Interactive Visualization of Surfaces in 4D

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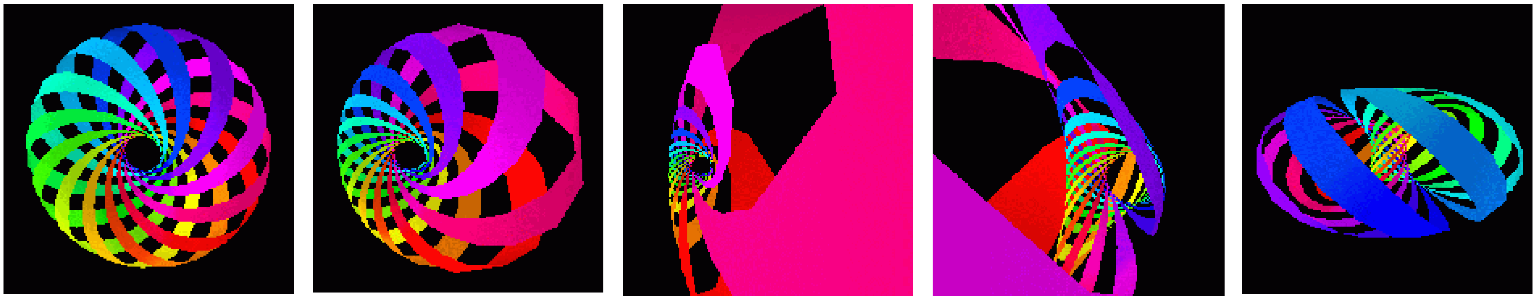

Figure 1 A Succession of frames as we move along the edges of the tetrahedron. Note how the 4-Torus evolves from one 3-Projection to the other. At each stage the user can stop and investigate the surface.

A variety of computer graphics techniques have enabled the display of surfaces (which exist in higher dimensions) on a 2D screen. In this project we have developed novel techniques to navigate and interact with a surface in 4D, in an immersive environment. We have successfully investigated if our visualization in the CAVE is any different from 'seeing' the surface on a computer screen and if it qualitatively helps the user to understand the surface better.

Introduction

The techniques of interactive 3-dimensional computer graphics can be extended so that users (particularly mathematicians) can examine surfaces that lie in 4-dimensional space. A significant body of literature exists in Topology and differential geometry that deals with the nature and properties of such surfaces. But unfortunately conventional computer graphics limits the user to view and interact with these fascinating shapes from outside. Even in immersive environments the viewer tends to lose track of his position in four-dimensional space and the particular projection he is looking at. What is missing is the notion of a control space which will give the user an intuitive understanding of his bearing vis-a-vis the surface. Our work provides the user with a tetrahedral control space (to change the three-dimensional projection) and an intuitive way to interact and navigate through the surface.

Method and Results

We started off with customizing the software StageManager to generate VRML files of a 4-torus rotating in 4 space as we move along the edges of the tetrahedron. Then we wrote a custom shape generator which, given an equation, would generate a 4D surface with standard visualization techniques like ribboning and color variation. Thereon we developed the tetrahedral control space, which gives us a choice of projection from 4D to 3D. We chose jot's interaction techniques for manipulating the object in the cave. So far mathematicians have been confined to moving along a single edge of the tetrahedron at a time. We have provided a means to move along the face and also into the tetrahedron. This has resulted in a marked improvement in the overall understanding of the object in 4 dimensions.

Finally we conducted a user-study amongst the graduate and undergraduate students who were taking MA106 (Differential Geometry) and validated that the visualization methodology indeed helped them to understand the surfaces better. The subjects were shown a video of the rotations of the Torus in 4 space and given a lecture on the mathematics involved by Professor Banchoff. Later on they were brought to the cave and exposed to the tetrahedral control space as a means to visualize and interact with the same transformations (which can be performed very simply by moving along a single edge in our model). They also experimented with complex interactions. Later on they were asked to subjectively evaluate if the tetrahedral control space helped them to gain further intuition into the mathematics. Although some users complained of "too many degrees of freedom" inside the tetrahedron, on the whole the response was positive.

Conclusions

We have been successful in demonstrating that our Tetrahedral control space model has made a 'qualitative' difference in our understanding of 4D surfaces.Since we have developed a toolkit for visualizing 4 dimensional surfaces, we can now tackle bigger problems which arise from visualization of 4D datasets. We hope to interactively 'fit' surfaces onto such datasets and try to see what class of surfaces the data corresponds to. Besides a more intuitive understanding of 4D visualization, we believe that this work could lead to development of interfaces which enable us to kinesthetically navigate through higher dimensional datasets.