

# CSCI 1820/2820: An overview

Spring 2022

- Ch. 1 BLAST Algorithm and Karlin-Altschul Statistics
- Ch. 2 Genome Assembly and Haplotype Assembly Algorithms
- Ch. 3 Hidden Markov Models (HMM) Algorithms: The Learning Problem
- Ch. 4 Recombination and Ancestral Recombination Graphs (ARGs)
- Ch.5 Rigorous Clustering and Spectral Clustering Algorithms
- Ch. 6 Algorithms for Constructing Suffix Trees in Linear Time
- Ch. 7 Protein Folding Algorithms (Introduction)



# The BLAST Algorithm

## Authors

- Stephen Altschul
- Warren Gish
- Webb Miller
- Eugene W. Myers
- David Lipman
- **“Basic Local Alignment Search Tool”**

Journal of Molecular Biology (1990) 215, 403-410

## Karlin Altschul Equation

$$E = kmNe^{-\lambda s}$$

$m$  Number of letters in query

$N$  Number of letters in db

$mN$  Size of search space

$\lambda s$  Normalized score

$k$  minor constant

# Gambler's Ruin problem



In Sir Ronald Fisher we trust!



Ronald Fisher  
(1890-1962)





**Dr. Margaret Oakley Dayhoff**  
**The Mother & Father of Bioinformatics**



# Smith and Waterman at Los Alamos, New Mexico

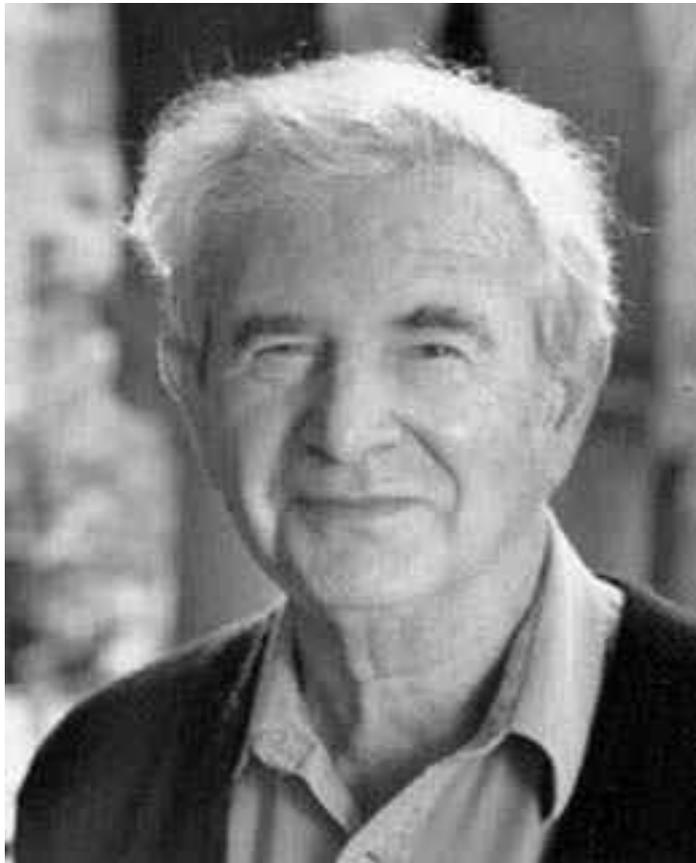
Photo by David Lipman, Taken Summer of 1980

Smith and Waterman



# Karlin-Altschul Statistics Theory

- Samuel Karlin and Stephen Altschul



# Ch. 2: Genome Assembly and Haplotype Assembly Algorithms



Questions: What algorithms to use to assemble DNA pieces into a contigs?  
How long are the contigs?  
How much the DNA target region is covered by the contigs?

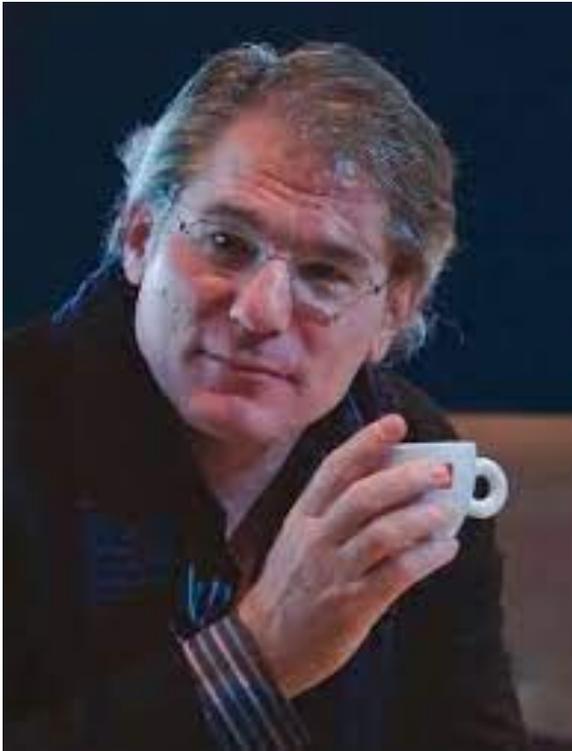
Examples of problems we need to solve along the way

