Particle Filters

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Outline

- Problem: Track state over time
  - State = position, orientation of robot (condition of patient, position of airplane, status of factory, etc.)
- Challenge: State is not observed directly
- Solution: Tracking using a model
  - Exact tracking (previous lecture) not always possible for large or continuous state spaces
  - Approximate tracking using sampling (this lecture)

Applications

- Activity recognition by mobile devices
  (hidden state is the activity)
- Robot self localization
  (hidden state is robot position, orientation)
- Tracking objects with limited observations
  (tracking pedestrians with/cars with surveillance cameras)

Toy Sampling Example
(no observations)

- Robot is monitoring door to the robotics lab
- D = variable for status of door (True = open)
- Initially we will ignore observations
- Define Markov model for behavior of door:

$$P(d_{t+1} \mid d_t) = 0.8$$
$$P(d_{t+1} \mid \bar{d}_t) = 0.3$$

Exact Solution

Suppose we believe the door was open with prob. 0.7 at time t.

What is the prob. that it will be open at time t+1?

$$P(d_{t+1} \mid d_t) = 0.8$$
$$P(d_{t+1} \mid \overline{d}_t) = 0.3$$

Staying open

Switching from closed to open

$$P(d_{t+1}) = P(d_{t+1} \mid d_t)P(d_t) + P(d_{t+1} \mid \overline{d}_t)P(\overline{d}_t)$$
$$= 0.8 * 0.7 + 0.3 * 0.3 = 0.65$$

Trivial, but in general:

- Suppose states are not binary:

$$P(S_{t+1}) = \sum_{S_t} P(S_{t+1} \mid S_t)P(S_t)$$

- Suppose states are continuous

$$p(S_{t+1}) = \int_{S_t} p(S_{t+1} \mid S_t)p(S_t)dS_t$$

- Issue: For large or continuous states spaces this may be hard to deal with exactly

Sampling Approximates the Integral/Sum

- We can approximate a nasty integral by sampling and counting:

$$p(S_{t+1}) = \int_{S_t} p(S_{t+1} \mid S_t) p(S_t) dS_t$$

- Repeat n times:
  - Draw sample from $p(S_t)$
  - Simulate transition to S_{t+1}
- Count proportion of states for each value of S_{t+1}

Sampling For Our Door Example

$$P(d_{t+1} \mid d_t) = 0.8$$
$$P(d_{t+1} \mid \bar{d}_t) = 0.3$$

- Pick n=1000
  - 700 door open samples
  - 300 door closed samples
- For each sample generate a next state
  - For open samples use prob. 0.8 for next state open
  - For closed samples use prob. 0.3 for next state open
- Count no. of open and closed next states

- Can prove that in limit of large n, our count will equal true probability (0.65)

Door Example With Observations

- D = Door status
- O = Robot's observation of door status
- Observations may not be completely reliable!

$$P(d_{t+1} | d_t) = 0.8$$
$$P(d_{t+1} | \overline{d}_t) = 0.3$$
$$P(o | d) = 0.6$$
$$P(o | \overline{d}) = 0.2$$

Rejection Sampling

$$P(d_{t+1} | d_t) = 0.8$$
$$P(d_{t+1} | \overline{d}_t) = 0.3$$
$$P(o | d) = 0.6$$
$$P(o | \overline{d}) = 0.2$$

- Suppose we observe door **closed** (O=false) at t+1
- Pick n=1000
  - 700 door open samples
  - 300 door closed samples
- For each sample generate a next state
  - For open samples use prob. 0.8 for next state open
  - For closed samples use prob. 0.3 for next state open
- For each next state sample an observation
- Discard samples where sampled observe != real observation
- Count proportion of remain states with door open/closed

Problems with Rejection Sampling

- Discarding samples is inefficient!
- Suppose a rare event occurs:
- -> most samples inconsistent with observation

- In continuous observation spaces, samples will have probability 0 of matching observation

Modified Sampling

- Problem: How do we adjust sampling to handle evidence?
- Solution: Weight each sample by the probability of the observations
- Called importance sampling (IS), or likelihood weighting (LW)

