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1 Introduction

Throughout history, humans have been observed to favor one hand over the other, with the majority of people exhibiting right-handedness. The dominance of one hand in an individual can be explained by cognitive functions — specialization of brain function generally leads to improved performance in critical tasks. However, the historically consistent dominance of right-handedness in the overall human population is less clear. Scientists typically attribute this dominance to a balance between cooperation and competition (Abrams and Panaggio, 2012). Sharing the same handedness as others in a society makes performing cooperative tasks easier, while having different handedness can give one a competitive edge in certain cases. By modeling these relationships, we can explain why society experiences a 90-10% split between right and left handedness. In addition, this model can be applied to situations outside of daily life such as a sport like baseball, where competition is much higher, and we see a significantly higher percentage of left-handed players.

In this paper, we first describe the model built by Daniel M. Abrams and Mark J. Panaggio on the population of right and left-handed people. Then, we add additional details to their original model by classifying bifurcations and plotting fixed point stability. After that, we extend the model by adding a dimension to account for ambidextrous individuals. We compare the original model to our extended model in the context of the general human population and present our conclusions.

2 Background

Daniel M. Abrams and Mark J. Panaggio's article "A model balancing cooperation and competition can explain our right-handed world and the dominance of left-handed athletes" presents a differential equation model for the proportion of left-handed people in a population (and right-handed by converse). Specifically, this equation describes the change in left-handed people over time in terms of the current proportion of left handed people and the probability that a left handed person will be replaced by a right handed person (over a long period of time) and vice versa. Abrams and Panaggio break down this probability function into two parts: cooperation and competition.

The probability that a right-handed person is replaced by a left-handed person due to cooperation increases with the proportion of left-handed people in a population because the more left-handed people there are, the easier it is share tools, participate in community activities, etc. The same probability due to competition decreases as the left-handed population increases because the fewer left-handed people in a population, the greater advantage that a left-handed person has in combat or other competitive activities. Abrams and Panaggio model both of these parts with sigmoid functions. The corresponding is true for the probability that a left-handed person would be replaced by a right-handed one. These relationships are described in detail in the following section.

Abrams and Panaggio found that the importance of cooperation, $0 \leq c \leq 1$, determines the stability of the model. For c below a "critical threshold," there is one stable equilibrium point at $l = 0.5$, indicating a population with roughly equal amounts of left and right-handed people. For c above such a threshold, there are 2 stable fixed points, which appear due to a supercritical pitchfork bifurcation or a subcritical pitchfork bifurcation depending on the cooperation and competition sigmoid functions. These stable points are likely to be in the $l = 0.7-1$ range, indicating strong "population lateralization" (Abrams and Panaggio, 2012).

The authors then apply this model to professional sports, with a focus on baseball. The relationship between cooperation and competition in a professional sport will not affect the underlying model for the general population, though, so they expand on their model. They define the professional proportion of left handed people, l_{pro}, to depend on the background proportion, l_{bg}; the proportion if the entire population operated under the cooperation parameter of the sport, $l*$; the number of interested individuals, n; and the number of players able to play professionally, N. They applied this model to 12 different sports, each with varying parameters. For example, c was assumed to be almost 0 for baseball and almost 1 for golf. The model performed fairly well when compared to actual data on the population of left-handed professional athletes.

3 Original Model

Abrams and Panaggio model the percent of left-handed people in the population, l, using a 1-D differential equation. We know that the change in left-handed people equals the number of right handed people that are replaced with left handed offspring minus the number of left handed people that are replaced with right handed offspring. Let P_{RL} be the probability that a right handed individual is replaced with left handed offspring, and P_{LR} be the inverse. Then the percent change in left-handed people can be modeled by

$$\frac{\partial l}{\partial t} = (1-l) * P_{RL}(l) - l * P_{LR}(l) \text{ (Abrams and Panaggio, 2012)}$$

