

# Modeling





## Context

Embedded systems involve SW + HW

We used FSM-based design for software

- Translates pretty easily to code

- Guides unit testing

- Helps us reason about the system?

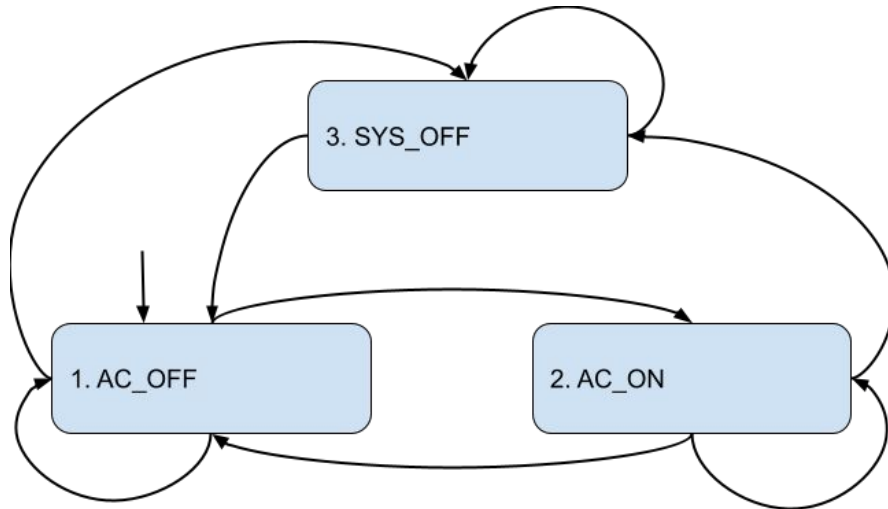
**Can we expand that reasoning? Can we incorporate HW/timing into that reasoning?**



# FSMs as models

FSM describes behavior of the system

Abstracts away some aspects of the system



“

*What do you think when you  
hear “all models are wrong,  
some models are useful”*

?



# Formalizing FSMs

We handwaved some aspects of FSMs

- Role and behavior of inputs and outputs

- Presence/absence of self-loops

- Distinction between FSMs and extended FSMS

# Back to the formal definition

(Lee/Seshia 3.3.3)

An FSM is a 5-tuple: (States, Inputs, Outputs, update, initialState)

- States is a finite set of states
- Inputs is a set of input valuations
- Outputs is a set out output valuations
- update: States  $\times$  Inputs  $\rightarrow$  States  $\times$  Outputs is an update function
- initialState is the initial state

Valuation: a set of values that a signal can take on or the assertion that the value is absent

**Numerical** signals:  $\mathbb{R} \cup \{\text{absent}\}$  or  $\mathbb{N} \cup \{\text{absent}\}$

**Pure** signals:  $\{\text{present}, \text{absent}\}$

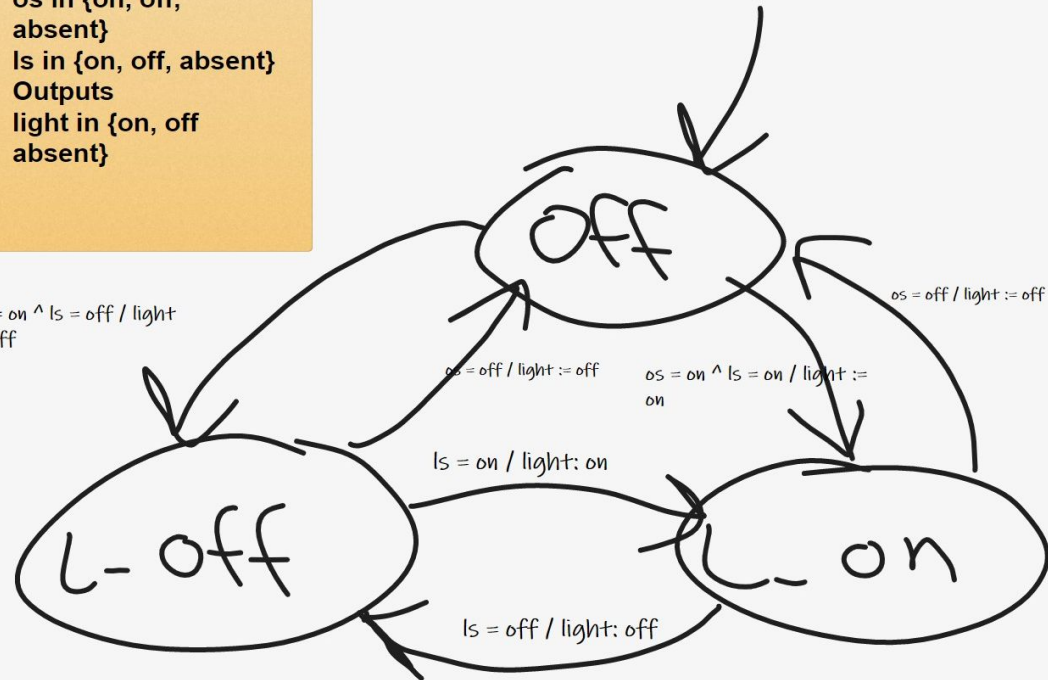
**Categorical** signals: examples  $\{1, 2, 3, \dots, 8, \text{absent}\}$  or  $\{\text{up}, \text{down}, \text{left}, \text{right}, \text{absent}\}$