Problem 1. Poisson statistics and DNA and Assembly

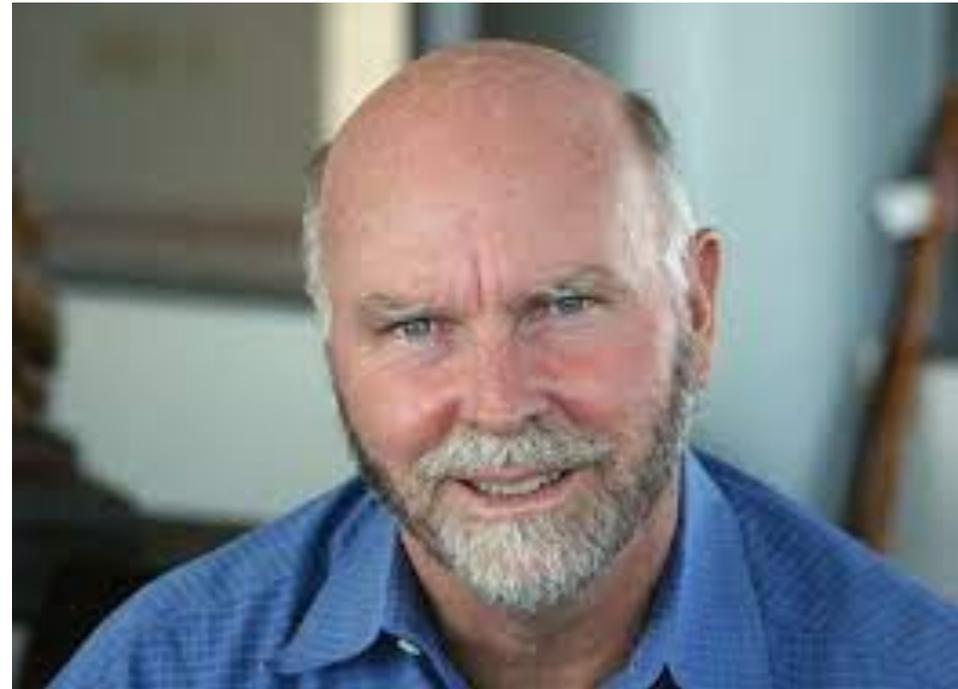
Problem 2. Ham Smith's DNA breaking in a Lab with no windows

# Hamiltonian Paths Algorithms for Genome Assembly

Gene Myers



Craig Venter



# Eulerian Paths Algorithms for Genome Assembly

Pavel Pevzner



Michael Waterman

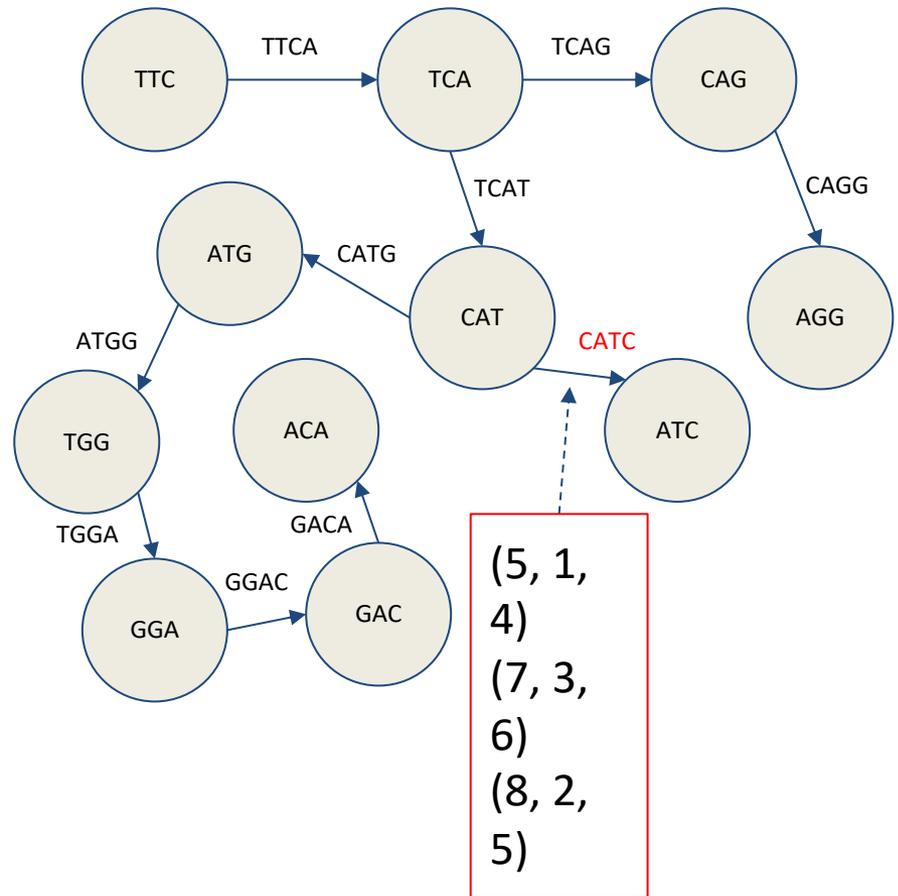


# Construct the sequence graph on (k-1)-mers

$f_1$  TTCAGG  
 $f_2$  TTCATGG  
 $f_3$  ATGGACA  
 $f_4$  TTCAT  
 $f_5$  **CATCGAC**  
 $f_6$  TCGAC  
 $f_7$  **GACATC**  
 $f_8$  **ACATCGA**