One reasonable assumption that the original authors make is that P_{RL} and P_{LR} are symmetrical. That is, $P_{RL}(l) = P_{LR}(1-l)$, since we don't expect any fundamental differences between a right hand dominated population and a left hand dominated one. This simplifies the equation to

$$\frac{\partial l}{\partial t} = (1-l) * P_{RL}(l) - l * P_{RL}(1-l) \text{ (Abrams and Panaggio, 2012)}$$

With this assumption we can immediately see that at $l = 0$, $\frac{\partial l}{\partial t} = 0.5 * P_{RL}(0.5) - 0.5 * P_{RL}(0.5) = 0$, meaning 0 is always a fixed point. To find other fixed points, we must break down P_{RL} further. As described above, in a cooperative setting, we know that people are likely to switch handedness if their handedness is different than the majority. Thus, P_{RL}^{coop}

is going to be near 1 when l is near 1, and near 0 when l is near 0. Abrams and Panaggio model this with a sigmoid function

$$P_{RL}^{coop} = (1 + e^{\frac{k_1(1-2l)}{l(1-l)}})^{-1}$$

Similarly, they model the competitive setting with

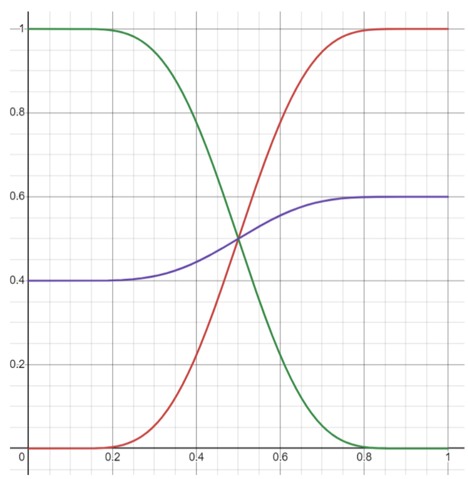
$$P_{RL}^{comp} = (1 + e^{\frac{-k_2(1-2l)}{l(1-l)}})^{-1}$$

(Abrams and Panaggio, 2012) Both k_1 and k_2 are constants larger than $\frac{\sqrt{3}}{2}$. In the human population, there is a mix of cooperative and competitive interactions, so the authors define $0 \leq c \leq 1$ as the "relative importance of cooperation in interactions." This gives us

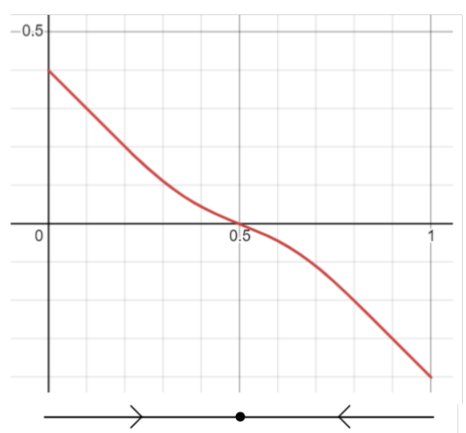
$$P_{RL} = c * P_{RL}^{coop} + (1-c) * P_{RL}^{comp}$$

4 Additional Bifurcation Analysis

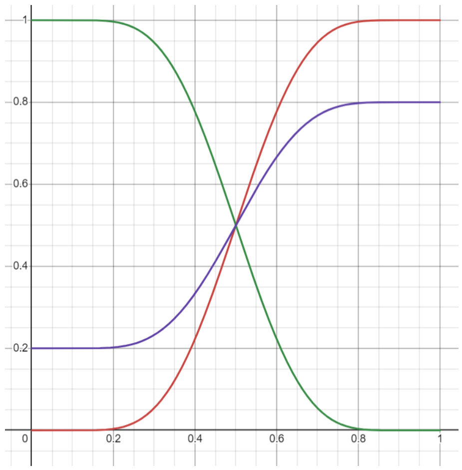
In this section, we go in to more detail than the original model's bifurcation analysis. We plot P_{RL} and its components, $\frac{\partial l}{\partial t}$ vs. l, and the phase portraits for each relevant case. We then define the bifurcations that occur within specific ranges of c. There are two cases of importance, the first being where $k_1 = k_2$. We will first set them both equal to 1.5.



