# Verification and invariants

*What are some limitations of software testing?*

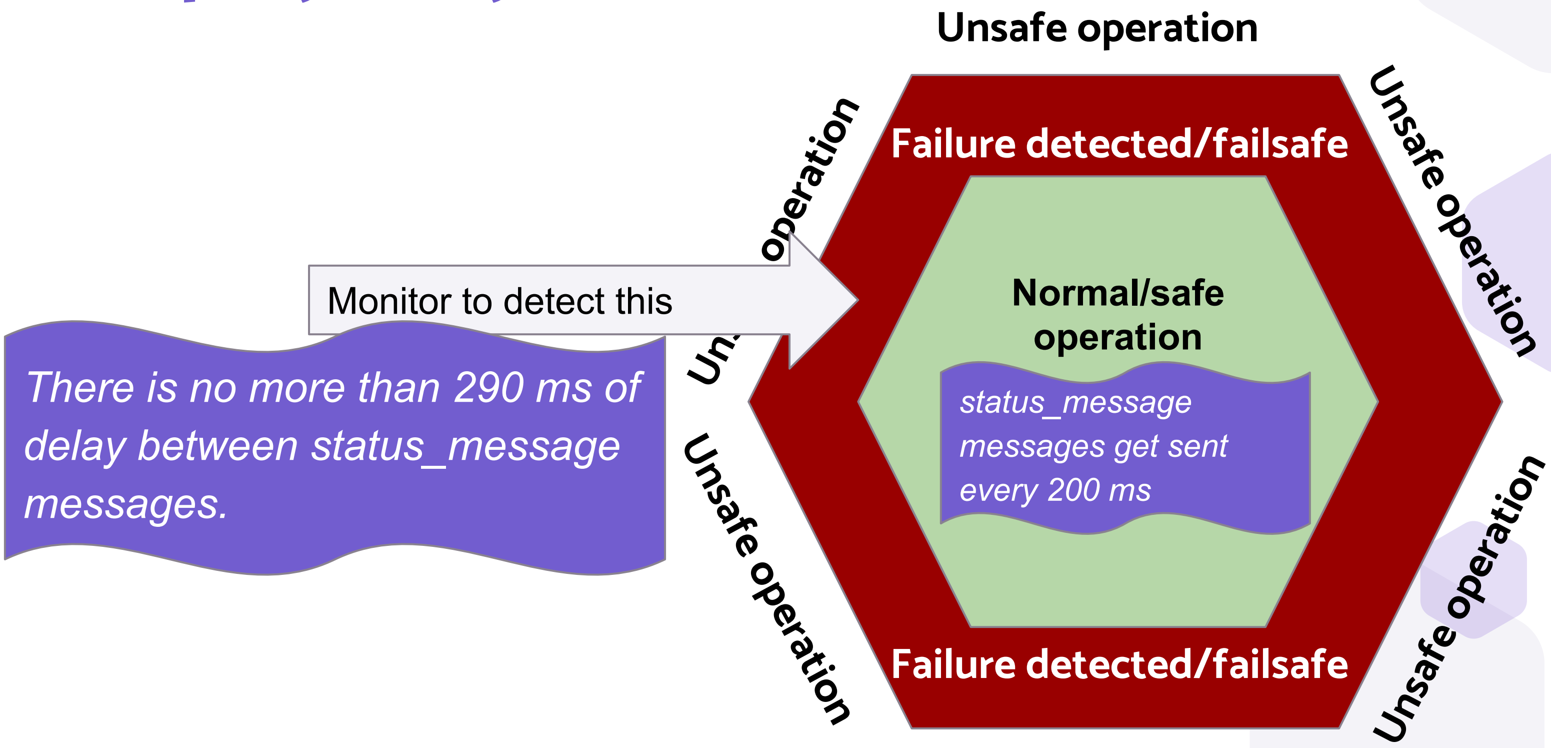
# Invariant

Invariant: some computable property of a system that we want to show always holds *(more precise definition later)*

# Working with invariants

- Runtime monitoring on a deployed system
- Testing (simulation or system logs)
- …formally proving?

# Runtime monitoring on a deployed system



# Invariants for testing

Our basic understanding of testing so far has been largely **transactional:**

- Give input, observe that output matches what is expected

Are embedded systems transactional?