TTCA  
 TCAG  
 CAGG  
 TCAT  
 CATG  
 ATGG  
 TGGA  
 GGAC  
 GACA  
**CATC**  
 ATCG  
 TCGA  
 CGAC  
 ACAT



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TTCAT  
TCAGG  
TCATG  
CATGG  
TGGAC  
GGACA  
GACATC  
CATCGA  
ATCGA  
TCGAC  
CGAC  
ACAT

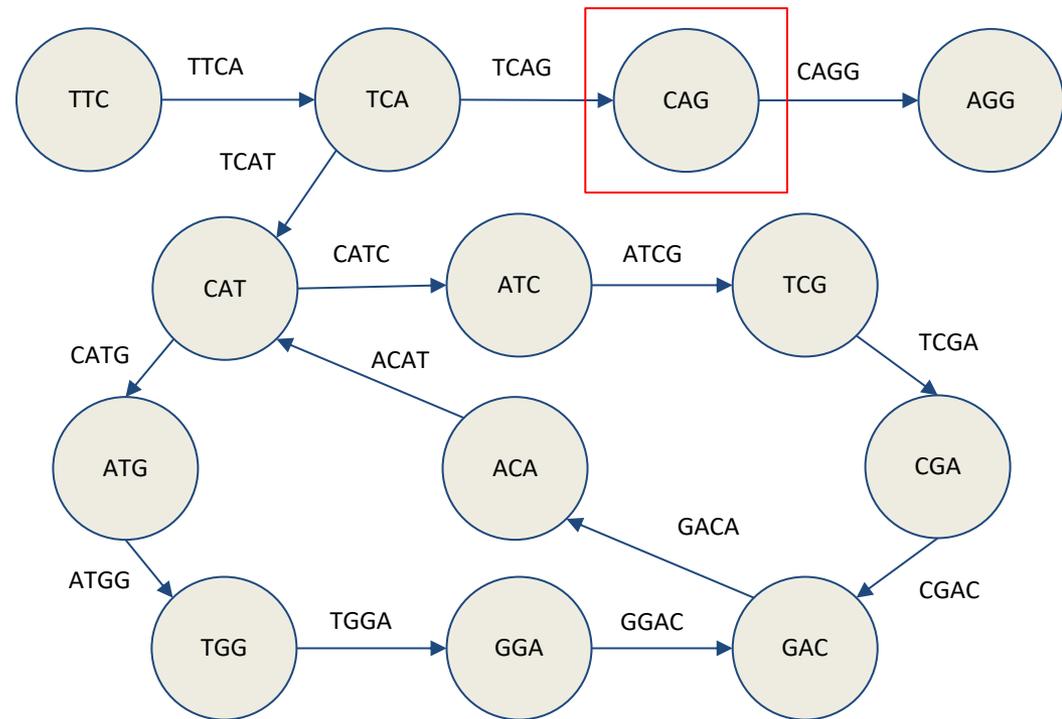
For each k-mer ( $a_1...a_k$ ), we create an edge between nodes labeled  $a_1...a_{k-1}$  and  $a_2...a_k$ .

If those nodes do not exist yet, we add them to the graph.

We label the edge by its k-mer,  $a_1...a_k$ .

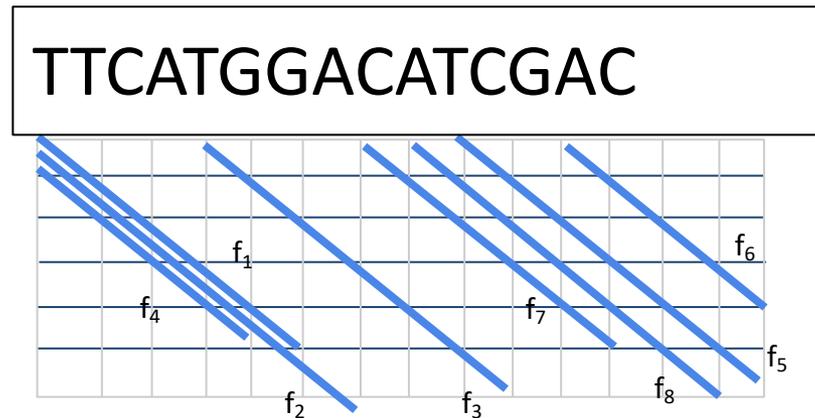
We also store the set of position values ( $f, i, j$ ) in each edge, which identify all occurrences of that k-mer by (fragment index, start position, end position)\*

# Graph reductions: singletons



# Align the reads to the assembled sequence

f<sub>1</sub> TTCAGG  
f<sub>2</sub> TTCATGG  
f<sub>3</sub> ATGGACA  
f<sub>4</sub> TTCAT  
f<sub>5</sub> CATCGAC  
f<sub>6</sub> TCGAC  
f<sub>7</sub> GACATC  
f<sub>8</sub> ACATCGA



First, we apply hashing methods to identify where each fragment might align well to the sequence.

This will produce “candidate diagonals.”

We can then perform alignment along those diagonals, which is more efficient than using the entire edit graph.

