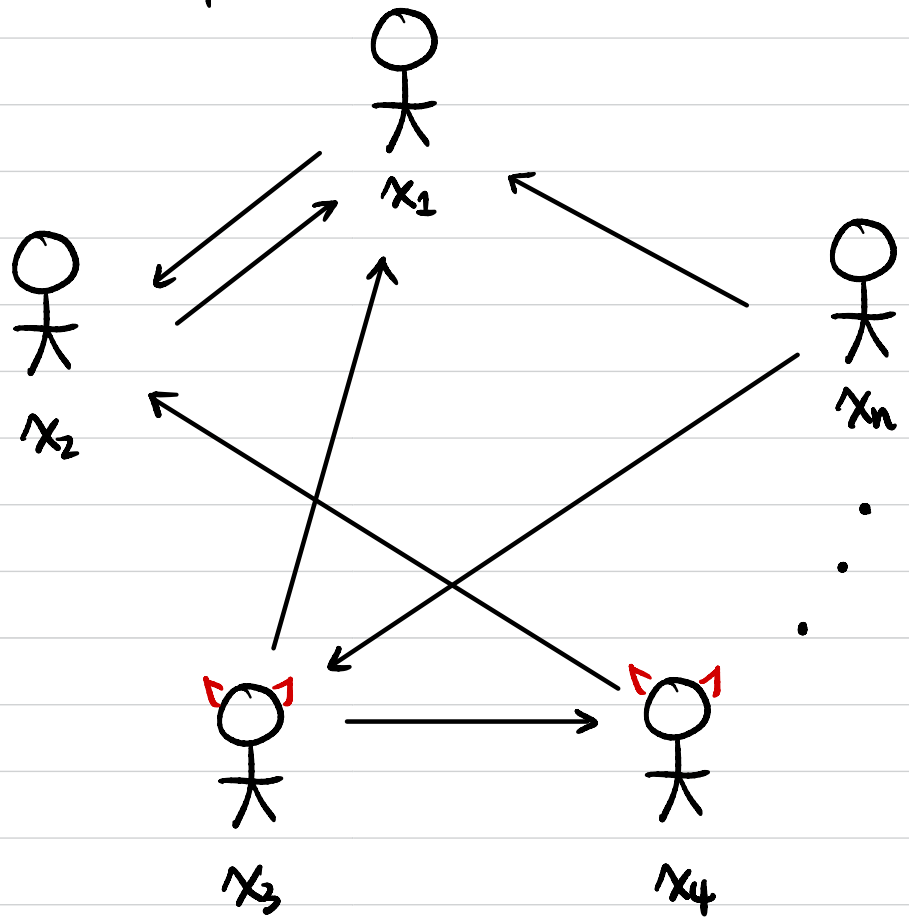
CSCI 1515 Applied Cryptography

This Lecture:

- GMW: Semi-Honest MPC for Any Function (continued)
- GMW Compiler: Malicious MPC for Any Funtion
- Privacy-Preserving Machine Learning

Secure Multi-Party Computation (MPC)

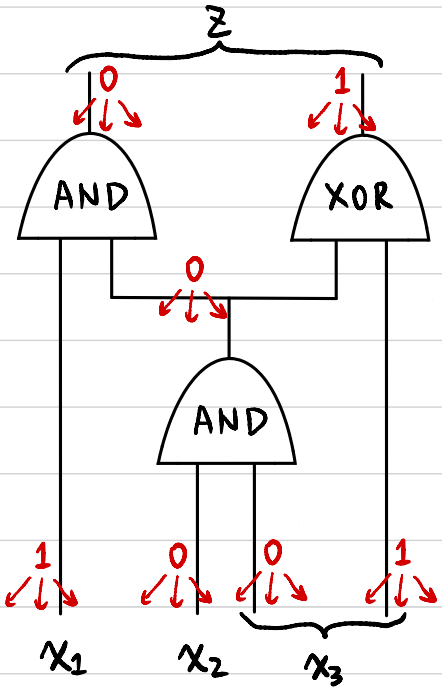


$$z = f(x_1, \cdots, x_n)$$

Allowed adversarial behavior:

- **Semi-honest:** Follow the protocol description honestly.
- **Malicious:** Can deviate arbitrarily from the protocol description.

MPC for any function with $t \le n-1$ (GMW)



Each party P_i holds a random share $v_i^w \in \{0,1\}$ s.t. $\bigoplus_{i=1}^{n} v_i^w = v^w$

Inputs:

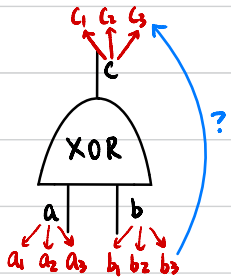
For each input wire w:

If it's from party P_k with input value $v^w \in \{0,1\}$,

P_k randomly samples $v_i^w \xleftarrow{\$} \{0,1\}$ s.t. $\bigoplus_{i=1}^{n} v_i^w = v^w$

↳ Sends v_i^w to party P_i.

