# Linkage Disequilibrium & Models of Population Genomics

**Models** = Mathematical models of evolution

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**Infinite Alleles Model** ≡ every mutation is new

**Random Mating**

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**Def** POPULATION

"population genetics"

"population genomics"

≈ a set of genomes & individuals

* SNP frequencies ≡ (single letter) allele DNA
* Haplotype frequencies ≡ sequences of SNPs alleles
* Genotype frequencies ≡ a pair of haplotypes

Conceptually, a population will be described mathematically as a family of allele, haps and genotype frequencies.

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The concept of "Population" is very difficult to define

population ≡ a subgroup of humans

subpopulations

Ex. Iceland people

geographically isolated
linguistically isolated
⋮

"local"

RANDOM MATING

- in sexual organisms genotypes are NOT transmitted to the next generation

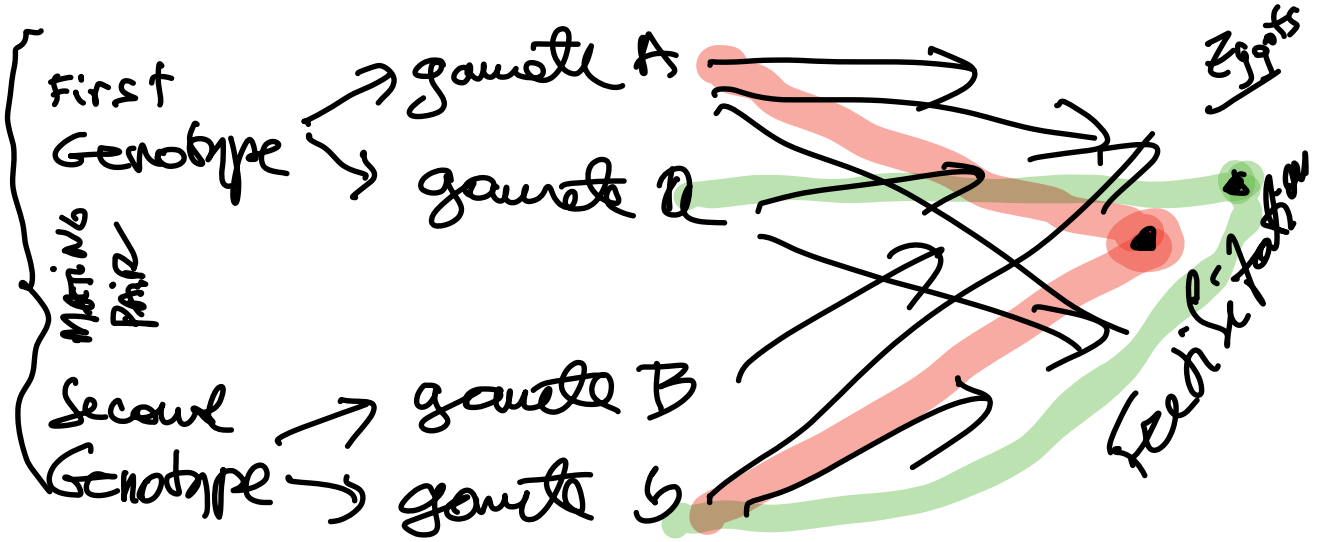
Genotype ≡ { Haplotype 1 = gamete 1
Haplotype 2 = gamete 2

Term equivalence:

gamete ≡ haplotype ≡ genomic seq

# The Mating process goes

GENOTYPE → GAMETES → GENOTYPE




The frequency of a specified genotype in a population is called the genotype frequency.

The genotypes of the mating pair determine the genotype of the progeny

There is a mathematical relation between the freq. of the mating pairs and the frequencies of the progeny genotypes. These frequencies all inferred based on models.

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**Def. RANDOM MATING MODEL**

≡ the mating pairs have same frequencies as if they were formed by random colisions between genotypes.

≡ the chance that an organism mates with another is based of its genotype frequencies in that population.

Ex. Population with genotype frequencies

| AA | Aa | aa : genotypes |
|---|---|---|
| 0.20 | 0.55 | 0.25 : genotype freq |

If mating is random then

AA males mate with females AA, Aa, aa in proportions 0.20, 0.55, 0.25

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Is human mating random?

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MODEL : HARDY-WEINBERG MODEL

single gene, or single SNP, or single locus

One SNP : biallelic SNP : two alleles A, a

# MODELS: HWM

"first the assumptions, then the conclusions."

## Models have assomptions

## Axioms / Assumptions of HW Model

1) The organisms are diploid
2) Reproduction is sexual
3) Generations are non overlapping
4) Gene / locus / SNP has exactly 2 alles
5) the allele freqs of males and females are the same
6) Mating is Random

7) Population is very large (in theory, infinite)

8) Migration is negligible

9) Mutation is ignored

10) Natural selection does not affect the alles at the single locus

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G. H. Hardy mathematician

W. Weinberg MD, physiologist

"The Hardy-Weinberg Law"

It is all about frequencies:

The Hardy-Weinberg model gives the mathematical relation between the allele frequencies and the genotype frequencies:

A, a the two alleles
AA, Aa, aa the genotypes

$AA: p^2$, $Aa: 2pq$, $aa: q^2$

$p$ = freq of A
$q$ = freq of a
$p+q=1$

$\text{freq}(AA) = p^2$
$\text{freq}(Aa) = 2pq$
$\text{freq}(aa) = q^2$

HW Law

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p, q are the freq of the alleles in the first generation, then in the next generation, the frequencies of AA, Aa, aa are $p^2, 2pq, q^2$.

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## The POWER OF MATH MODELS

THEOREM 1

The Hardy-Weinberg frequencies are attained after one generation of random mating irrespective of the genotype frequencies in in the parental generation.

THEOREM 2 [Constancy of allele frequencies]

The Hardy-Weinberg law implies the constancy of allele frequency in every generation after the first, and therefore the genotypic composition of the population.