# Statistics of Sequence Graphs: vertices

$$\begin{aligned}
 \mathbb{E}(\text{True}) &= L' \sum_{i=1}^{\infty} (1 - R^i) \mathbb{P}(X = i) \\
 &= L' \sum_{i=1}^{\infty} \left( \frac{e^{-c} c^i}{i!} - \frac{e^{-c} (cR)^i}{i!} \right) \\
 &= L' (1 - e^{-c(1-R)}).
 \end{aligned}$$

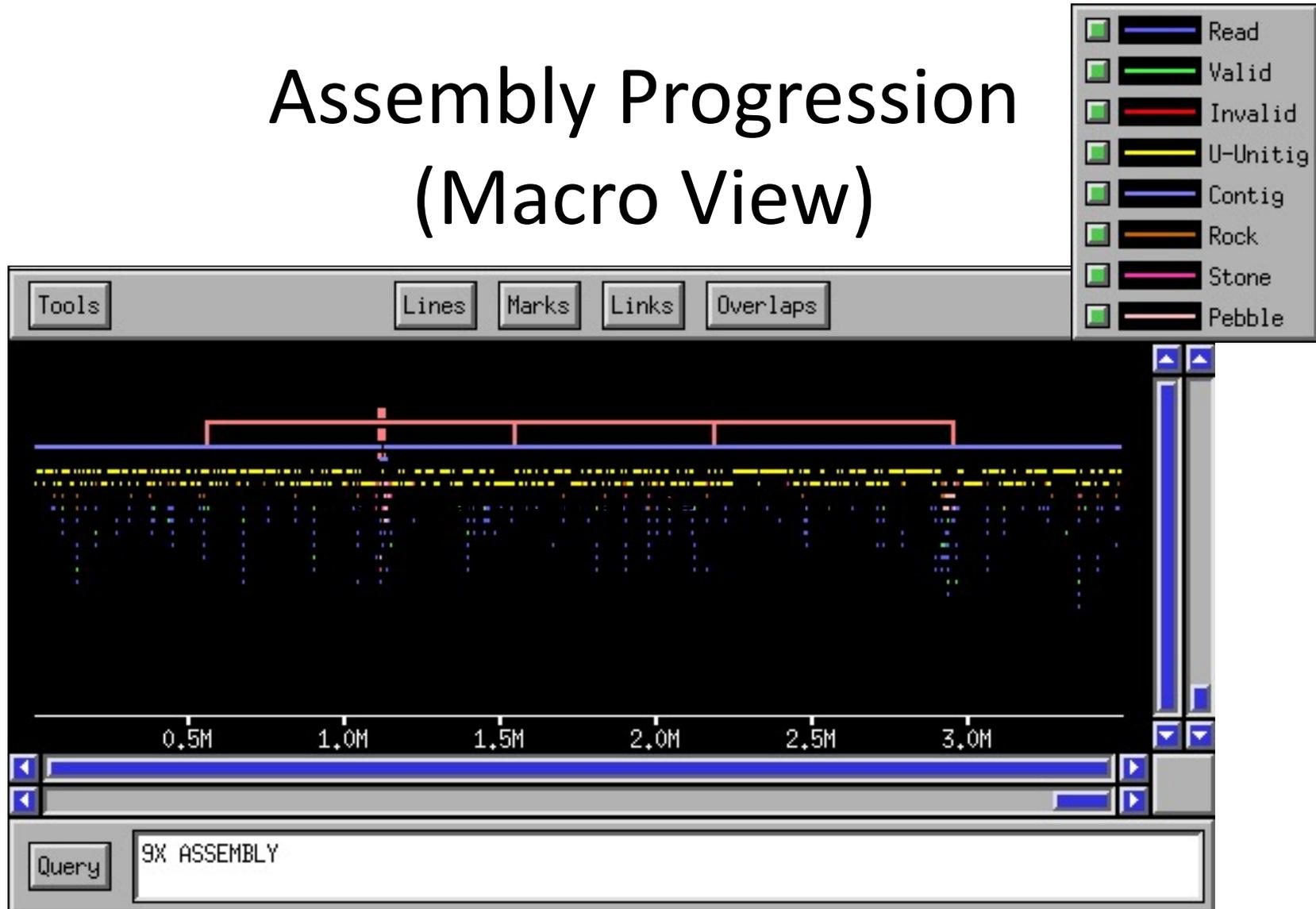
pmf of Poisson  $\Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$

using Taylor expansion for e:  $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

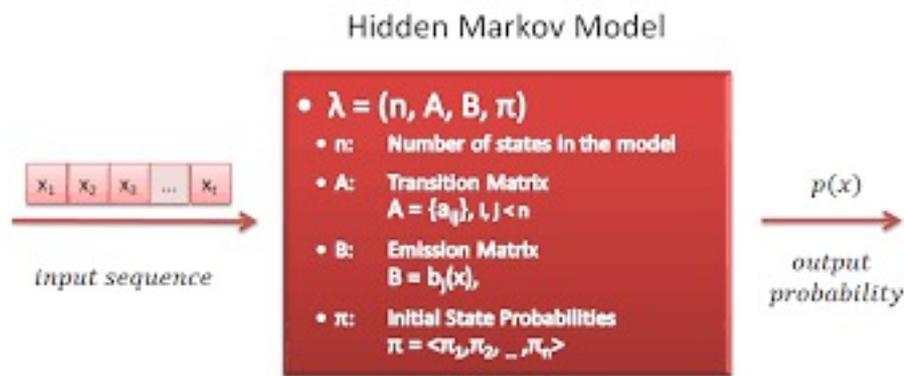
Summing the number of false vertices and true

vertices. The expected number of vertices  $\mathbb{E}(|V|) = RT + [1 - e^{-c(1-R)}]L'$ .