XOR gates:

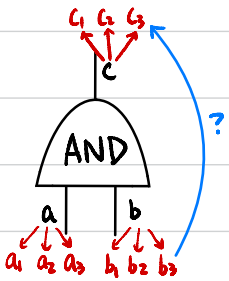


GIVEN: $\bigoplus_{i=1}^{n} a_i = a$ $\bigoplus_{i=1}^{n} b_i = b$

WANT: $\{c_i\}$ s.t. $\bigoplus_{i=1}^{n} c_i = c = a \oplus b$

$c_i = a_i \oplus b_i$

AND gates:



GIVEN: $\bigoplus_{i=1}^{n} a_i = a$ $\bigoplus_{i=1}^{n} b_i = b$

WANT: $\{c_i\}$ s.t. $\bigoplus_{i=1}^{n} c_i = c = a \cdot b$

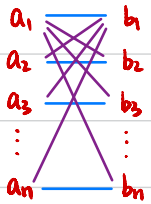
$c_i = a_i \cdot b_i \oplus \sum_{j \neq i} r_{i,j}^{(1)} \oplus \sum_{j \neq i} r_{j,i}^{(2)}$

$$a \cdot b = \left(\sum_{i=1}^{n} a_i\right) \cdot \left(\sum_{i=1}^{n} b_i\right) \mod 2$$

$$= \left(\sum_{i=1}^{n} a_i \cdot b_i\right) + \left(\sum_{i \neq j} a_i \cdot b_j\right) \mod 2$$

P_i locally

Reshare $r_{i,j}^{(1)}$ $r_{i,j}^{(2)}$



Outputs:

For each output wire w:

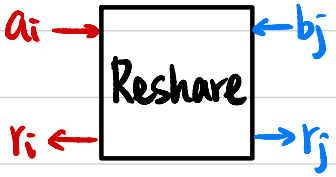
Each party P_i holds a random share $v_i^w \in \{0,1\}$

↳ Sends v_i^w to all parties

Each party computes the value $v^w = \bigoplus_{i=1}^{n} v_i^w$

MPC for any function with $t \le n-1$ (GMW)

Reshare:



P_i

Input: $a_i \in \{0,1\}$

P_j

Input: $b_j \in \{0,1\}$

Output: $r_i \in \{0,1\}$

Output: $r_j \in \{0,1\}$

WANT: Random $r_i, r_j \in \{0,1\}$ s.t. $r_i \oplus r_j = a_i \cdot b_j$

1) P_i randomly samples $r_i \xleftarrow{\$} \{0,1\}$

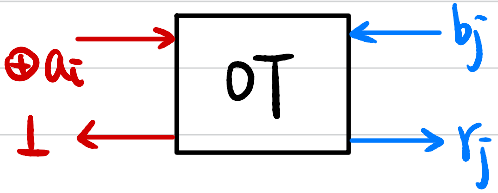
2) How to let P_j learn r_j s.t. $r_i \oplus r_j = a_i \cdot b_j$?