# Board example: system with two switches

System with two switches and one indicator light  
on/off switch: if off, no response to light switch and light is off  
light switch: light on if on, off if off

Inputs  
os in {on, off, absent}  
ls in {on, off, absent}  
Outputs  
light in {on, off, absent}

$os = on \wedge ls = off / \text{light} := off$

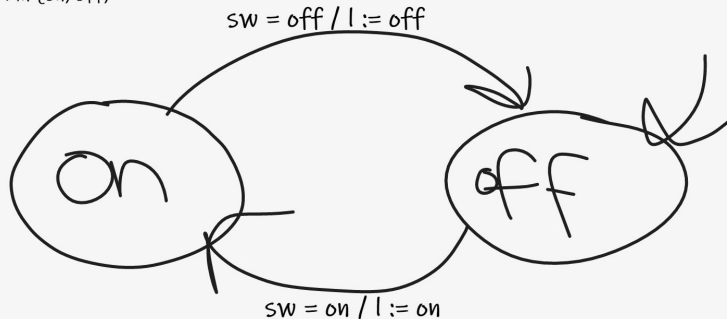


# Self loops, present/absent I/O

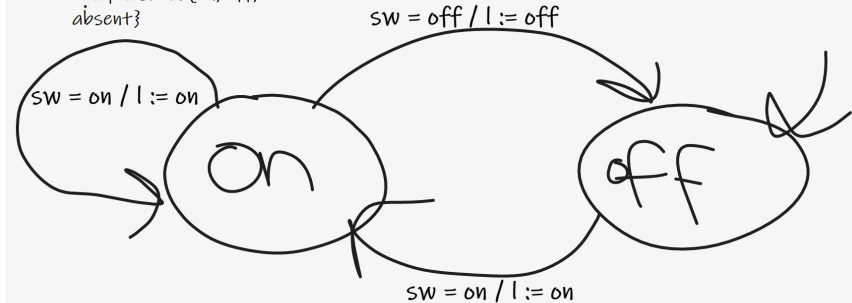
Difference between the below two FSMs?

FSM on the right: sw keeps being read and output keeps being asserted

Inputs: sw in {on, off, absent}  
Outputs: l in {on, off, absent}



Inputs: sw in {on, off, absent}  
Outputs: l in {on, off, absent}







## Keeping track of data

An FSM is a 5-tuple: (States, Inputs, Outputs, update, initialState)

How do we keep track of internal data?

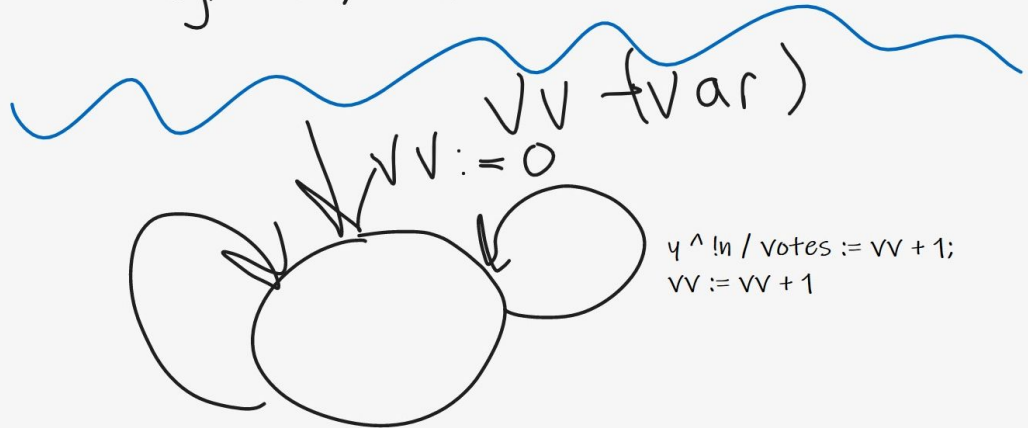
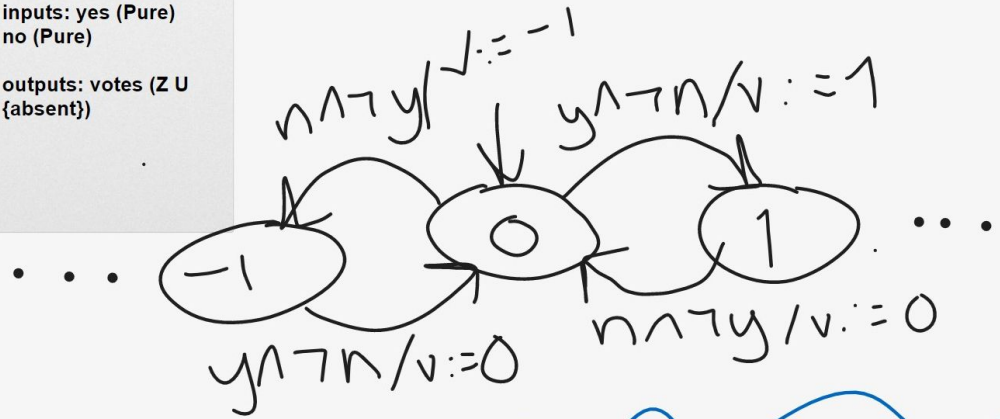
Example: system with yes/no vote buttons, keep track of difference in votes (board example)