# Assembly Progression (Macro View)



# Ch. 3: HMM - the Learning Problem

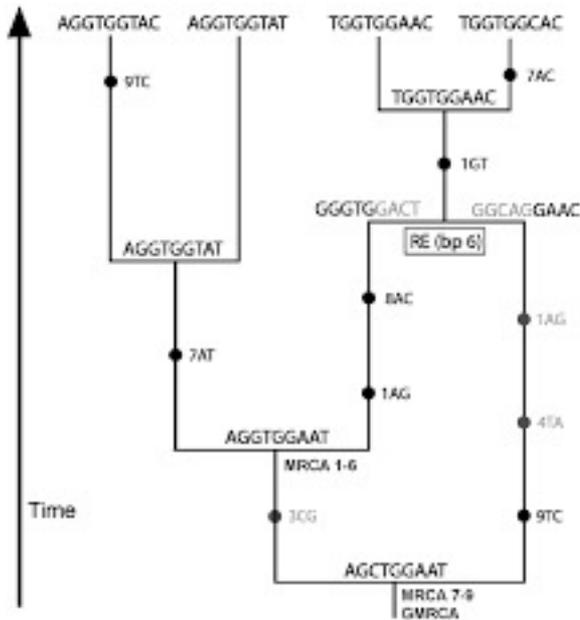


What does machine learning an HMM model mean?

Maximum Likelihood and the Expectation-Maximization problem

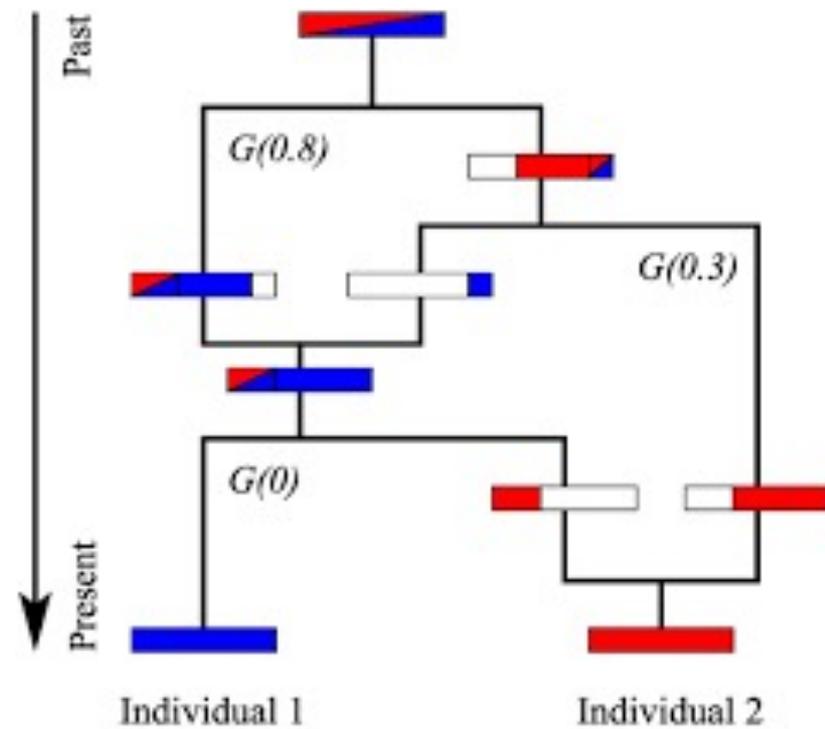
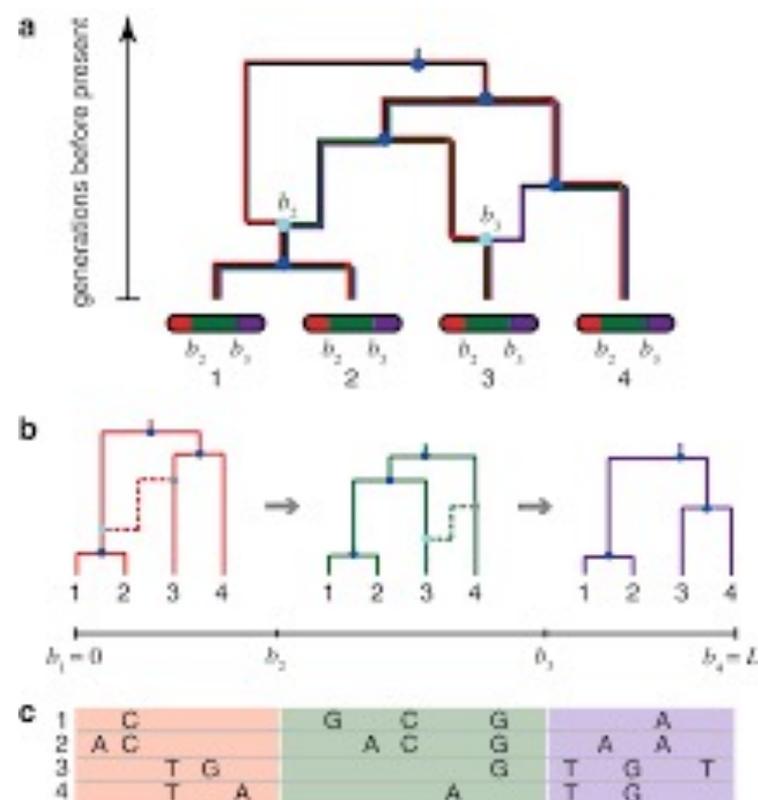
# Ch. 4 Recombination and Ancestral Recombination Graphs (ARG)

## Algorithms



How do we reconstruct genealogies of a sample of individuals incorporating past mutations and recombinations?

