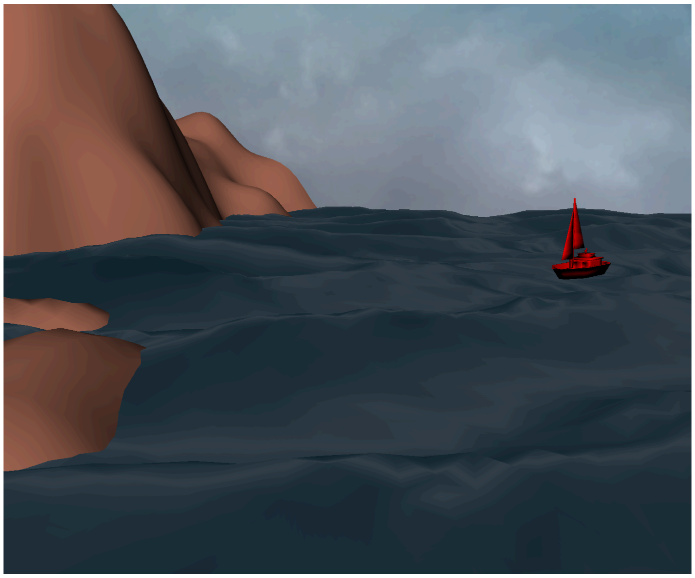
Water Surface Wavelets: CSCI 2240 Capstone Abstract

In Collaboration With: Max Guo, Mohammed Khan, and Kelvin Jiang

Computer graphics researchers have been investigating methods to simulate the behavior of ocean water for decades. With obvious applications in CGI and video games, this problem is as fascinating as it is challenging. The overarching goal is to solve for the change in water height along a simulated surface using established equations that describe the dynamics of ocean water. Two popular approaches to this end are (1) using numerical approximations, and (2) using Fourier transforms to solve for a closed-form solution. However, both approaches have flaws: the former becomes computationally expensive as the level of detail (namely, the frequency of the waves) increases, and the latter does not readily support environmental reactions, such as with wind or buoyant objects.



A third approach pioneered in 2018 by Jescke et al. uses wavelets, wave-like motions that oscillate between a positive/negative polarity and zero polarity, to embed more information about wave motion in less frequency domain space, thereby creating a solution with the fidelity of closed-form approaches, even with large simulation domains, while also supporting environmental interactions. These wavelets are realized as discretized wave amplitudes in a four-dimensional grid, where the first two dimensions are in X and Y, the third is in theta, and the fourth is in wavenumber, which encodes the frequency of the wave. In other words, this approach involves discretizing wave amplitudes as a function along space, direction, and frequency, all at once.

Implementing this approach requires four components: a wavelet-based discretization of wave amplitude, functions for updating the motion of waves every timestep, sampling water heights using wave profiles, and rendering a mesh using the water heights.

Our implementation of discretization involved writing functions for converting a set of four-dimensional indices into a one-dimensional index for access into a one-dimensional buffer, and vice-versa. In addition, to support motion updates, we wrote methods for interpolating continuous values along the dimensions of our grid, using both bilinear interpolation and the Catmull-Rom spline, a form of higher-order interpolation using control points.

To update wave motion every timestep, we implemented functions simulating advection, the movement of ocean water via wind, and diffusion, the tendency for waves to stretch out as they advance, using equations described by Jescke et al. Both functions involve sampling at continuous points on the discretized amplitude grid, which is why the interpolation described previously was vital to implement.

To convert these amplitudes into water heights, we accumulate values in a sampling function that essentially weights directional contribution by amplitude. However, there is one more required component, as amplitude alone does not describe wave motion: the profile buffer. This is essentially an integral over what is known as the spectrum function over wavelet frequency that is computed every timestep, and effectively computes the "profile", or cross-section, of the wave. The spectrum function describes how wind stress varies over time across the water surface, and is typically sinusoidal.

Lastly, rendering the sampled water heights involves offsetting discrete points on a plane mesh by the sampled height at each vertex. In addition, we used the Gerstner wave equations, which take sampled amplitudes and wave directions as input and supply horizontal wave offsets. To create an appealing visual scene, we implemented a skybox and reflection and refraction sampling of this skybox on the water surface.

Finally, to test the simulation's visual fidelity and ability to support environmental interactions, we implemented a simulation of wind that periodically changes direction and intensity, procedurally-generated terrain that interacts with the water at its boundaries, and two-way buoyancy and rigid-body wave interactions on a user-controllable boat mesh.