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Def <u>dominating set</u>

$G = (V, E)$ an undirected graph

V = vertex set

E = edge set

A subset $D \subseteq V$ is called a <u>dominating set</u> if every vertex outside D is connected by at least one edge to a vertex in D.

A reduction between DOMINATING SET and SET COVER

$$\boxed{\text{(1) From DOMINATING SET to SET COVER}}$$

Take a general graph $G = (V, E)$ with $V = \{1, 2, \dots, n\}$

We construct a SET COVER for G as follows.

$U = V$ universe of elements

consider a family of subsets of U defined as follows:

$$S = \{S_1, S_2, \dots, S_n\}$$

one for each vertex of G.

S_v = {the set of vertices adjacent to v together with v}

Consider <u>D any dominating</u> set of G.

let $C = \{S_v \mid v \in D\}$.

want to show that C is a set cover. Indeed!

Moreover: $|C| = |D|$

Conversely: $W \subseteq V$

<u>if</u> $C = \{S_v : v \in W\}$ is

<u>a set Cover</u>

then want to show that

W is a dominating set for G.

indeed!

One-to-one correspondence between

dominating sets in G and

Set covers in (U, S).

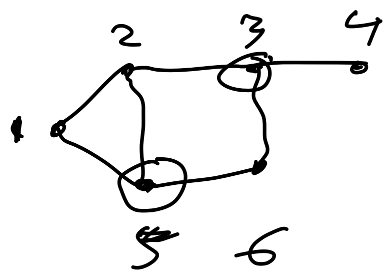
min — min

A very simple algorithm to construct

a set Cover solution from

a dominating set in G.

An example : From Dominating Set to Set Cover

G: 

example of dominating set

$D = \{3, 5\}$ min DS

Construct the Set Cover Instance

$S_1 = \{1, 2, 5\}$, $S_2 = \{1, 2, 3, 5\}$, $S_3 = \{2, 3, 4, 6\}$

$S_5 = \{1, 2, 5, 6\}$, $S_6 = \{3, 5, 6\}$

In G $D = \{3, 5\}$ is a dominating set

This corresp. to $C = \{S_3, S_5\}$

a set cover

② From SET COVER to DOMINATING SET

A general Set Cover instance

(U, S) $\qquad$ U = universe
S = collection of subsets of U

$$S = \{ S_i : i \in I \}$$

Assume U and I are disjoint

Construct a graph $G = (V, E)$ as follows:

- the set of vertices $V = U \cup I$
- the set of edges:

type I $\quad \{i, j\} \in E$
for every $i, j \in I$

type II $\{i, u\} \in E$

for every $i \in I$

$u \in S_i$

Note. I is a clique (completely connected graph)

Suppose that C is a set cover solution to (U, S) pb.

$C = \{ S_i : i \in Z \}$

want to show that Z is a dominating set in G.

$Z \subseteq I$

then Z is a dominating set for G.

. First for each $u \in U$ there

is $i \in Z$ such that $u \in S_i$
and by construction

u and i are adjacent in G.

hence u is dominated by i

- Second, since Z must be nonempty each $i \in I$ is adjacent to a vertex in Z.

So Z is a dominating set.

Conversely: let D be a dominating set for G

Then it is possible to construct another dominating set X such that $|X| \leq |D|$

and $X \subseteq I$:

simply replace each $u \in D \cap V$ by a neighbor $i \in I$ of u.

Then $C = \{S_i : i \in X\}$ is a set cover solution for (U, S)

and $|C| = |X| \le |D|$.

Example: From Set Cover $\Rightarrow$ Dominating Set

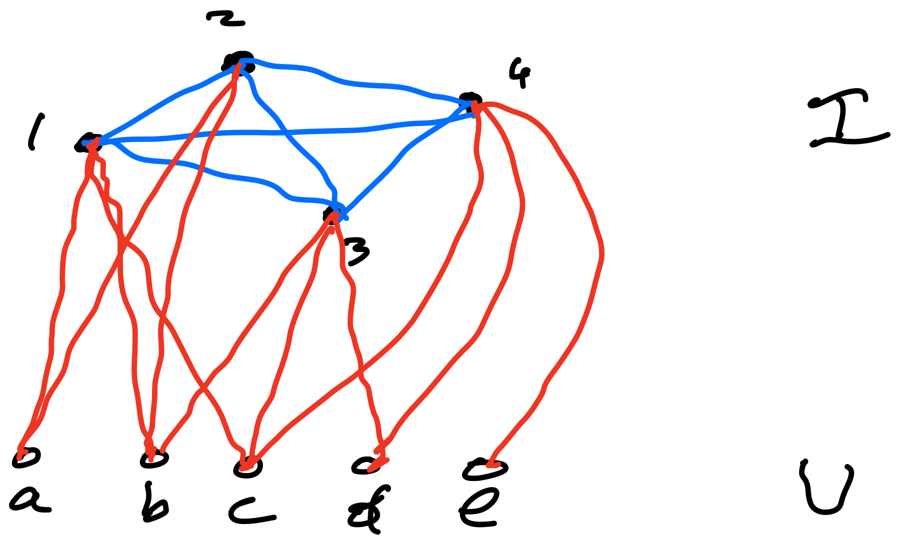
- $U = \{a, b, c, d, e\}$

$S_1 = \{a, b, c\}$, $S_2 = \{a, b\}$,

$S_3 = \{b, c, d\}$, $S_4 = \{c, d, e\}$

Construct G: $V = U \cup I$

$I = \{1, 2, 3, 4\}$



In this example:

$C = \{S_1, S_4\}$ is a set cover

corresponding $D = \{1, 4\}$ it is a dominating set

· $D = \{a, 3, 4\}$ is also a dominating set

X constructed as above

$X = \{1, 3, 4\}$ a dominating set

$\subseteq I$.