Recombination + Phylogenetic Trees = ARG

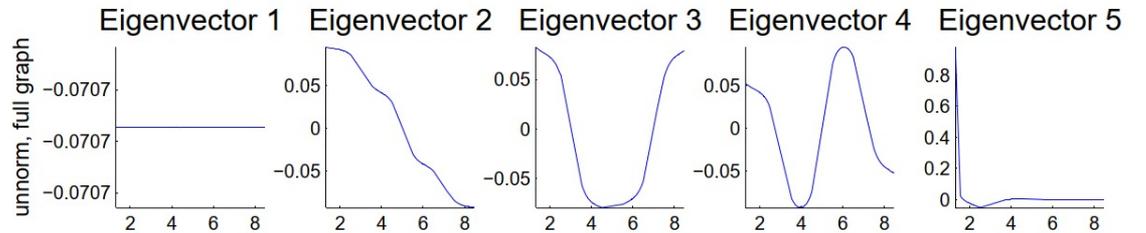
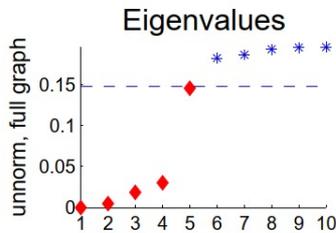
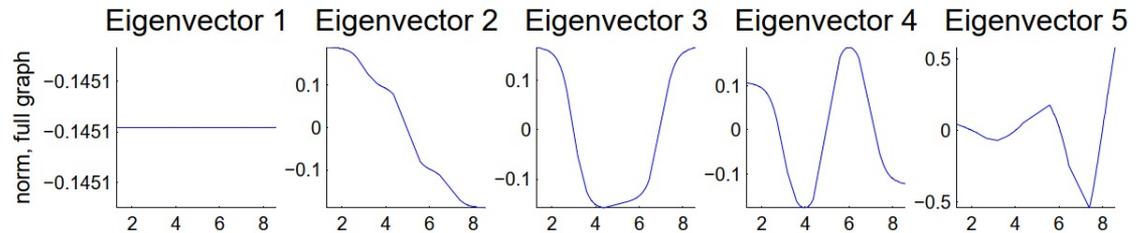
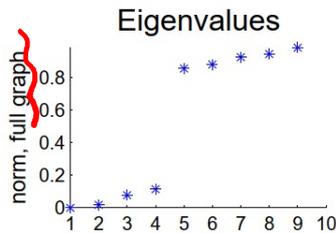
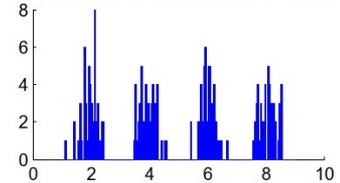


# Ch. 3: Spectral Clustering

## GRAPH LAPLACIANS

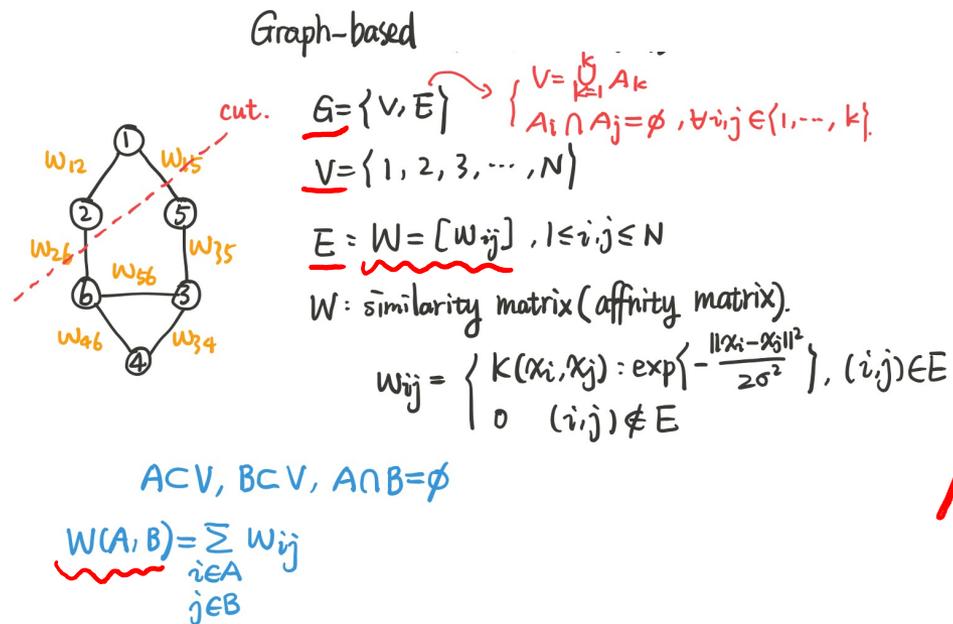
- Quick example

Histogram of the sample



Sentences in red and graphs are cited from A Tutorial on Spectral Clustering (Ulrike von Luxburg). See reference list at the end for detail.

# GRAPH CUT POINT OF VIEW



Sentences in red and graphs are cited from A Tutorial on Spectral Clustering (Ulrike von Luxburg). See reference list at the end for detail.