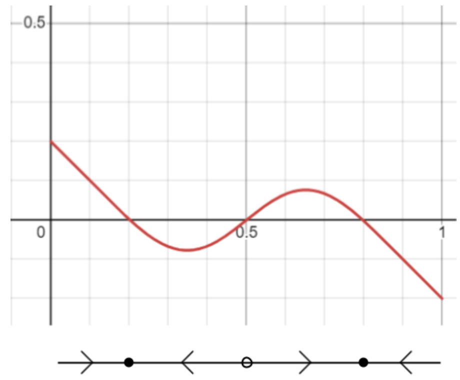
(a) Graph of P_{RL}^{coop}(red), P_{RL}^{comp}(green), P_{RL}(purple) with c = 0.6. l on x-axis, P_{RL} on y-axis



(b) Graph of $\frac{\partial l}{\partial t}$ with c = 0.6 and corresponding phase portrait. l on x axis, $\frac{\partial l}{\partial t}$ on y-axis



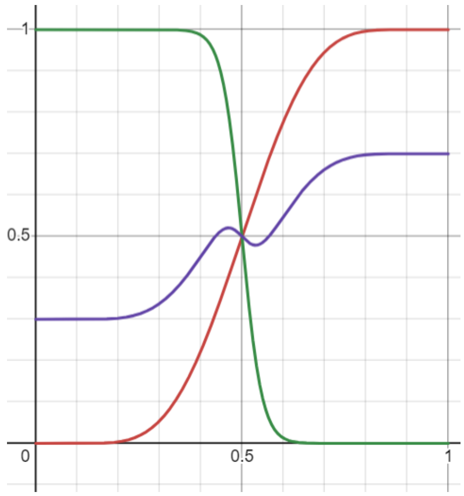
(a) Graph of P_{RL}^{coop}(red), P_{RL}^{comp}(green), P_{RL}(purple) with c = 0.8. l on x-axis, P_{RL} on y-axis



(b) Graph of $\frac{\partial l}{\partial t}$ with c = 0.8 and corresponding phase portrait. l on x axis, $\frac{\partial l}{\partial t}$ on y-axis

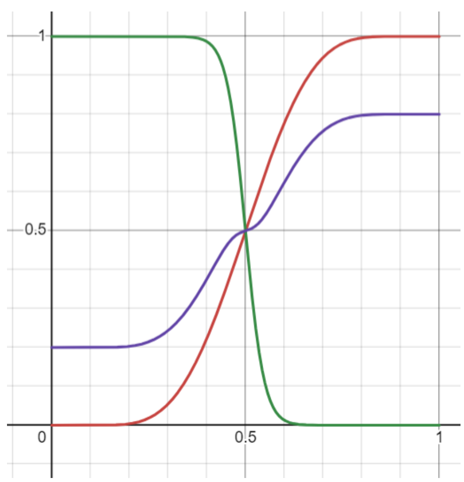
In the graph of $\frac{\partial l}{\partial t}$, we see 1 fixed point when c = 0.6 and three when c = 0.8, an indication of a pitchfork bifurcation. Specifically in the c = 0.6 case, $\frac{\partial l}{\partial t} > 0$ when $l < 0.5$ and $\frac{\partial l}{\partial t} < 0$ when $l > 0.5$ meaning $l = 0.5$ is an attractor. Meanwhile in the c = 0.8 case, $\frac{\partial l}{\partial t}$ is negative for a small range of values below $l = 0.5$ and positive for a range above it before crossing $l = 0$ and mirroring the previous case. This means that 0.5 is now a repeller, and we have 2 new fixed points that are attractors. Thus, a supercritical pitchfork bifurcation occurs between $c = 0.6$ and $c = 0.8$.