- *Does the right thing* for large n

Example with evidence

$P(d_{t+1} | d_t) = 0.8$

$P(d_{t+1} | \bar{d}_t) = 0.3$

$P(o | d) = 0.6$

$P(o | \bar{d}) = 0.2$

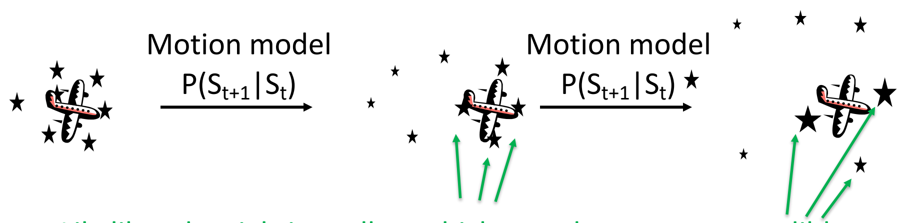
- Suppose we observe door **closed** (O=false) at t+1
- Pick n=1000
  - 700 door open samples
  - 300 door closed samples
- For each sample generate a next state
  - For open samples use prob. 0.8 for next state open
  - For closed samples use prob. 0.3 for next state open
  - If next state is open, weight by 0.4
  - If next state is closed, weight by 0.8
- Compute weighted sum of no. of open and closed states to estimate state probabilities at time t+1

Problems with IS (LW)

- What happens when we repeat this for many time steps?
- Sequential importance sampling (SIS) does the right thing for the limit of large numbers of samples
- Problems for finite numbers of samples:
  - *Effective* sample size (total weight of samples) drops
  - Eventually
    - Something unlikely happens, or
    - A sequence of individually somewhat likely events has the effect of a single unlikely event, and
    - Population of samples ***drifts*** away from reality
- Over time: Estimates become unreliable

Example of "Drift"

- Suppose you're tracking an aircraft
- Each sample corresponds to a possible aircraft position
- You have a physics based simulation model that predicts:
  next_pos = current_pos + velocity*time + noise
- Over time, samples can drift from reality



Likelihood weighting tell us which samples are more credible

But doesn't fix underlying drift problem, i.e., that most become not very credible over time

Solution: SISR (PF)

Sequential **I**mportance **S**ampling with ***R**esampling* = **P**article **F**ilter

- Maintain n samples for each time step

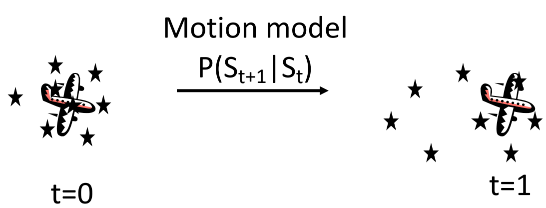
- Repeat n times:
  - Draw sample from $p(S_t)$
    (according to current weights)
  - Simulate transition to S_{t+1}
  - Weight samples by evidence & normalize

- Note: Works for continuous as well as discrete vars!
- AKA: Condensation, Monte Carlo Localization