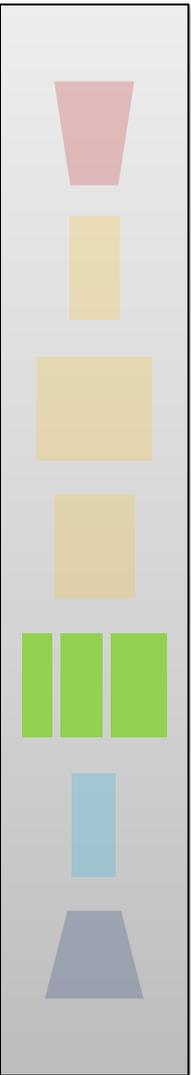
# RANDOM WALK POINT OF VIEW

- What is random walk?
  - A random walk on a graph is a stochastic process which randomly jumps from vertex to vertex.
- How does it walk?
  - Formally, the transition probability of jumping in one step from vertex  $\mathbf{v}_i$  to vertex  $\mathbf{v}_j$  is proportional to the edge weight  $\mathbf{w}_{ij}$  and is given by  $\mathbf{p}_{ij} := \mathbf{w}_{ij}/\mathbf{d}_i$ .
  - The transition matrix  $P = (p_{ij})_{i,j=1,\dots,n}$  of the random walk is thus defined by

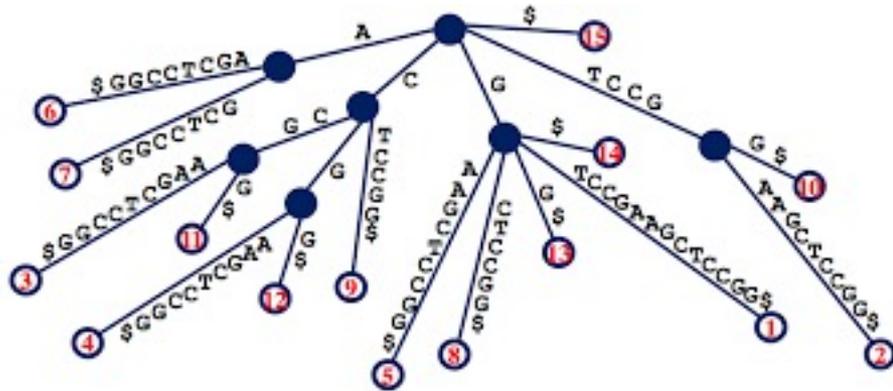
$$P = D^{-1}W.$$

- Initial condition?
  - a unique stationary distribution  $\boldsymbol{\pi} = (\boldsymbol{\pi}_1, \dots, \boldsymbol{\pi}_n)$ , where  $\boldsymbol{\pi}_i = \mathbf{d}_i / \text{vol}(V)$ .
- Clustering in random walk?
  - Finding a partition of the graph, such that the random walk stays along within the same cluster and seldom jumps between clusters.
  - Intuitively, it is the same as the graph cut.

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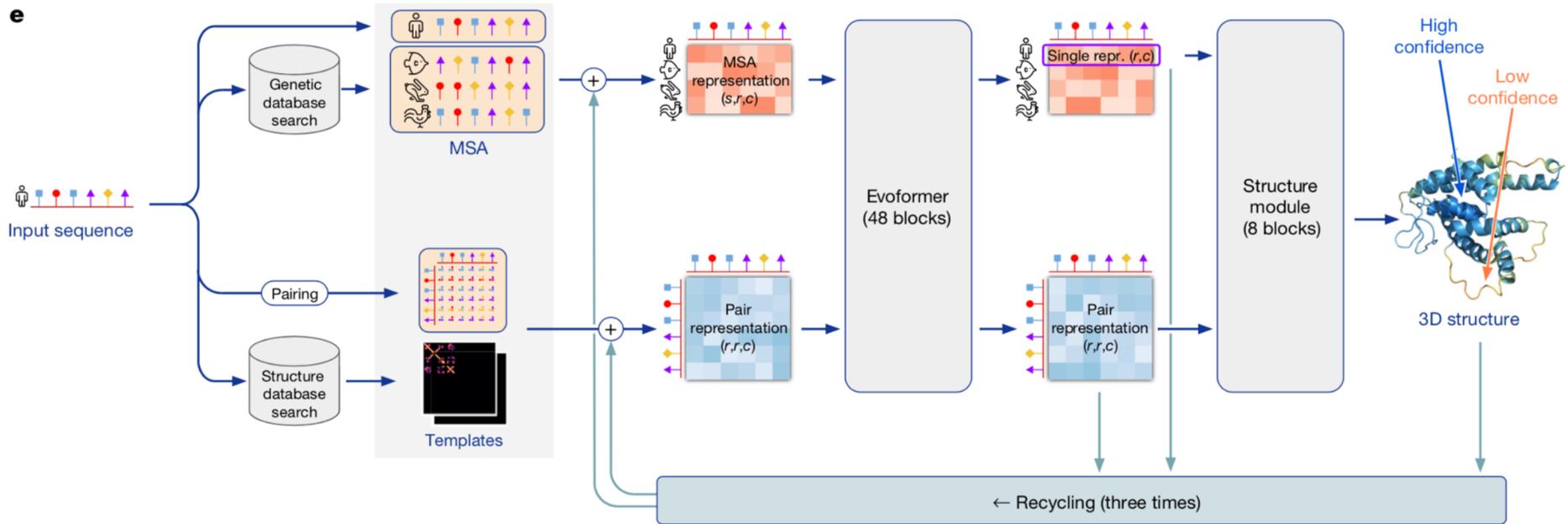
# Ch. 6 Suffix Trees in Linear Time