- Robot asked to navigate to a goal point

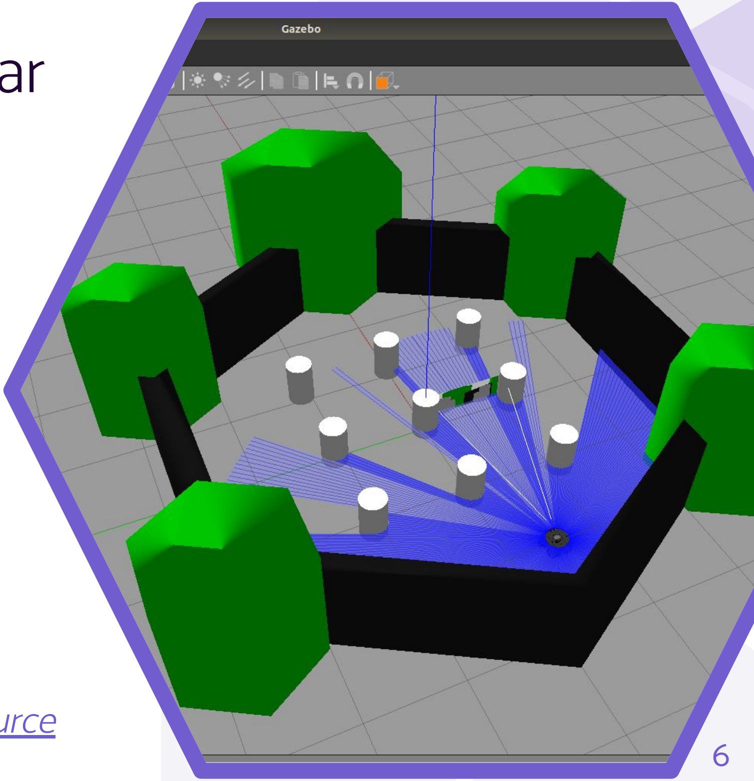



*Image source*

# Safety properties and invariants

Invariant: some computable property of a system that always holds

Safety property (or safety requirement): assertion that nothing bad ever happens

*How are invariants and safety properties related?*

# Safety properties can be expressed as invariants

Define "bad thing" computably

Invariant: not(bad thing) always holds

**Example for AC from lab 8**

1) *There is no more than 290 ms of delay between status_message messages.*

→ "bad thing": two consecutive status_message messages come more than 290 ms apart

→ invariant: bad thing is not true

→ your monitor checked if the invariant always held

# Another example from lab 8

*8) If the system is on and the control knob hasn't changed for 290 ms, the desired temperature as sent by status message obeys the formula 5400 + 25 * (control knob reading) / 8 with an error of at most 3 degrees F (300 centidegrees).*

What is the “bad thing?”

# Formalizing invariants

...back to FSMs!

Board discussion: reachability

# Propositional logic

Composed of terms (“a”, “b”, “c”), where a term can be:

**p(x), q(x), r(x,y)**: propositions (evaluate to either true or false)

x > 0

x + y = 2

robot x has not hit obstacle y

fsm x is in state y *(abbreviated as y if y is a state)*

**a ∧ b** : a and b (true if term a is true and term b is true)

**a ∨ b**: a or b (true if term a is true or term b is true or both)

**¬a**: not a (true if term a is false)

**a ⇒ b**: a implies b (true if term b is true or if term a is false)

# Formal definition of an invariant

A property *p* of a transition system* *S* is an *invariant* of *S* if every reachable state of *S* satisfies *p*

**For our class, think of a transition system as an FSM*

*[Alur, chapter 3]*

# Inductive invariants

A property $p$ of a transition system $S$ is an *inductive invariant of* $S$ if:

1. The initial state $s$ satisfies $p$, and
2. If a state $s$ satisfies $p$, and $(s, t)$ is a transition, then the state $t$ also satisfies $p$

(Board discussion: Prove $(x >= 0 \land y > 0) \lor (x > 0 \land y >= 0)$)

# Proving non-inductive invariants

To establish that a property *p* is an invariant of the transition system *S*, find a property *q* that:

1. *q* is an inductive invariant of *S*, and
2. the property *q* implies the property *p* (that is, a state satisfying *q* is guaranteed to satisfy *p*)

(Board discussion: Prove B => x > 0 ^ y > 0)

# How would you deal with this invariant?

*8) If the system is on and the control knob hasn’t changed for 290 ms, the desired temperature as sent by status message obeys the formula 5400 + 25 * (control knob reading) / 8 with an error of at most 3 degrees F (300 centidegrees).*

# Stateful invariants

For a transition system *S*, Create a *safety monitor FSM* called *M* where:

- inputs of *M* are a subset of the inputs and outputs of *S*
- Some subset *E* of the states of *M* are designated as “error” states
- The behavior of *M* is designed such that if the sequence of inputs to *M* leads *M* to an error state in *E*, this is an invariant violation

Compose *M* and *S*. The invariant becomes that any state in *E* is not reachable

*What similarities do you see between the safety monitor FSM definition and the runtime monitor you wrote in lab 8?*