# Vote buttons

inputs: yes (Pure)  
no (Pure)

outputs: votes (Z U  
{absent})



# FSM vs Extended SM

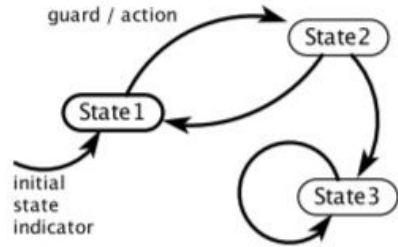


Figure 3.3: Visual notation for a finite state machine.

variable declaration(s)  
input declaration(s)  
output declaration(s)

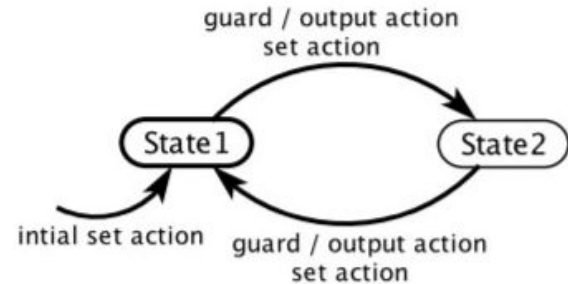


Figure 3.9: Notation for extended state machines.

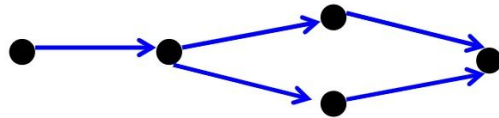
“

*What are we missing out on  
when we tell time by using  
“mils” as an input?*

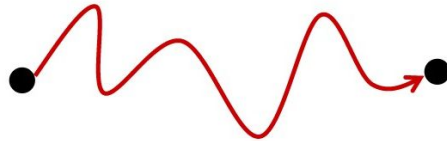


# Hybrid systems

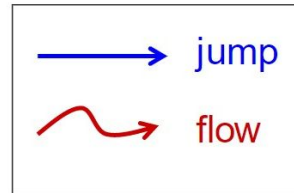
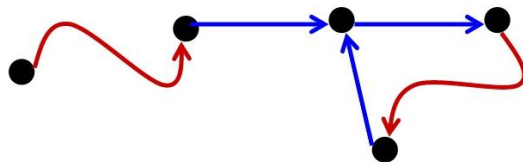
Discrete System (FSM)



Continuous System



Hybrid System



Slide from Prabal Dutta and  
Sanjal A. Seshia, 2019

# Timed automata

Distinction between discrete and continuous variables

Continuous behavior defined in “states”

Now called “modes”

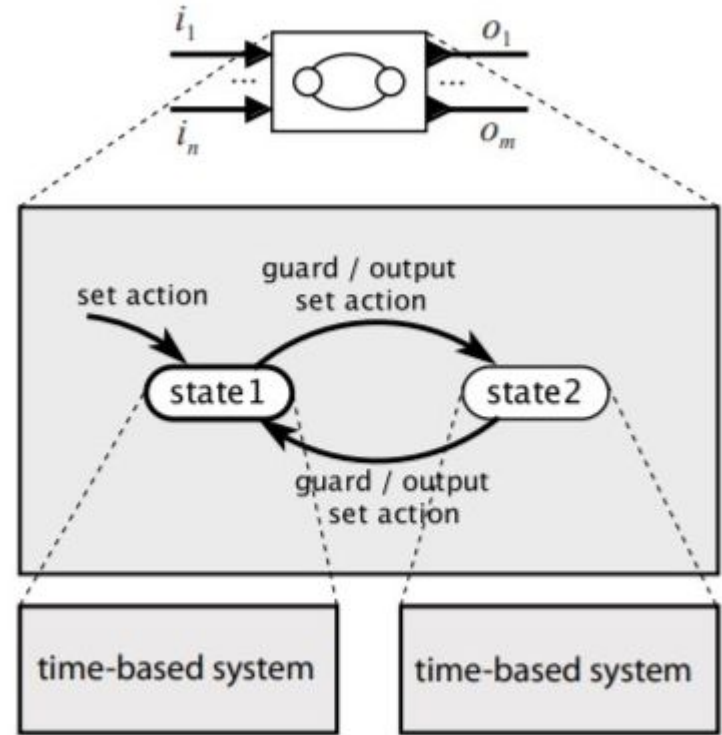


Figure 4.4: Notation for hybrid systems.