We now consider the case where the k values are not identical. For this example we look at $k_1 = 1.5$, $k_2 = 5.2$.

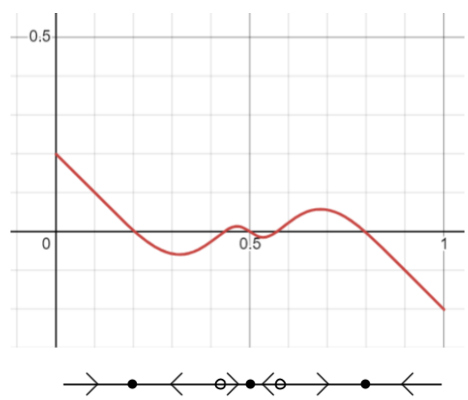


(a) Graph of P_{RL}^{coop}(red), P_{RL}^{comp}(green), P_{RL}(purple) with c = 0.7. l on x-axis, P_{RL} on y-axis

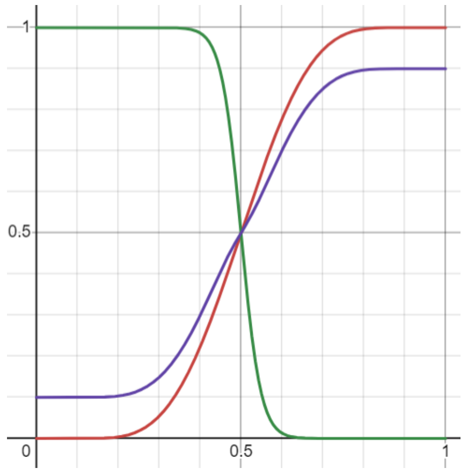
(b) Graph of $\frac{\partial l}{\partial t}$ with c = 0.7 and corresponding phase portrait. l on x axis, $\frac{\partial l}{\partial t}$ on y-axis



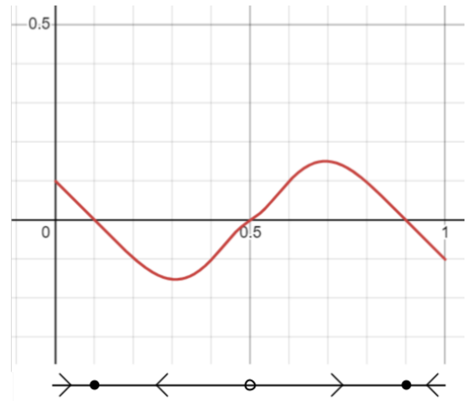
(a) Graph of P_{RL}^{coop}(red), P_{RL}^{comp}(green), P_{RL}(purple) with c = 0.8. l on x-axis, P_{RL} on y-axis



(b) Graph of $\frac{\partial l}{\partial t}$ with c = 0.8 and corresponding phase portrait. l on x axis, $\frac{\partial l}{\partial t}$ on y-axis



(a) Graph of P_{RL}^{coop}(red), P_{RL}^{comp}(green), P_{RL}(purple) with c = 0.9. l on x-axis, P_{RL} on y-axis



(b) Graph of $\frac{\partial l}{\partial t}$ with c = 0.9 and corresponding phase portrait. l on x axis, $\frac{\partial l}{\partial t}$ on y-axis

In the graph of $\frac{\partial l}{\partial t}$, we see 1 fixed point when c = 0.7, five when c = 0.8, and three when c = 0.9. In the c = 0.7 case, we see $l = 0.5$ is an attractor. Meanwhile in the c = 0.8 case, $l = 0.5$ is still stable, and so are the two outermost fixed points. The two fixed points adjacent to $l = 0.5$ are unstable. The amount of fixed points changed from 1 to 5 without the stability of $l = 0.5$ remaining the same, meaning two saddle node bifurcations occur between $c = 0.7$ and $c = 0.8$. Finally, at $c = 0.9$, we can see that $l = 0.5$ is now an unstable fixed point, with two adjacent fixed points that are still stable. These stable outer fixed points remain for both $c = 0.8$ and $c = 0.9$. Since $l = 0.5$ changes from stable to unstable and its adjacent unstable fixed points are destroyed, a subcritical pitchfork bifurcation occurs between $c = 0.8$ and $c = 0.9$.