If $b_j = 0 \Rightarrow r_j = r_i$

If $b_j = 1 \Rightarrow r_j = r_i \oplus a_i$

$m_0 = r_i$

$m_1 = r_i \oplus a_i$

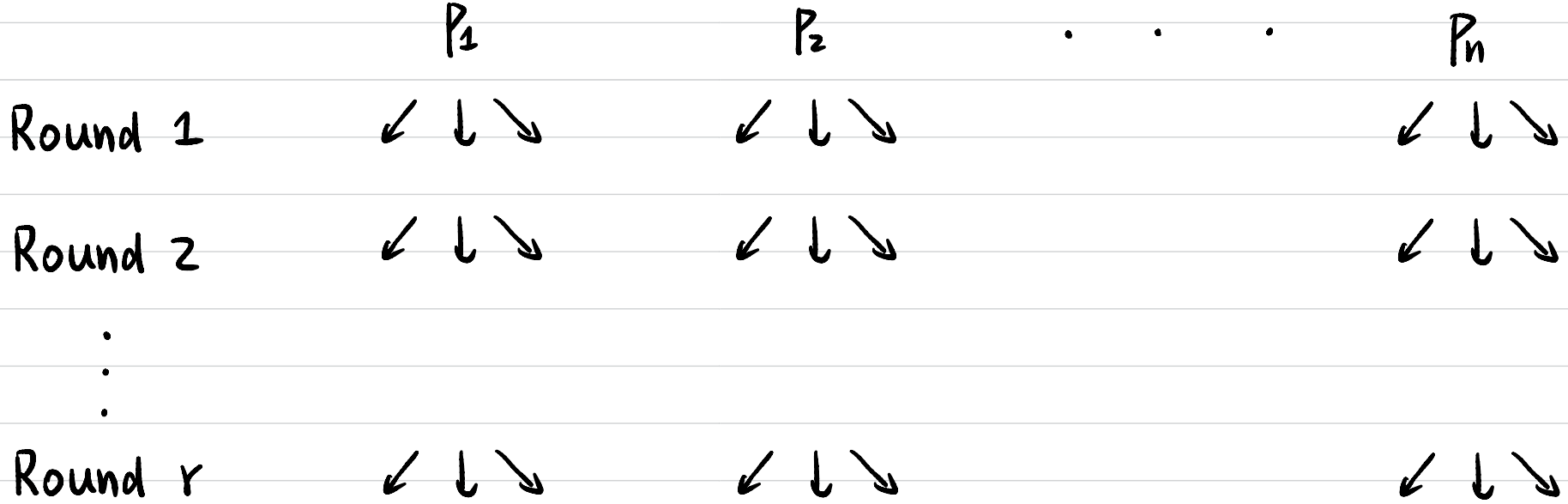


MPC for any function with $t \leq n-1$ (GMW)

Computational Complexity?

Communication Complexity?

Round Complexity?



What could go wrong against malicious adversaries?

GMW Compiler

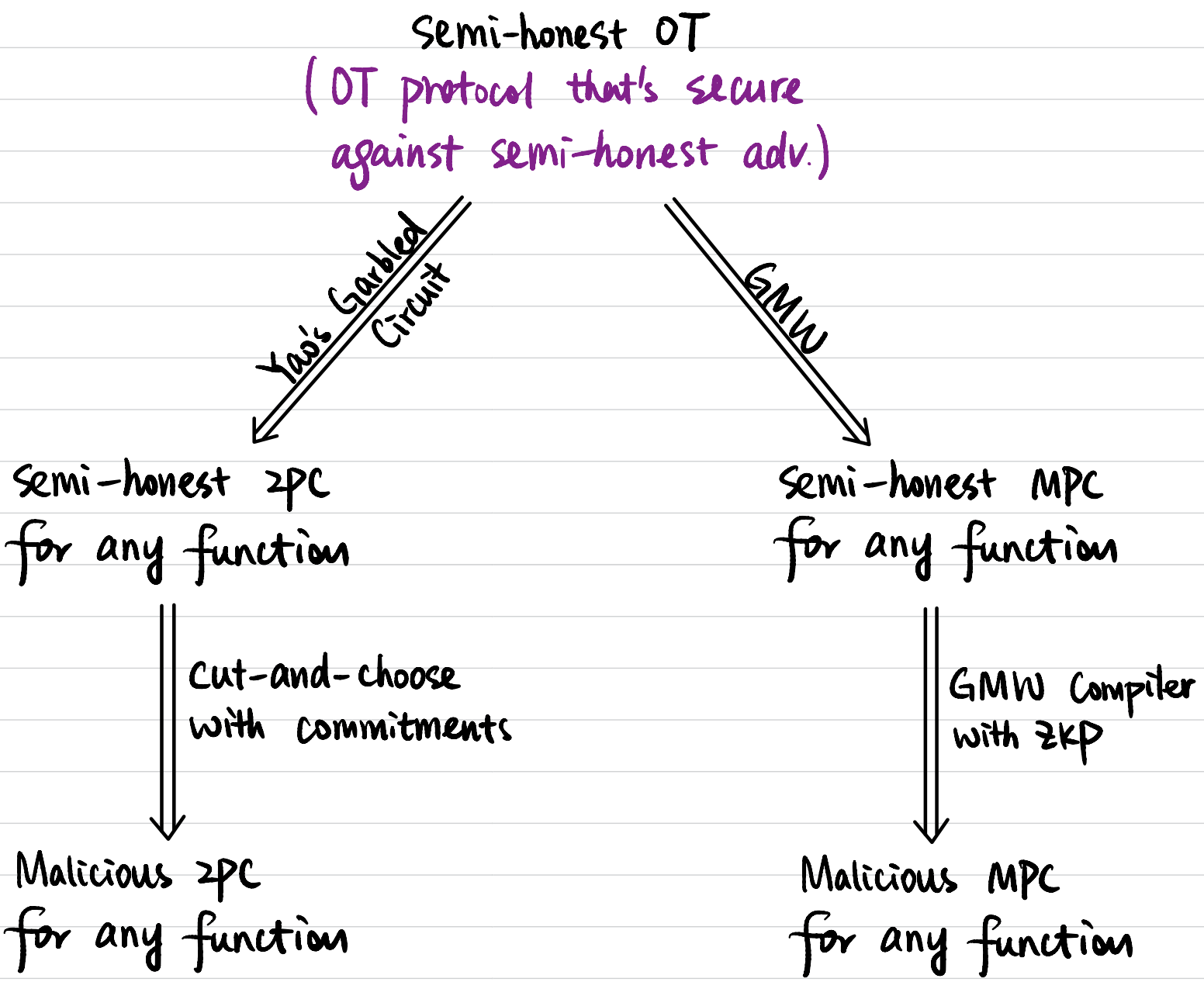
Given a semi-honest protocol:

