

# Linear Temporal Logic





# AC Liveness Requirements

**Safety:** nothing bad ever happens

**Liveness:** something good eventually happens

Eventually reaching the desired temperature (as long as desired temperature hasn't changed and the system is on)

Eventually a message gets sent (or saying messages keep being sent eventually)

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*How would you **monitor** that  
a liveness requirement is  
fulfilled?*



## Verifying some liveness properties

Saying something *eventually* happens is the same thing as saying that it is *not* the case that it always *doesn't* happen

Can we use invariant verification to check this?

# Linear Temporal Logic (LTL)

Assume you have *some* execution trace

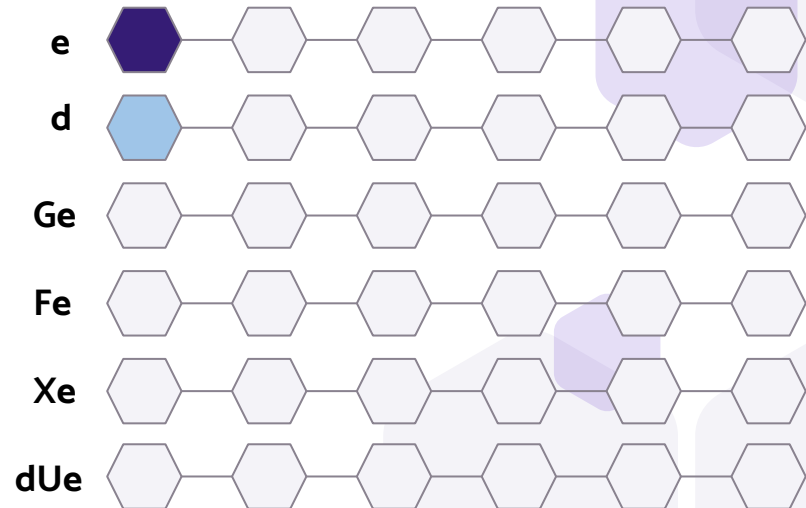
LTL operators are propositional logic operators PLUS:

G (globally/always)

F (eventually/finally)

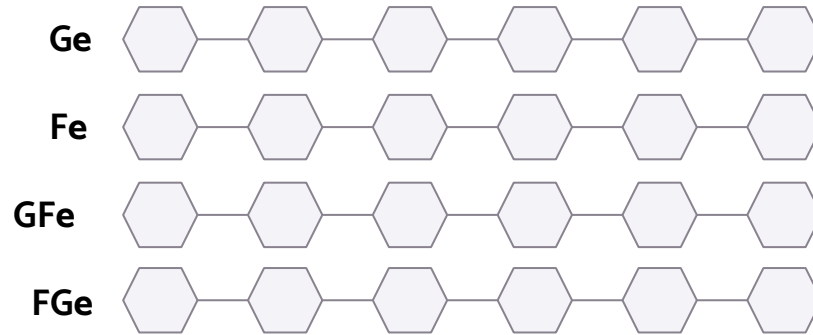
X (next state)

U (until)





# FG vs G, GF vs F, FG vs GF





## LTL examples on FSMs

Safety property is an *invariant* if property  $p$  holds for all reachable states of  $S$

Liveness property *holds* for  $S$  if it holds for all possible traces of  $S$

Lee/Seshia Chapter 13, exercise 2

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*LTL means we can specify  
liveness properties with  $F$ .  
Can we specify safety  
properties more easily with  
LTL, too?*





## Safety properties with LTL

Use “G” to say a property holds for every state

Can use “X” to express statefulness/history  
without a monitor state machine



## Limits of LTL

$G(\text{even}(x) \rightarrow ((X\text{-even}(x)) \wedge (XX\text{even}(x))))$

But if you don't know if you started a sequence with an odd number or even number, you cannot write

$(\text{even}(x) \wedge X\text{-even}(x))$



## Repeatability

A property  $p$  over the state variables of a transition system  $S$  is said to be *repeatable* if there exists *some* trace  $q$  of  $S$  such that  $q$  satisfies the recurrence LTL-formula  $GFp$ .



## Buchi automata

Automata which “accept” a given LTL formula  
(see Alur’s textbook for examples)



# Automated LTL verification

Buchi automata construction can be automated  
(discussion in optional reading)

Compose Buchi automata of negated property  
with system and produce a counter-example of  
repeatability to prove property

Done using a DFS + cycle detection (“nested DFS”)



# More verification techniques

## Automated verification

Symbolic model checking: represents a set of states symbolically as a logic formula and does symbolic (algebraic) computation

## What about timed/hybrid automata?

Symbolic model checking for a different kind of logic (signal temporal logic)

Assisted proof engines (differential dynamic logic)

An active area of research!

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*Summary: pros/cons of  
verification?*