# ODEs

Sometimes it is more desirable to describe a variable in terms of how it changes rather than its explicit form

Useful for: modeling, reasoning

Define a function in terms of its derivative and possibly initial conditions

Ordinary Differential Equation, or ODE

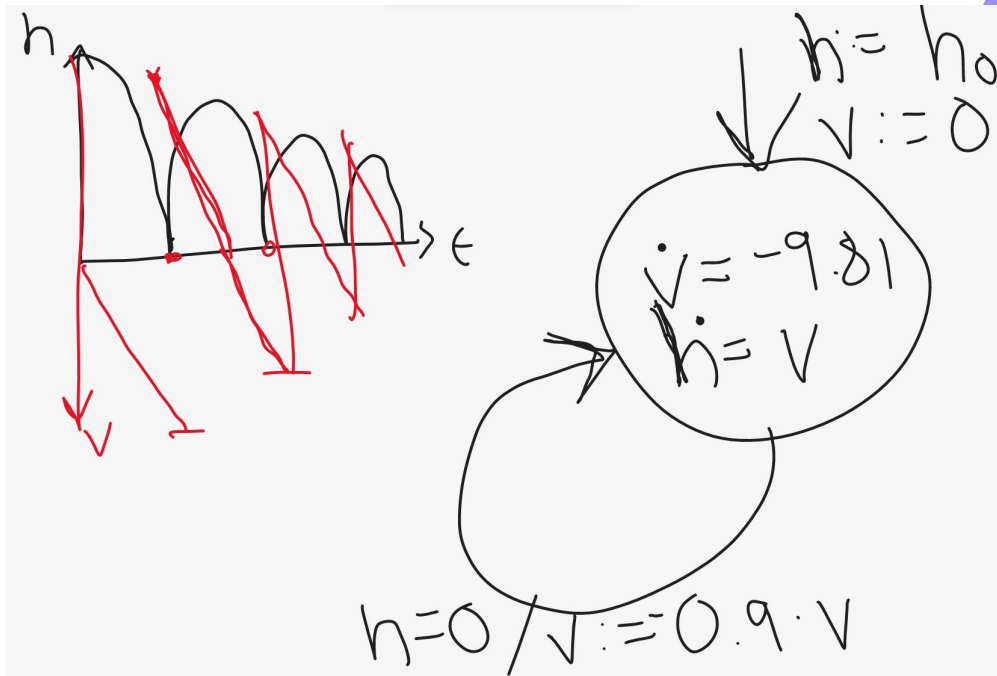
Solving general ODEs is beyond the scope of the class, but we will discuss some patterns here