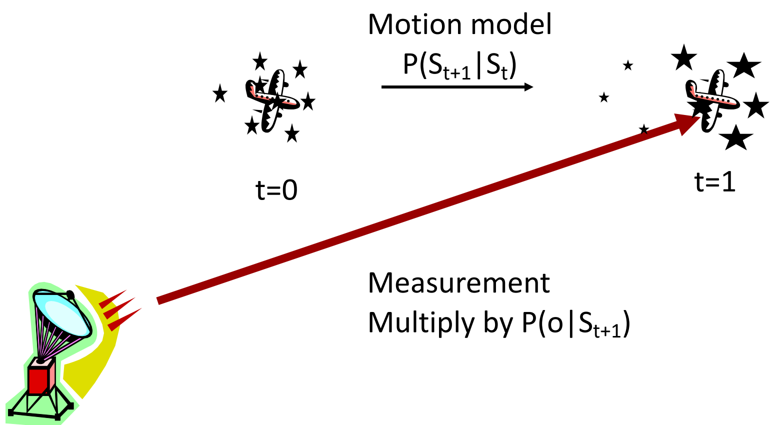
Particle Filter for Trajectory Tracking

t=0

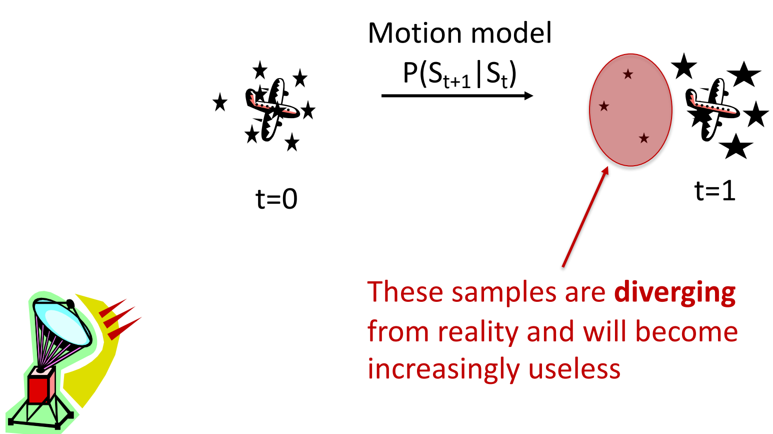
Particle Filter for Trajectory Tracking



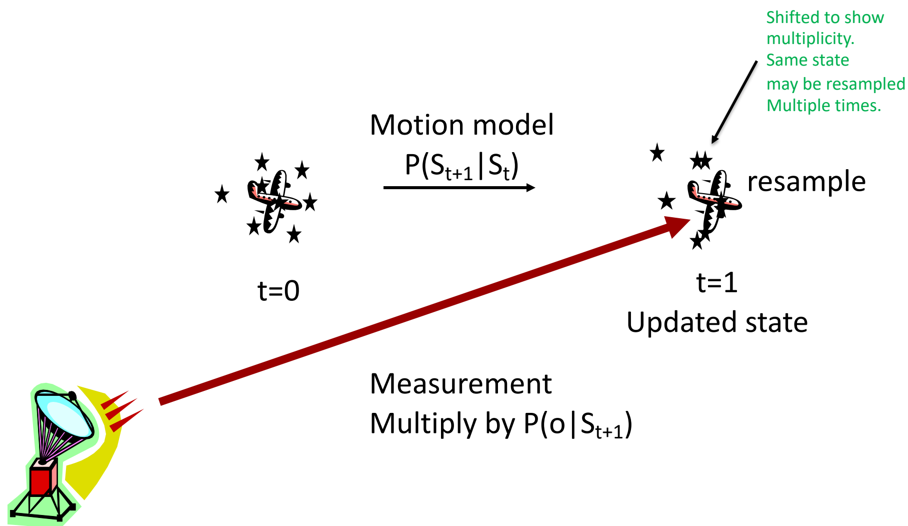
Particle Filter for Trajectory Tracking



Particle Filter for Trajectory Tracking



Particle Filter for Trajectory Tracking



Key Points About Particle Filters

- Given a finite budget of samples:
- PF reallocates resources to samples that better match reality

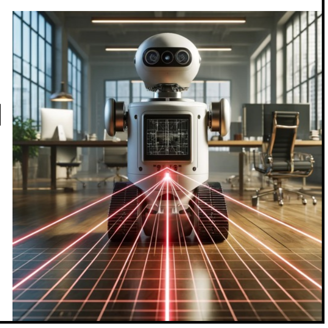
- Leads to more relevant samples
- Less concern about drift

Example: Robot Localization

- Particle filters combine:
  - A model of state change
  - A model of sensor readings
- To track objects with hidden state over time

- Robot application:
  - Hidden state: Robot position, orientation
  - State change model: Robot motion model
  - Sensor model: Sonar/LiDAR error model

- Note: Robot is tracking itself!



Main Loop

- Sample n robot states
- For each state
  - Simulate next state (action model)
  - Weight states (observation model)
  - Normalize
- Repeat

Details included for completeness Not emphasized this semester

Robot States

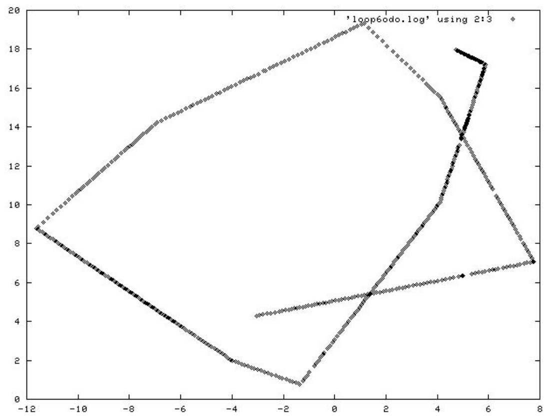
- Robot has X,Y,Z,θ
- Usually ignore z
  - assume floors are flat
  - assume robot stays on one floor
- Form of samples
  - $(X_i, Y_i, \theta_i, p_i)$
  - $\sum_i p_i = 1$

Main Loop

- Sample n robot states
- For each state
  - Simulate next state (action model)
  - Weight states (observation model)
  - Normalize
- Repeat