5 Evaluation

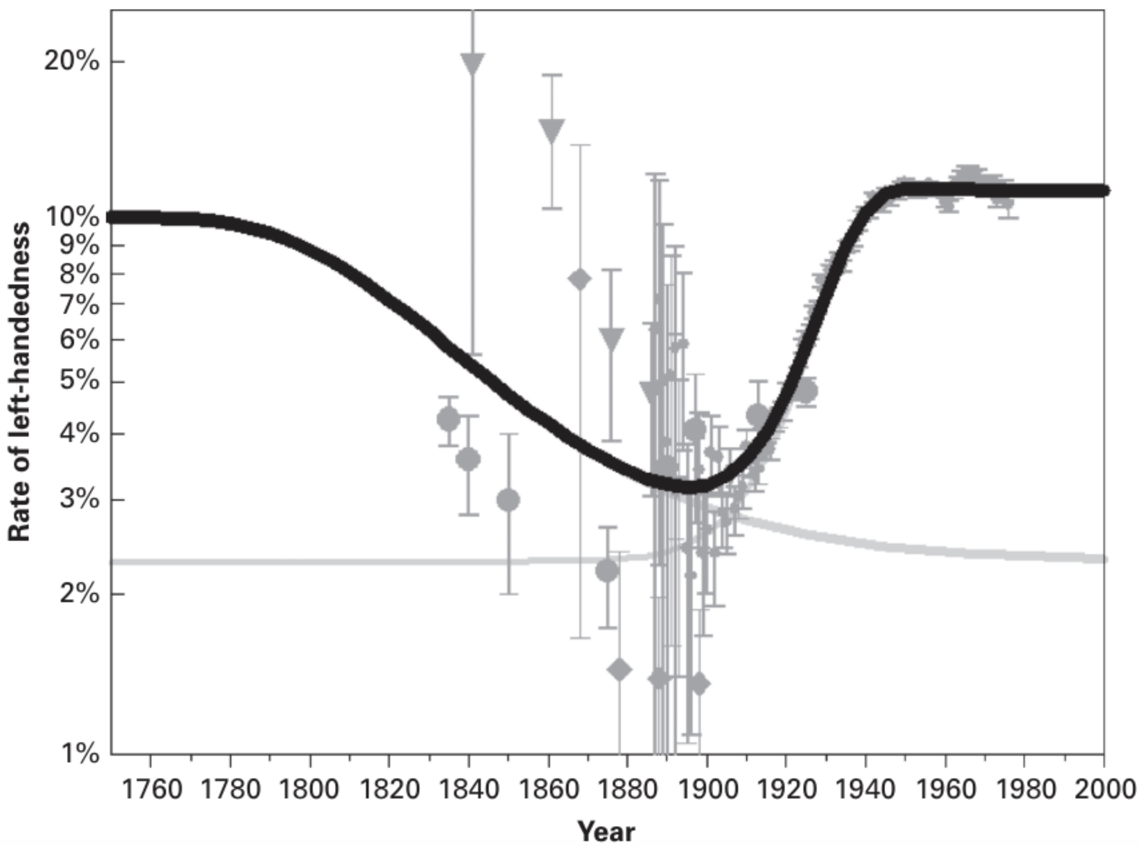


Figure 6: Rate of left-handedness over time (McManus, n.d.)