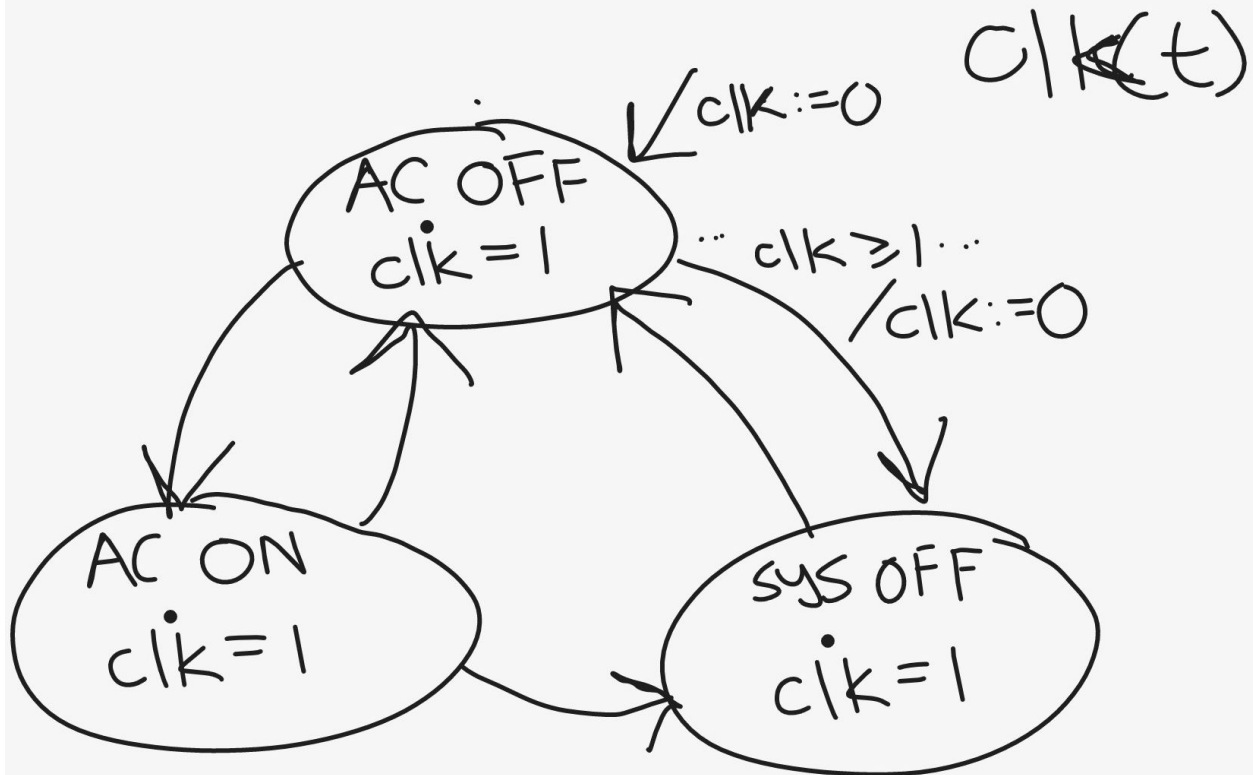
# Discussion of homework problems



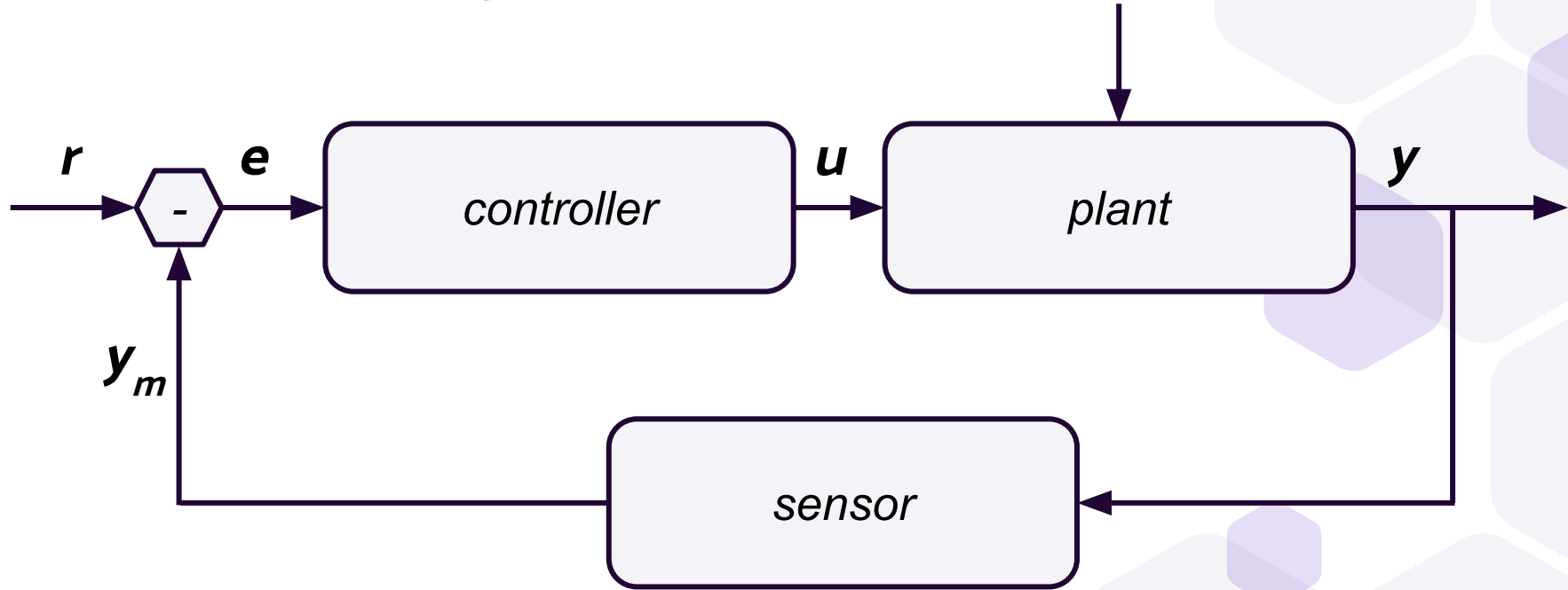