Motion Model

- How far has the robot traveled?
- Robots have (noisy) odometers:

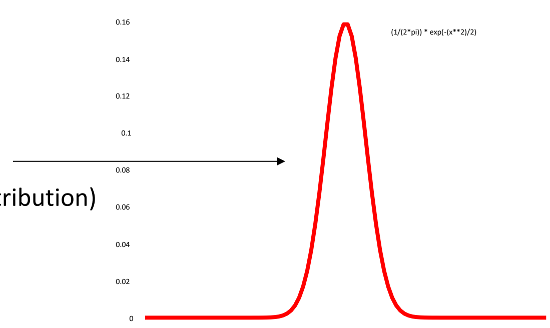


Actual path was a closed loop on the second floor of Duke CS dept!

Odometer Model

- Odometer is:
  - Relatively accurate model of wheel turn
  - Very inaccurate model of actual movement
- Actual position = odometer X,Y,θ + random noise

Classic,
Bell-shaped
Curve
(normal distribution)



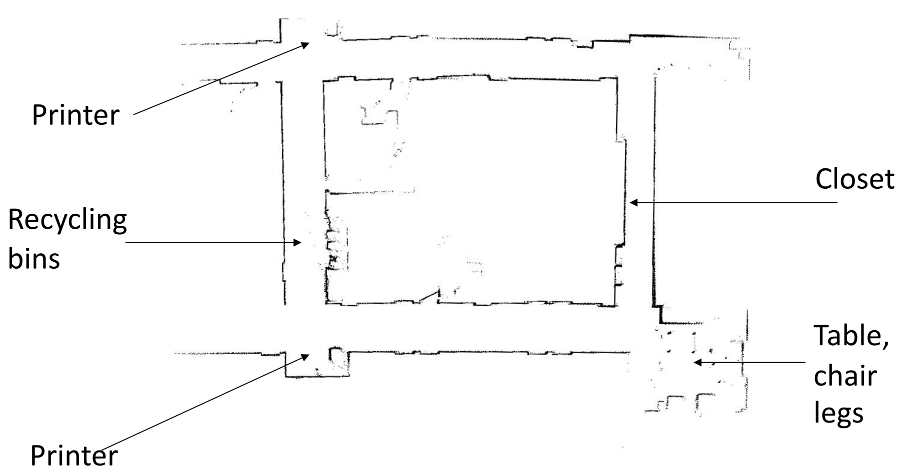
Simulation Implementation

- Start with odometer readings
- Add linear correction factor
  - $X = a_x*X+b_x$
  - $Y = a_y*Y+b_y$
  - $\theta = a_\theta*\theta+b_\theta$

  Linear correction (determined experimentally)
- Add noise from the normal distribution
  - $X = X + N(0,s_x)$
  - $Y = Y + N(0,s_x)$
  - $\theta = \theta + N(0,s_\theta)$

  $N(\mu,s)$ returns random noise from normal distribution with mean μ and standard deviation s (standard deviation determined experimentally)

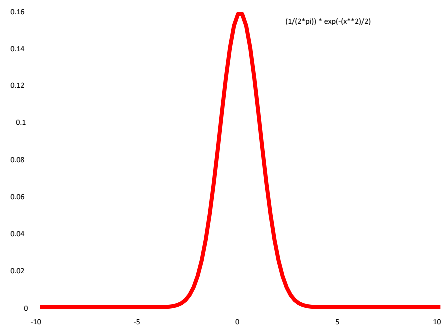
Internal Map Representation



Laser Error Model

- Laser measures distance in ~1 degree increments in front of the robot (height is fixed)
- Laser rangefinder errors also have a normal distribution

Prob. of measurement



Distance from closest occupied square to endpoint of laser cast

Laser Error Model Contd.

- Probability of error in measurement k for sample i (normal)

$$p_{ik}(x_k) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x_k^{\,2}}{2\sigma^2}}$$

- x_k is distance of laser endpoint to closest obstacle
- σ is standard deviation in this measurement (estimated experimentally), usually a few cm.

Laser Error Model Contd.

- Laser measurements are independent
- Weight of sample is product of errors:

$$p_i = \prod_k p_{ik}$$

- Note: Good to bound x to prevent a single bad measurement from making p_i too small

Summary

- HMMs provide mathematical basis for tracking
- Exact solution intractable for large state spaces
- Particle filters approximate the exact HMM solution using **sampling**, **simulation, weighting**