HIDDERN MARKAY MODELSCHAMPLE SLEMATIN:suppose ALCE MAS 2 coins (C1 and C2) with different biases: R_c [H]: 507. R_c [T]: 707. R_c [R]: 717. R_c [R]: 717.<
Example scenaris suppose Auce has a coins (c1 and c2) with different biases: $\begin{bmatrix} R_{c_1}[T] = 50. \\ R_{c_1}[T] = 70. \\ P_{c_2}[T] = 70. \\ P_{c_3}[T] = 70. \\ P_{c_$
suppose Aide may 2 coins (C2 and C2) with different biases: [fr_[T]*f07. $P(_{1}[T]*f07. P(_{1}[T]*f07. nidden states: which can Aide finds i.e. C, $
bidden states: which with an Alike that bidden states is the resummy heads tails i.e. $T H T T T T$ there are also different probabilities of transitioning from 1 hidden state (win) to another: 287 387 507. Suppose we only see the observations. Given a long enough sequence of observations, we can use statistical algorithms is techniques to infer the underlying sequence of hidden states. bHMMS have powerful applications in genomics i.e. the hidden states can be various genes or genetic structures that we can infer from the observed sequence of bases 5° set of states A° summy raining $[0.7 \ 0.5]$ \rightarrow i.e. P(sumy miny transition] 0.3 $rainy [0.5 \ 0.5]$ \rightarrow i.e. P(sumy miny transition] 0.3 B° contains A° summy $[0.7 \ 0.5]$ \rightarrow i.e. P(sumy miny transition] 0.3 B° states A° summy A° summy $[0.7 \ 0.5]$ \rightarrow i.e. P(sumy miny transition] 0.3 B° states A° summy A° summy $[0.7 \ 0.5]$ A° summy $[0.5 \ 0.5]$ B° miny transition A° summy A° summethes A° summy A° summethes A° sum
there are also different probabilities of transitioning from 1 hidden state (uoin) to another: $\frac{292}{10} + \frac{292}{501} + \frac{507}{501} + 5$
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Suppose we only see the abservations. Given a long enough sequence of observations, we can use statistical algorithms is techniques to infer the underlying sequence of hidden states. bHMMS have powerful applications in genomics i.e. the hidden states can be various genes or genetic structures that we can inter from the observed sequence of bases Markov Chains S^{*} set of states A = slate transition probability notrix S^{*} set of states A = slate transition probability notrix S^{*} set of states A^{*} summy rainy S^{*} [summy, rainy] A = summy [0.7 0.3] $0.5 0.5$] \rightarrow i.e. P[sumy-rainy transition]: 0.3 P[crowing = summy transition]: 0.5 P[crowing = summy transition]: 0.5 P[crowing = summy transition]: 0.5 P[crowing = summy transition]: 0.5 S^{*} set of states i vect on added component of different states having different observation states A^{*} probability to state $\rightarrow 0$ is $= P[Q_{t+1}^{*}S_{i}]Q_{i}^{*}S_{i}]$ (probability of state the being S_{s} given state B^{*} emission probability matrix $\rightarrow 0$ is $= P[Q_{t+1}^{*}S_{i}]Q_{i}^{*}S_{i}]$ (probability of state the being S_{s} given state B^{*} emission probability distribution A^{*} signability of state probability distribution A^{*} signability of structure S^{*} is S^{*} . A^{*} probability distribution A^{*} signability of structure A^{*} signability distribution A^{*} signability distribution A^{*} probability distribution A^{*} signability
genes or genetic structures that we an inter from the observed sequence of bases Markov Chains S= set of states A= shate transition probability notrix state 1 ex] providence Weather stotes from the observed sequence of bases A= shate transition probability notrix state 1 summy rainy rainy S= [summy, rainy] A= summy [0.7 0.3] \rightarrow i.e. R[summy=rainy transition]: 0.3 Ring [0.5 0.5] R[rainy=stanny transition]: 0.5 Hidden Markov Models: have an added component of different states having different observation [emission a S= set of states; V=set of possible observation symbols A= probability matrix \rightarrow 0; = R[q_{t+1} s; $[q_t=s_i]$ (probability of state tel being s; given state B= emission probability matrix \rightarrow b; (L) = probability of emitting symbol K when in state ; TI= initial state probability distribution Alices coin fixp example: C, Cz, H T
$\begin{array}{c c} \underline{Mar Kov Chains} \\ S = set of states \\ A = shote transition probability notifix \\ \underline{ex} \\ providence Weather \\ S = [sunny], rainy] \\ A = sunny [0.7 0.3 \\ 0.5 0.5] \\ \hline miny [0.5 0.5] \\ $