## Example: bouncing ball



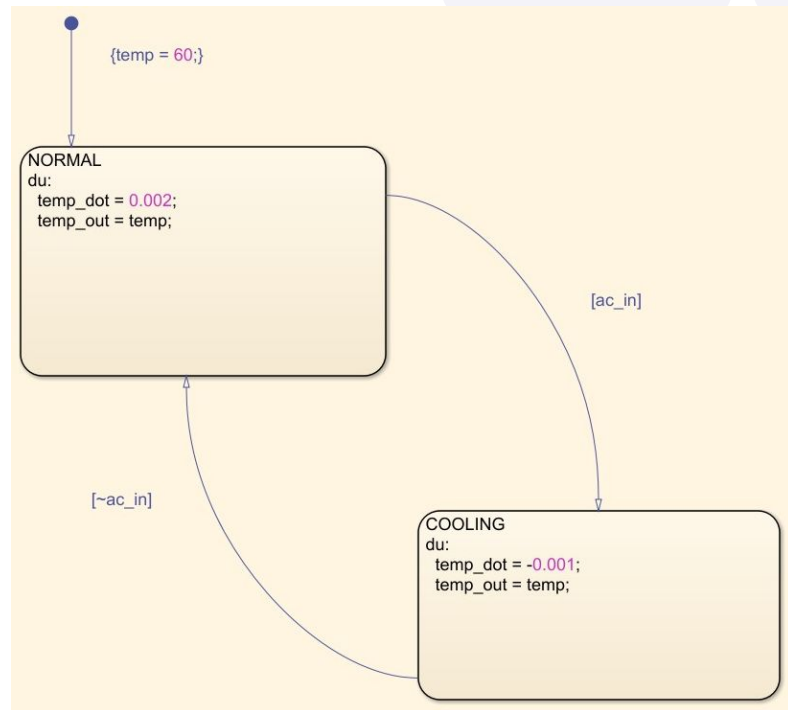
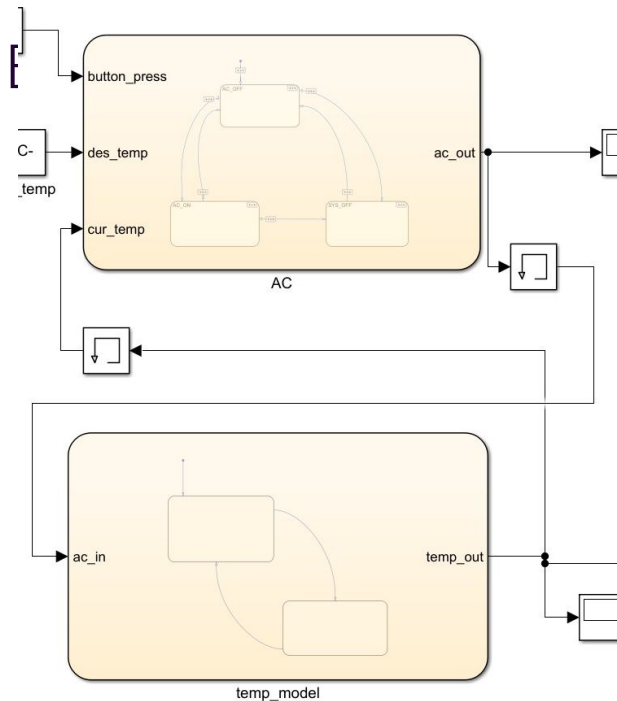
## Air conditioner with continuous time