Once inputs & randomness are fixed, protocol is deterministic.

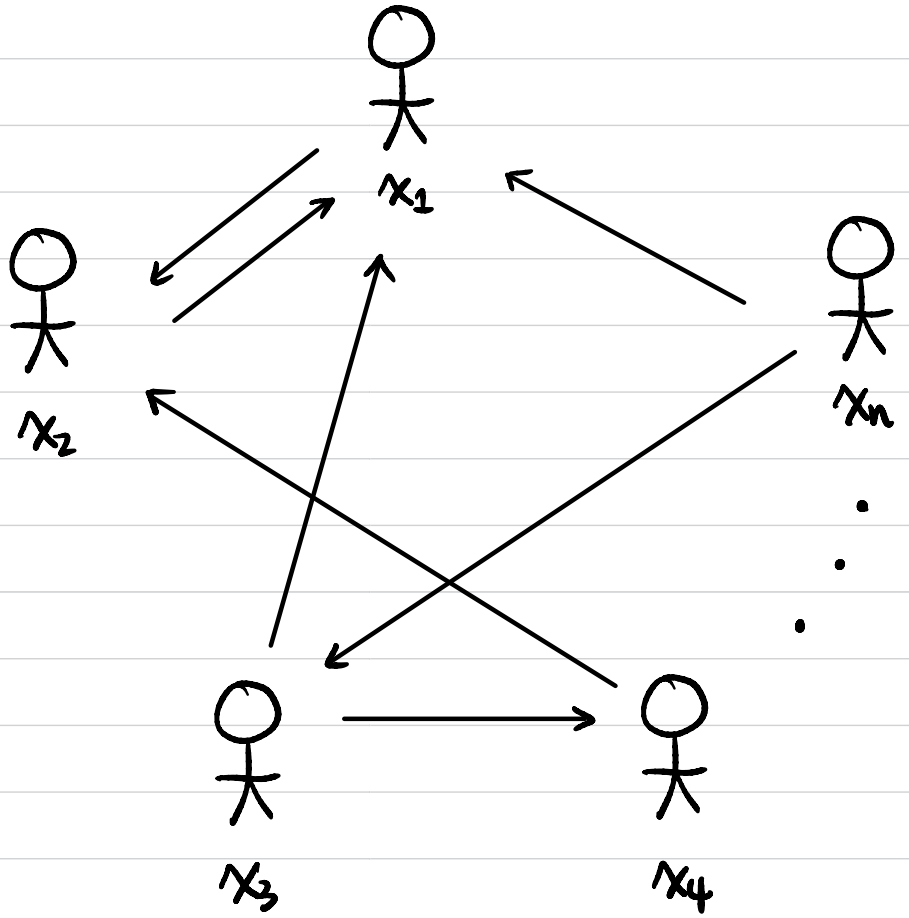
Step 1: Each party P_i commits to its input x_i & randomness r_i to be used in the semi-honest protocol.

Step 2: Run semi-honest protocol.

Along with every message, prove in ZK that the message is computed correctly (based on its input, randomness, transcript so far)


Semi-honest OT
(OT protocol that's secure against semi-honest adv.)
Yao's Garbled Circuit
GMW
Semi-honest 2PC for any function
Semi-honest MPC for any function
Cut-and-choose with commitments
GMW Compiler with ZKP
Malicious 2PC for any function
Malicious MPC for any function

Privacy-Preserving Machine Learning (PPML)