S= set of states A= slate transition probability notifix State 2 (x) providence Weather S= [sunny, rainy] A= sunny [0.7 0.3] \rightarrow i.e. R[sunny=rainy transition]: 0.3 rainy [0.5 0.5] \rightarrow i.e. R[sunny=rainy transition]: 0.3 R[rainy \rightarrow sunny transition]: 0.3 R[rainy \rightarrow sunny transition]: 0.3 R[rainy \rightarrow sunny transition]: 0.5 Hidden Markov Models: have an added component of different states having different observation (mission ~ S= set of states; V=set of possible observation symbols A= probability transition matrix \rightarrow $a_{ij} = iR[q_{t+1} s_j](q_t=s_i](probability of state the being s_j given state B= emission probability matrix \rightarrow b_j(L) = probability of emitting symbol K when in state ; TI= initial state probability distribution Alices coin fisp example: C_1 C_2 H T$
A = slote transition probability matrix e_{X} providence Weather $S = [sunny], rainy]$ A = sunny $[0.7 + 0.3]$ \rightarrow i.e. $\mathbb{P}[sunny + rainy transition] = 0.3$ $rainy [0.7 + 0.3]$ \rightarrow i.e. $\mathbb{P}[sunny + rainy transition] = 0.3$ $rainy [0.5 + 0.5]$ $\mathbb{P}[rainy + sunny transition] = 0.3$ $\mathbb{P}[rainy + sunny transition$
$\begin{array}{c c} \underline{ex} & providence \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$
$S = [sunny], rainy] A^{s} sunny [0.7 0.3] \rightarrow i.e. R[sunny rainy transition] = 0.3rainy [0.5 0.5] R[rainy = sunny transition] = 0.3R[rainy = sunny transition] = 0.5Hidden Markov Models: have an added component of different states having different observation [emission aS = set of states; V=set of possible observation symbolsA= probability to state the being s; given stateB = emission pobability matrix = b; (L) = probability of emitting symbol K when in state ;TI = initial state probability distributionAlice's coin flip example: C, C_1 = H T$
<u>Hidden Markov Models</u> : have an added component of different states having different observation emission A So set of states; Veset of possible observation symbols A = probability taission matrix $\rightarrow a_{ij} = R\left[q_{t+1} \cdot s_{j}\right] q_{t} \cdot s_{i}$] (probability of state the being s_{j} given state B = emission probability matrix $\rightarrow b_{j}(L) = probability of emitting symbol K when in state jT = initial state probability distributionAlices coin flip example: c_{i} \cdot c_{i} H T Alice is equally$
So set of states; Voiset of possible observation symbols A = probability transition matrix $\rightarrow a_{ij} = i R [q_{t+1} \cdot S_j q_t \cdot S_i] (probability of state the being s_given state B = emission probability matrix \rightarrow b_j(L) = probability of emitting symbol K when in state j t is s_i) TI = initial state probability distribution Alice's coin flip example: C_CH T Alice is equally$
A= probability then site and the state of t
B= emission probability matrix -> b; (L) = probability of emitting symbol k when in state ; t is sit TI = initial state probability distribution Alices coin flip example: C, C, H T Alice is equally
T = initial state probability distribution Alices coin flip example: C, C2 H T Alice is equally
Alice's coin flip example: C1 C2 H T 4 And is equally
$S = \{c_{1}, c_{2}\} A = c_{1}[a, 75, b, 25] \qquad B = c_{1}[0, 3, 0, 7] \qquad T_{1} = [0.5, 0.5] \text{likely to start} \\ V = \{H, T\} \qquad c_{2}[b, 5, 0.5] \qquad B = c_{1}[0, 3, 0, 7] \qquad T_{2} = [0, 5, 0, 5] \text{likely to start} \\ C_{2}[0, 5, 0, 5] \qquad C_{2}[0, 5, 0, 5] \qquad C_{3}[0, 7] \qquad$
Fundamental Computational HMM Problems
> Problem 1: The evaluation problem
input: HMM defined by $\lambda = \{A_1, B_1, T_1\}$ and an observation sequence $O = 0, 0, 0, 0, \dots, 0_T$ <u>output:</u> the probability of observing the sequence O given the HMM $\lambda : R(O \lambda)$
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<u>autput</u> : the most likely sequence of states Q=q.qzqt given the observation sequence O. The solution can be found using the <u>Viterbi Algorithm</u>
Problem 3: The Learning Problem * not too worried about this problem in this class
input: observation sequence of
output: find the most likely HMM defined by $\lambda = \{A, B, IL\}$ given O
Solution to problem 1 ex: $absolutions: 0 \ a \ b \ c \ c \ \Rightarrow O = 6$
if we have N states, there are N ¹⁶ possible sequences of states underlying, the observation sequence C
lut's dutive a few hundamental probabilities:
$ \mathbb{P} \left[\text{abserving O. given. a sequence of states Q: } \mathbb{P} \left[O Q \right] = \mathbb{P} \left(O_1 Q_1 \right) \mathbb{P} \left(O_2 Q_2 \right) \dots \mathbb{P} \left(O_T Q_T \right) \right] $
$s \ b_{q_1}(o_1) \ b_{q_2}(o_2) \ \dots \ b_{q_n}(o_n)$
$\mathbb{R}[\operatorname{naving} a \operatorname{particular} \operatorname{sequence} oc. \operatorname{states} Q]$; $\mathbb{R}[Q]$ = $\mathbb{T}_{q_1} Q_{q_2} Q_{q_2} Q_{q_2} Q_{q_3} \cdots \tilde{Q}_{q_{T-1}} Q_{T}$
Joint probability of Q and Q: R[O, Q] = R[O[Q] · R[Q]
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