# Feedback loops and control

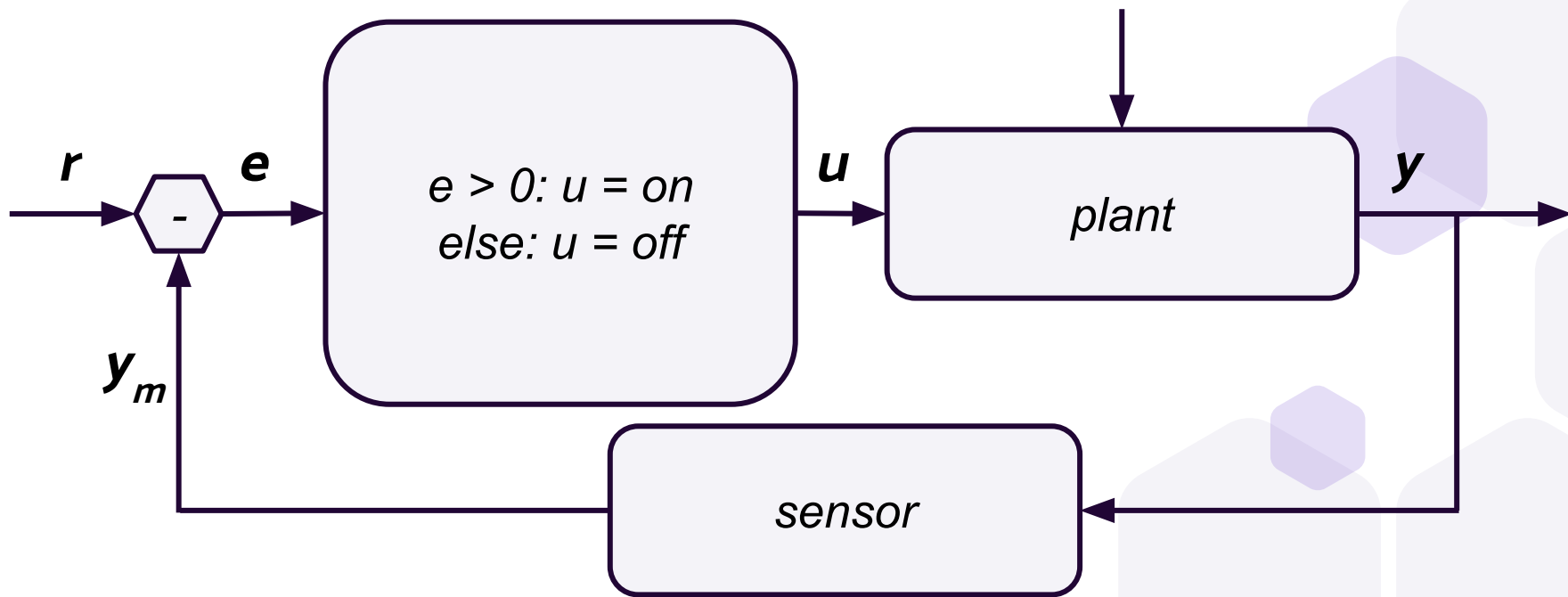


# Air conditioner temperature model



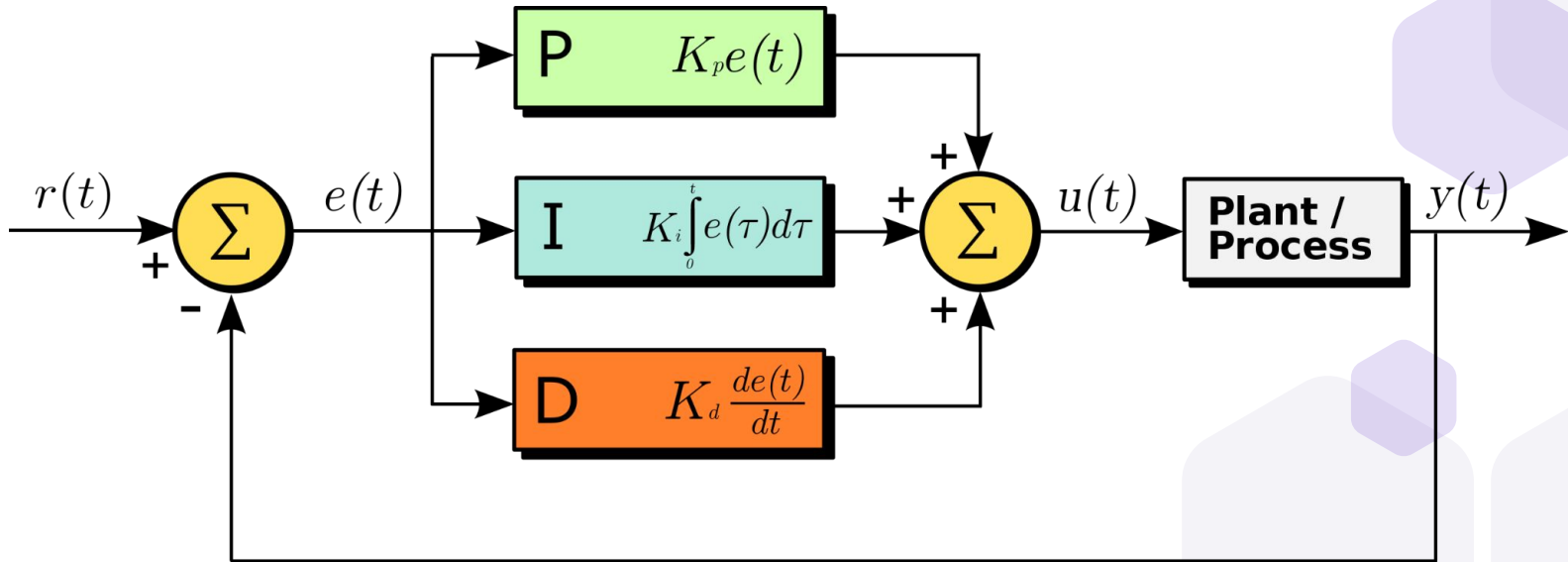
# Bang-Bang controller

Controller output is 2-state ({on, off}, {up, down}, etc)