While data on the historic rate of left-handedness is difficult to obtain, the above figure is an estimate over the last few hundred years given available data. If this data is accurate, we can see that the percentage of left handed people seems to eventually converge to around 11 percent. After 1900, this behavior seems to be consistent with the case where $c = 0.8, k_1 = k_2 = 1.5$ since in both cases there is a stable point in the range of $10-20\%$, and a presumably unstable point at 50%. However, before 1900, we see the amount of left-handed people decreasing to a trough at around 3%. Our model suggests that there was a shift of parameters, specifically an increase in c to a value close to 1, shortly before 1900 before returning to the previous parameters. However, since our model assumes a relatively long period of time between shifts of handedness, it likely does not do a good job of modelling this quick dip in left-handedness. That being said, McManus recognizes that due to increasing industry at the time and tools being built primarily for right handed people, left-handed people may have lied about their innate left-handedness or tried to cover it up, and the population of left-handed people perhaps did not drop off.

6 Application

First we will apply this model to daily life. We currently observe about a 90-10 (McManus, n.d.) split between right handed, left handed, and ambidextrous people respectively. From our model we see that this is indicative of a c value close to 1 for 0.1 to be a fixed point for l. This makes sense, since in modern society cooperation between individuals or safety measures are much more prevalent than situations where competition involving physicality arises.

We can also use this system to model handedness in baseball. In baseball, competition dominates over cooperation, meaning c = 0, and the ideal breakdown of players is 50/50 between right and left handed. However, since baseball players are sampled from the right hand dominated population, we still expect more right handed players. Thus, we can model the skill of players with a normal distribution, with left handed players having a higher mean due to their advantage, before picking the top x players. In the MLB, this means picking the top 1200 players off a pool of 2,629,000 people (Abrams and Panaggio, 2012). Then the predicted number of baseball players is

$$l_{pro} = \frac{1}{2}\frac{N l_{bg} erfc(\hat{s_c} - \Delta\hat{s})}{r}$$

(Abrams and Panaggio, 2012)

where N is the total player pool, r is the number selected into the MLB, l_{bg} is the percent of left handed people in the population, 0.1, $\Delta\hat{s}$ is found through data to be 0.3003 and represents the advantage left handed players have, and $\hat{s_c}$ satisfies the equation

$$r = \frac{1}{2}(N l_{bg} erfc(\hat{s_c} - \Delta\hat{s})) + \frac{1}{2}(N(1 - l_{bg}) erfc(\hat{s_c}))$$

(Abrams and Panaggio, 2012)

Plugging in r = 1200, we get $\hat{s_c} = 2.401$. Solving for l_{pro} we get 0.325 which is very close to the observed baseball player level of 0.329 (Abrams and Panaggio, 2012).

We can also flip the argument for a sport where conformity is good, such as football. Quarterbacks on the team having the same handedness makes it easier to coordinate and practice, and so here the right handed individuals would have the easier time.

7 Ambidexterity Model

We can extend our model to account for ambidexterity in individuals. We note that ambidexterity succeeds in both a competitive and cooperation environment since the individual can simply switch to the handedness most appropriate for the situation. So, we don't expect ambidexterity's differential equation to depend on c. However, specialization in one hand improves cognitive function, and so we expect ambidextrous individuals to suffer from this. We let d represent this cognitive detriment, where d is between 0 and 1. This yields the following system, in which the subscript A represents a shift to or from ambidexterity.

$$\frac{\partial l}{\partial t} = r * P_{RL}(l,r,a) - l * P_{LR}(l,r,a) + a * P_{AL}(l,r,a) - l * P_{LA}(l,r,a)$$
$$\frac{\partial r}{\partial t} = l * P_{LR}(l,r,a) - r * P_{RL}(l,r,a) + a * P_{AR}(l,r,a) - r * P_{RA}(l,r,a)$$
$$\frac{\partial a}{\partial t} = l * P_{LA}(l,r,a) - a * P_{AL}(l,r,a) - a * P_{AR}(l,r,a) + r * P_{RA}(l,r,a)$$