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ML model

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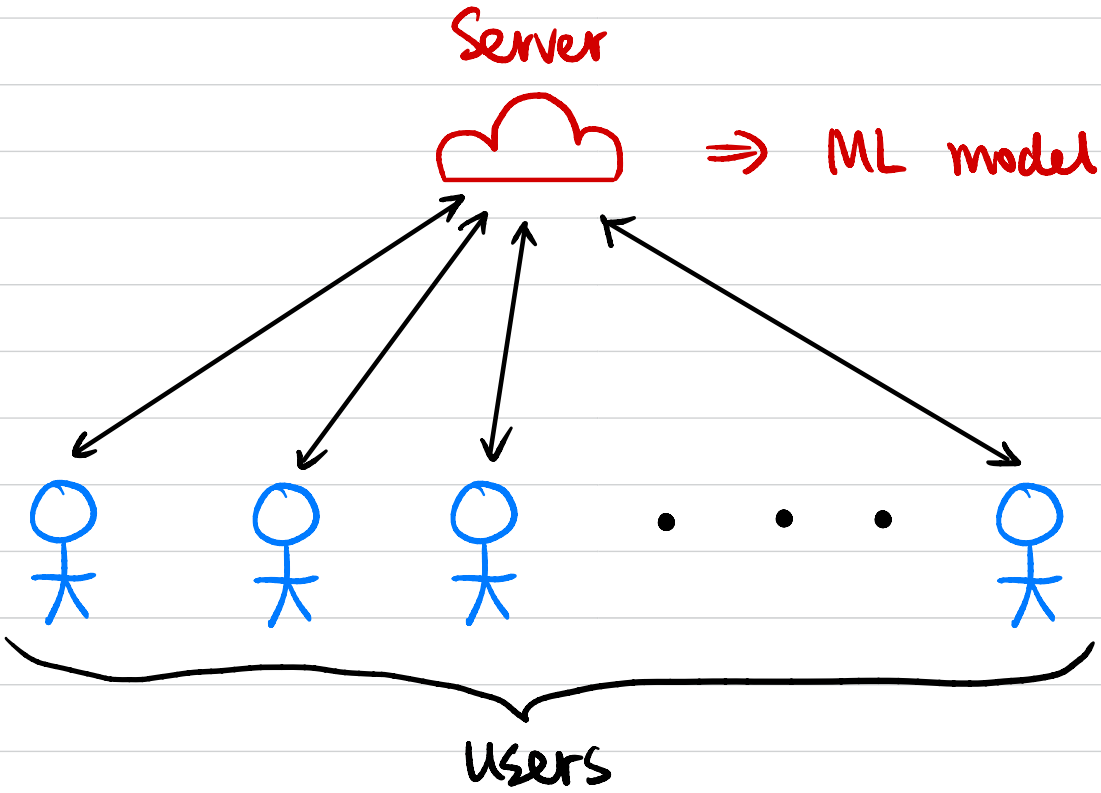
ML inference

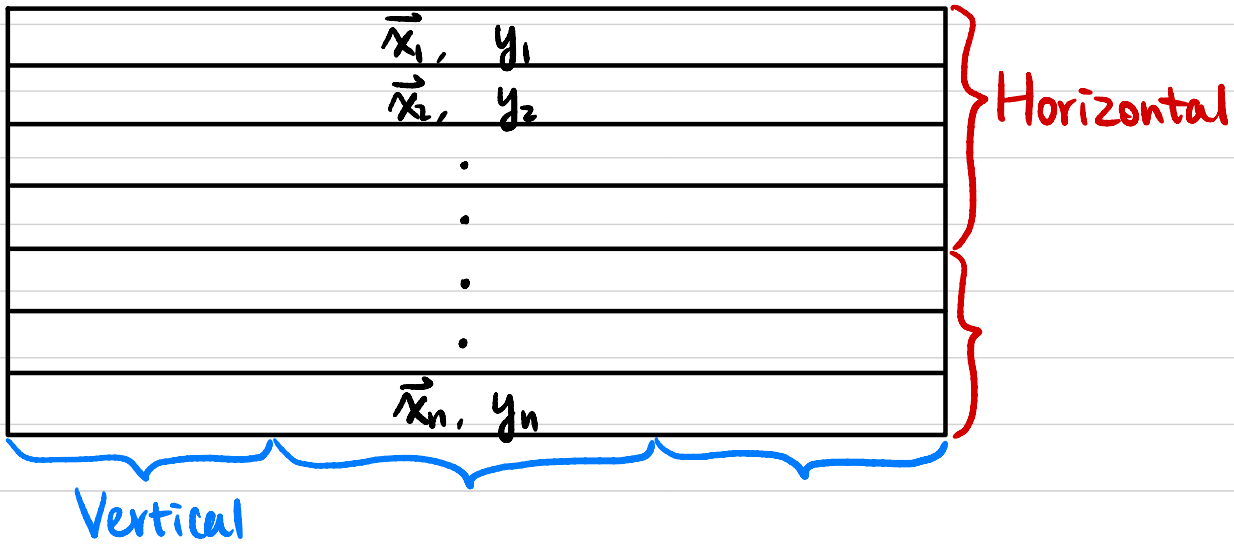
Federated Learning (FL)