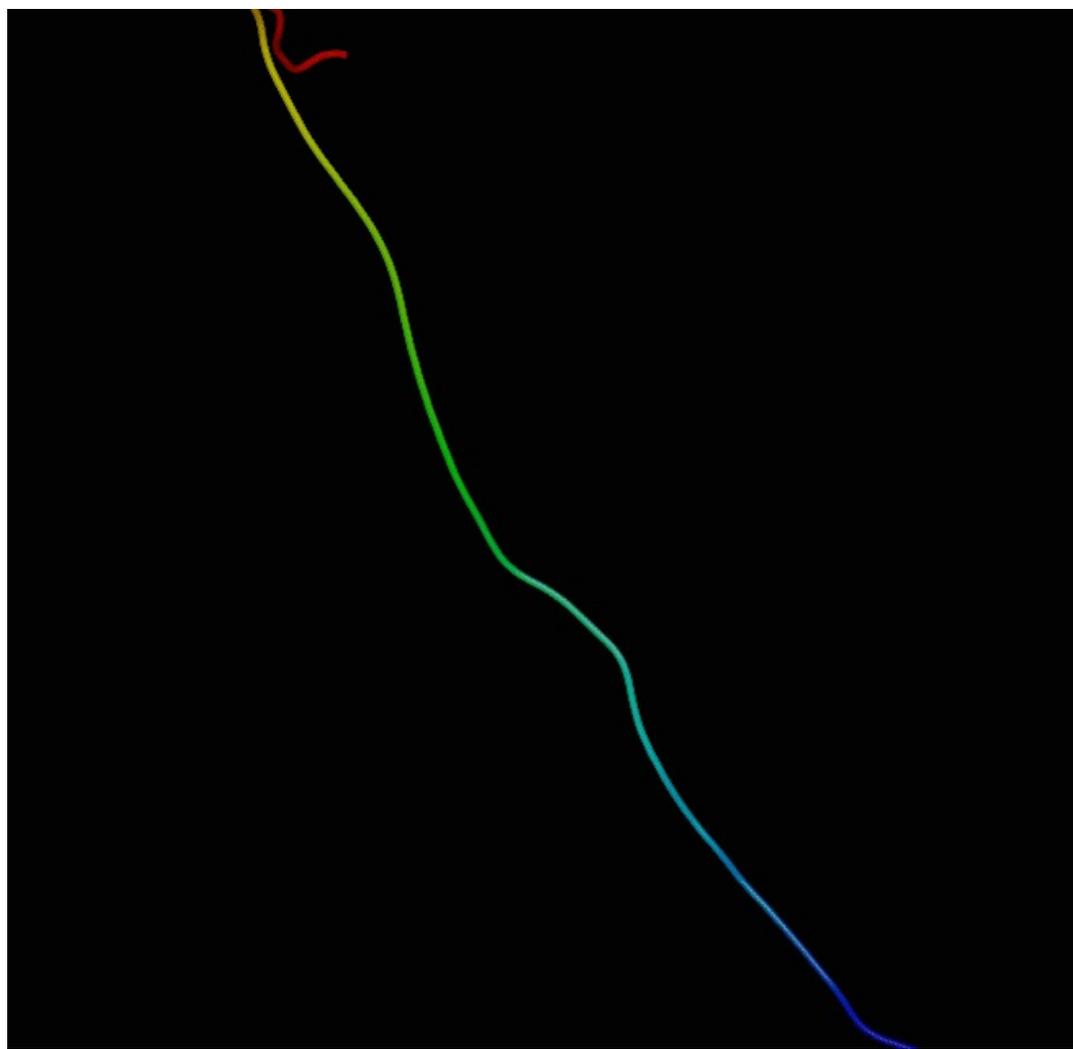
# Ch. 7 Protein Folding Algorithms (Intro)

- Protein Folding on Lattice Models
- AlphaFold and Deep Learning

# High-level Overview of Architecture of AlphaFold



Deep learning uses sequential modules (layers) to progressively extract information (learn) from the input data.

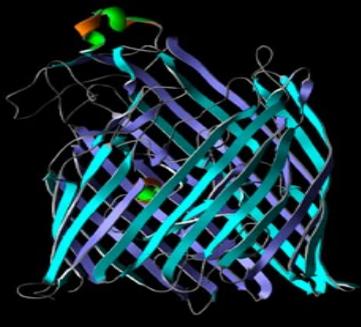


# The Protein Folding Problem

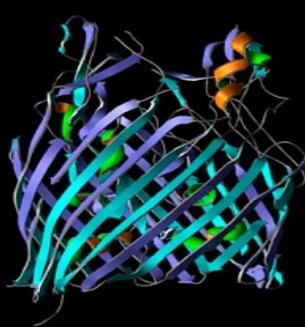
Statistical Mechanics models

Mixed character of the problem :

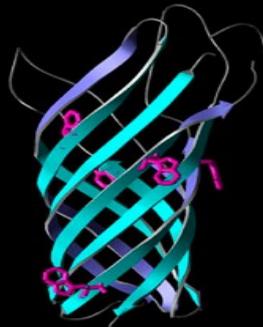
**continuous** mathematics -- geometry of surfaces &  
**discrete** mathematics -- combinatorics of folds



**FhuA**  
(Ferguson et al., 1998)



**FepA**  
(Buchanan et al., 1999)



**OmpA**  
(Pautsch & Schulz, 1998)

Illustration © 1999 JHK