The total population, $l + r + a$, is conserved, so we can simplify this model into

$$\frac{\partial l}{\partial t} = r * P_{RL}(l,r,a) - l * P_{LR}(l,r,a) + a * P_{AL}(l,r,a) - l * P_{LA}(l,r,a)$$
$$\frac{\partial r}{\partial t} = l * P_{LR}(l,r,a) - r * P_{RL}(l,r,a) + a * P_{AR}(l,r,a) - r * P_{RA}(l,r,a)$$

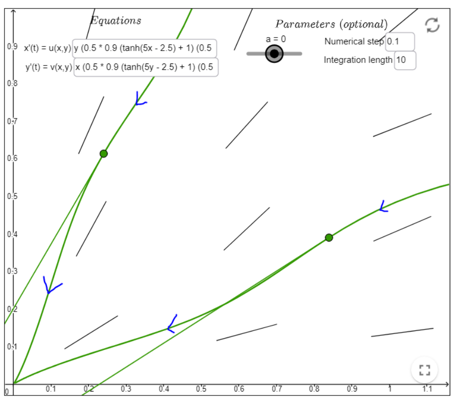
where $\frac{\partial a}{\partial t} = -\frac{\partial l}{\partial t} - \frac{\partial r}{\partial t}$. where l, r, and a are the percent of the population that are left handed, right handed, and ambidextrous respectively. We now need to define our probability transitions.

$$P_{RL}^{coop}(l,r,a) = \frac{tanh(5l - 2.5) + 1}{2} * (0.5 * d + 0.5)$$
$$P_{LR}^{coop}(l,r,a) = \frac{tanh(5r - 2.5) + 1}{2} * (0.5 * d + 0.5))$$
$$P_{LA}(l,r,a) = P_{RA}(l,r,a) = (1 - d)/2$$
$$P_{AL}(l,r,a) = P_{AR}(l,r,a) = d/2$$
$$P_{RL}(l,r,a) = P_{RL}^{coop}(l,r,a) * c + P_{RL}^{coop}(1 - l,r,a) * (1 - c)$$
$$P_{LR}(l,r,a) = P_{LR}^{coop}(l,r,a) * c + P_{LR}^{coop}(l,1 - r,a) * (1 - c)$$

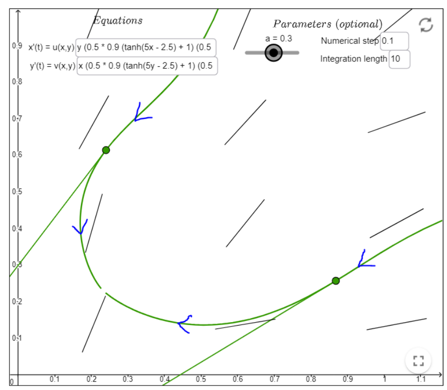
Here, we use hyperbolic tangent as a simpler sigmoid function. Furthermore, the probability that an ambidextrous person would switch to right or left handed is our cognitive detriment, d, and $1 - d$ for the other direction. We also recognize that $P_{RL}^{comp}(l,r,a) = P_{RL}^{coop}(1 - l,r,a)$, and is similar for the left to right transition.

$$\frac{\partial l}{\partial t} = r * (\frac{tanh(5l-2.5)+1}{2} * c * (0.5*d+0.5) + \frac{tanh(5(1-l)-2.5)+1}{2} * (1-c) * (0.5*d+0.5))$$
$$- l * (\frac{tanh(5r-2.5)+1}{2} * c * (0.5*d+0.5) + \frac{tanh(5(1-r)-2.5)+1}{2} * (1-c) * (0.5*d+0.5))$$
$$+ (1-l-r) * d/2 - l * (1-d)/2$$

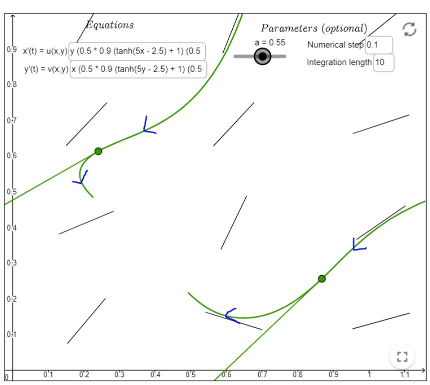
Below we plot the portraits as we fix c at 0.9 and vary d. Each figure includes two lines that are tangent to the starting points chosen and are differentiated by not including a blue arrow on them.