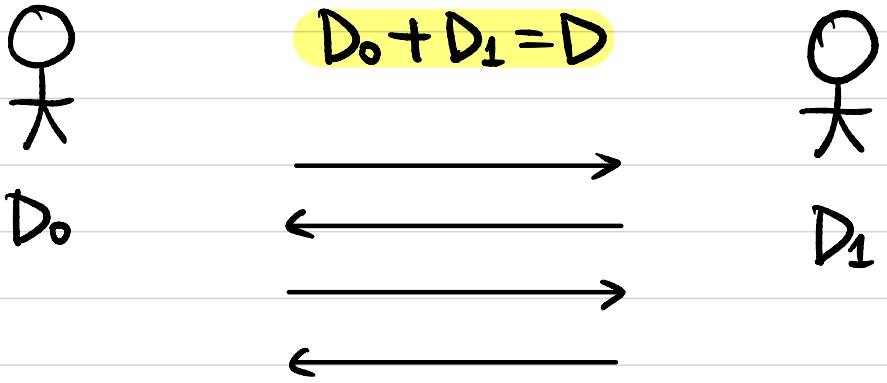
Application: Google mobile keyboard prediction

Privacy-Preserving Machine Learning (PPML)

Horizontal / Vertical Data Partitioning



PPML Framework



Invariant: Two parties hold secret shares of all inputs & intermediate values.

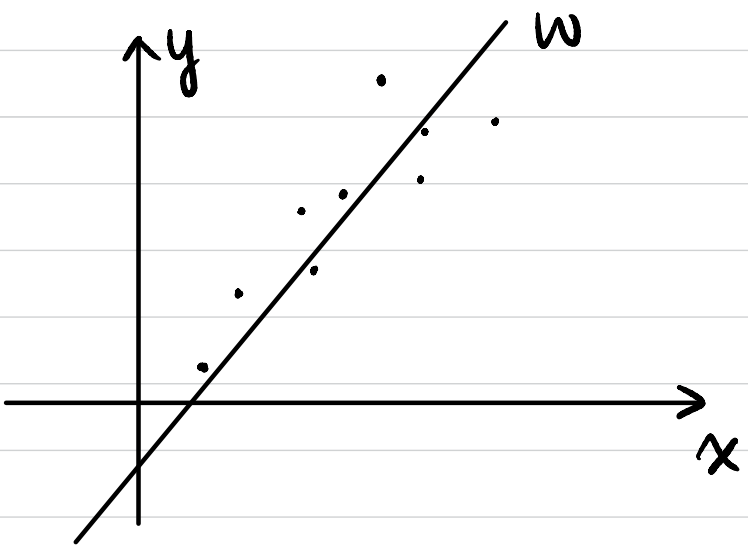
Machine Learning Background

Linear Regression

Data Points $(\vec{x}, y)$

ML Model: coefficient vector $\vec{w}$

$$g(\vec{x}) = \langle \vec{x}, \vec{w} \rangle$$



For each data point $(\vec{x}_i, y_i)$,

Define loss function

$$L_i(\vec{w}) := \frac{1}{2}(\langle \vec{x}_i, \vec{w} \rangle - y_i)^2$$

Total loss: $L(\vec{w}) := \frac{1}{N} \sum_{i \in [N]} L_i(\vec{w})$

Goal: Find $\vec{w}$ that minimizes $L(\vec{w})$.

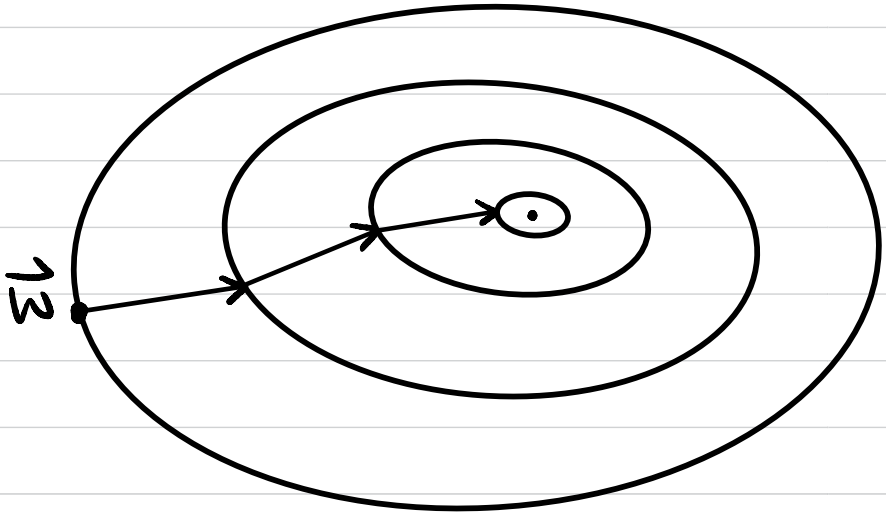
Machine Learning Background

Stochastic Gradient Descent (SGD)