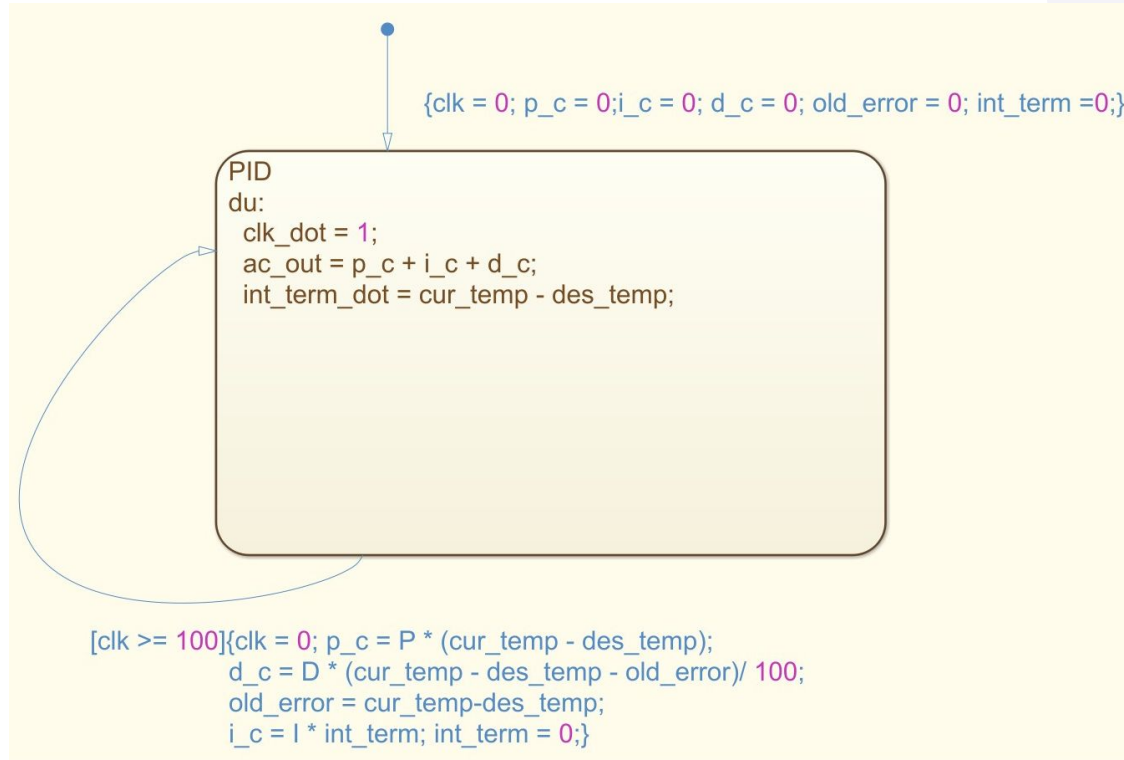


# PID controller

For continuous controller outputs



# Board and simulink discussion of PID



## Bonus/if time: composition of automata

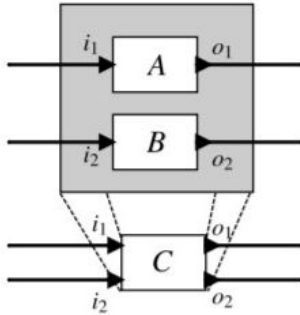


Figure 5.2: Side-by-side composition of two actors.

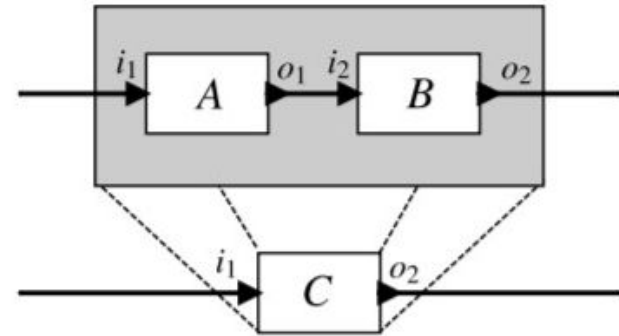


Figure 5.7: Cascade composition of two actors.



## Bonus/if time: feedback loops in automata

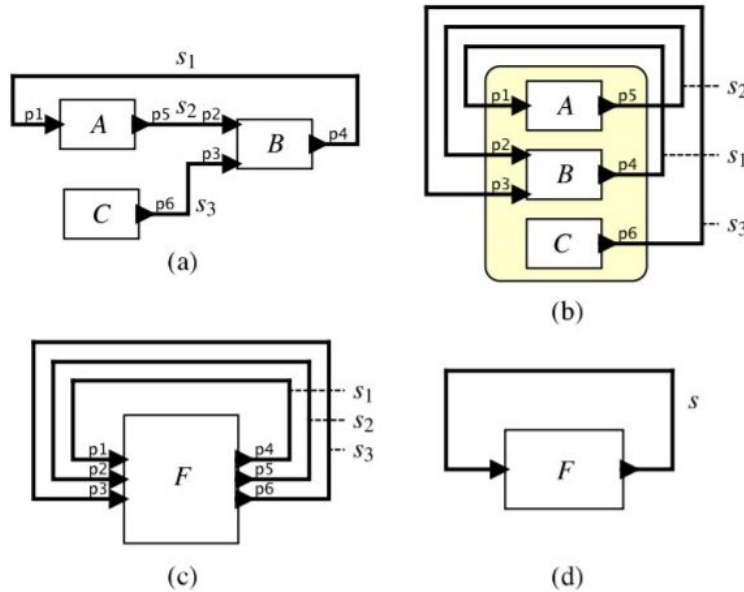


Figure 6.1: Any interconnection of actors can be modeled as a single (side-by-side composite) actor with feedback.