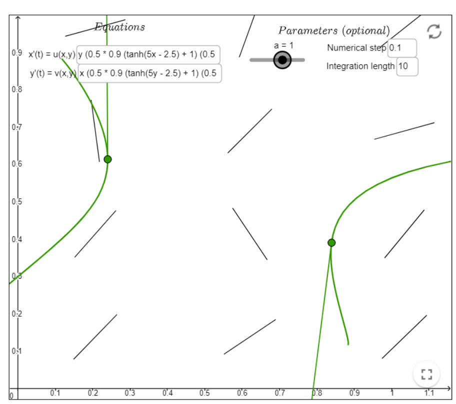
(a) Graph of phase portrait, with l on x axis and y on axis. c = 0.9, d= 0



(b) Graph of phase portrait, with l on x axis and y on axis. c = 0.9, d= 0.3



(a) Graph of phase portrait, with l on x axis and y on axis. c = 0.9, d= 0.55



(b) Graph of phase portrait, with l on x axis and y on axis. c = 0.9, d= 1

We see that when $d = 0$, we initially have a population consisting entirely of ambidextrous people since l and r are both 0 at the only fixed point in the top left figure. This makes sense because in this case there is no cognitive downside to being ambidextrous, and it is thus strictly better than both being left handed and right handed. As d increases, the singular fixed point turns into two fixed points that are mirror images of each other. That is, every two fixed points are in the form (m, n, 1-m-n) or (n,m,1-m-n) since left and right-handed populations are symmetric. Eventually, as d gets to 1, the ambidextrous population gets reduced to 0 in the steady state, and we return back to the fixed points of the original model, (0.9, 0.1) and (0.1, 0.9).

8 Conclusion and Evaluation

In this paper, we investigated Abrams and Panaggio's model for the population of right and left-handed people. We expanded upon their model by plotting specific components of their differential equation model, plotting phase portraits for varying k_1, k_2, and c values, and defining the bifurcations that occur as specific ranges of c. Their model seemed to perform reasonably well when compared with historical data on left-handed individuals, though more research should be done on the left-handed population dip that seems to occur in the given figure. We extended Abrams and Panaggio's model by adding a dimension to account for ambidexterity. While this certainly made the model more complex, we simplified certain aspects by using a hyperbolic tangent function as the sigmoid function. We plotted the phase portraits of our extended model by changing d and found that as d increased from 0 to 1, one fixed point broke into two, which separated further apart. This indicates that the larger value of d, the smaller the ambidextrous population, which is consistent with what one would expect.

One of the problems with our current model is that it lacks data to be validated on, since it is difficult to find reliable data that describes the amount of ambidextrous people. Due to the small percentage of the population that is ambidextrous, accounting for ambidexterity might not create a noticeable change in the dynamical system.

Overall, we hope to see continued research on the handedness of the human population. Our extended model serves as an effective benchmark to begin more in-depth research on ambidexterity, and further research can help us better understand why humans experience ambidexterity and why it is such a uncommon phenomenon. These mathematical models serve not only as explanations of human traits but as platforms from which to ask further questions that ultimately help us better understand ourselves.

References

Abrams, D. M., & Panaggio, M. J. (2012). A model balancing cooperation and competition can explain our right-handed world and the dominance of left-handed athletes. *Journal of the Royal Society, Interface*, *9*(74), 3502–3506.

McManus, I. C. (n.d.). The history and geography of human handedness [http://www.med.mcgill.ca/epidemiology/hanley/bios601/CandHchapter06/HistoryGeographyHumanHandedness.].