- $\vec{w}$ initialized with arbitrary value
- Given a data point $(\vec{x}_i, y_i)$:

$$\vec{w} \leftarrow \vec{w} - \eta \cdot \nabla L_i(\vec{w})$$

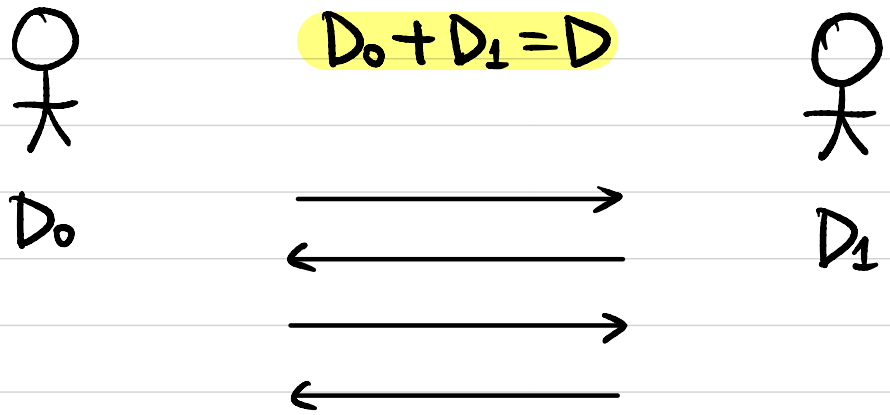
learning rate, gradient



$$\vec{w} \leftarrow \vec{w} - \eta \cdot (\underbrace{\langle \vec{x}_i, \vec{w} \rangle}_{y_i^* = \langle \vec{x}_i, \vec{w} \rangle \text{ (forward propagation)}} - y_i) \cdot \vec{x}_i$$

backward propogation

PPML for Linear Regression



Invariant: Two parties hold secret shares of all inputs & intermediate values.

Initialization: Randomly sample $\vec{w}_0, \vec{w}_1$ respectively.

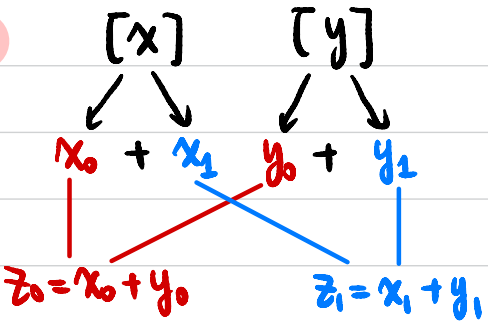
SGD: $\vec{w} \leftarrow \vec{w} - \eta \cdot (\langle \vec{x}_i, \vec{w} \rangle - y_i) \cdot \vec{x}_i$

(η: public)

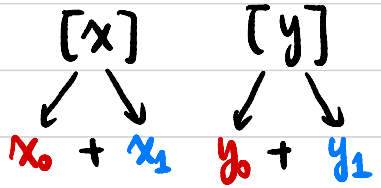
How to get a secret share of the updated $\vec{w}$?

PPML for Linear Regression

ADD:



MULT:



$$(x_0 + x_1) \cdot (y_0 + y_1) = x_0 \cdot y_0 + x_1 \cdot y_1 + x_0 \cdot y_1 + x_1 \cdot y_0$$

Reshare Reshare

Machine Learning Background

Logistic Regression

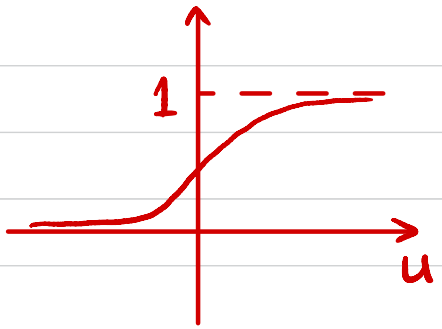
Data Points $(\vec{x}, y)$

ML Model: coefficient vector $\vec{w}$

$$g(\vec{x}) = f(\langle \vec{x}, \vec{w} \rangle)$$

f: activation function

Sigmoid $f(u) = \frac{1}{1+e^{-u}}$



For each data point $(\vec{x}_i, y_i)$,

Define loss function $L_i(\vec{w}) := -y_i \cdot \log y_i^* - (1-y_i) \cdot \log(1-y_i^*)$

$y_i^* := f(\langle \vec{x}_i, \vec{w} \rangle)$

Total loss: $L(\vec{w}) := \frac{1}{N} \sum_{i \in [N]} L_i(\vec{w})$

Goal: Find $\vec{w}$ that minimizes $L(\vec{w})$.

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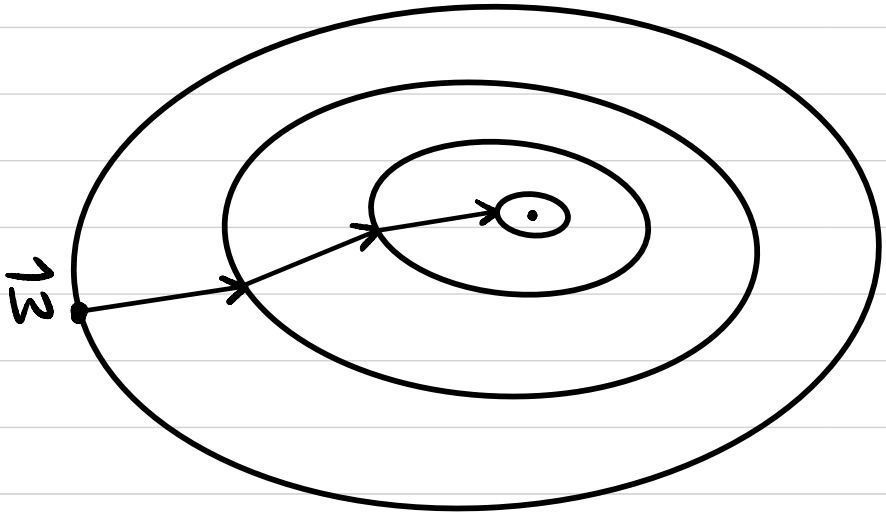

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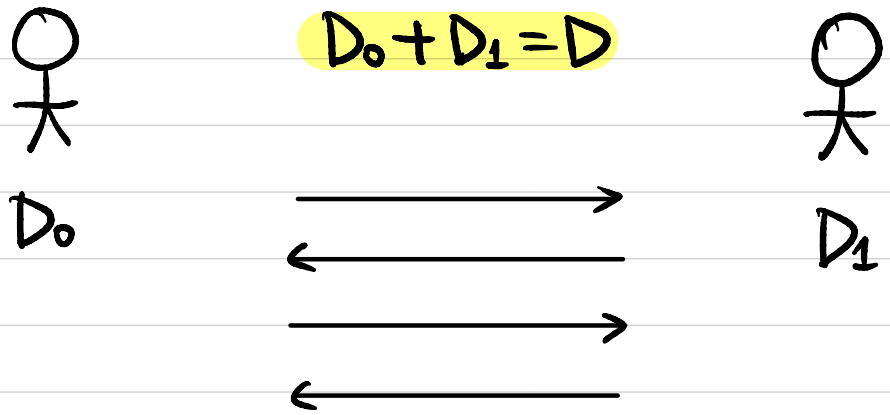
backward propogation

PPML for Logistic Regression



Invariant: Two parties hold secret shares of all inputs & intermediate values.

Initialization: Randomly sample $\vec{w}_0, \vec{w}_1$ respectively.

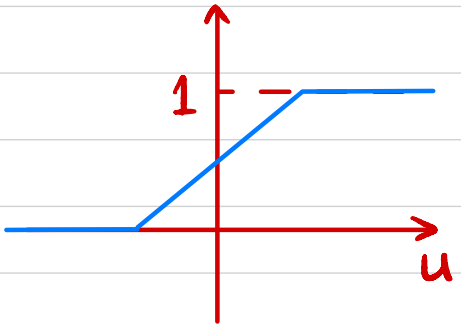
SGD: $\vec{w} \leftarrow \vec{w} - \eta \cdot (f(\langle \vec{x}_i, \vec{w} \rangle) - y_i) \cdot \vec{x}_i$

(η: public)

How to get a secret share of the updated $\vec{w}$?

PPML for Logistic Regression

Approximating $f(u)$



Piecewise polynomial $f(u) = \begin{cases} 0 & \text{if } u < -\frac{1}{2} \\ u + \frac{1}{2} & u \in [-\frac{1}{2}, \frac{1}{2}] \\ 1 & u > \frac{1}{2} \end{cases}$

$b_1 := \begin{cases} 1 & \text{if } u < -\frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$

$b_2 := \begin{cases} 1 & \text{if } u > \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$

$f(u) = ?$