# CSCI 1515 Applied Cryptography

This Lecture:

- SNARGs from PCP (Continued)
- Blockchain

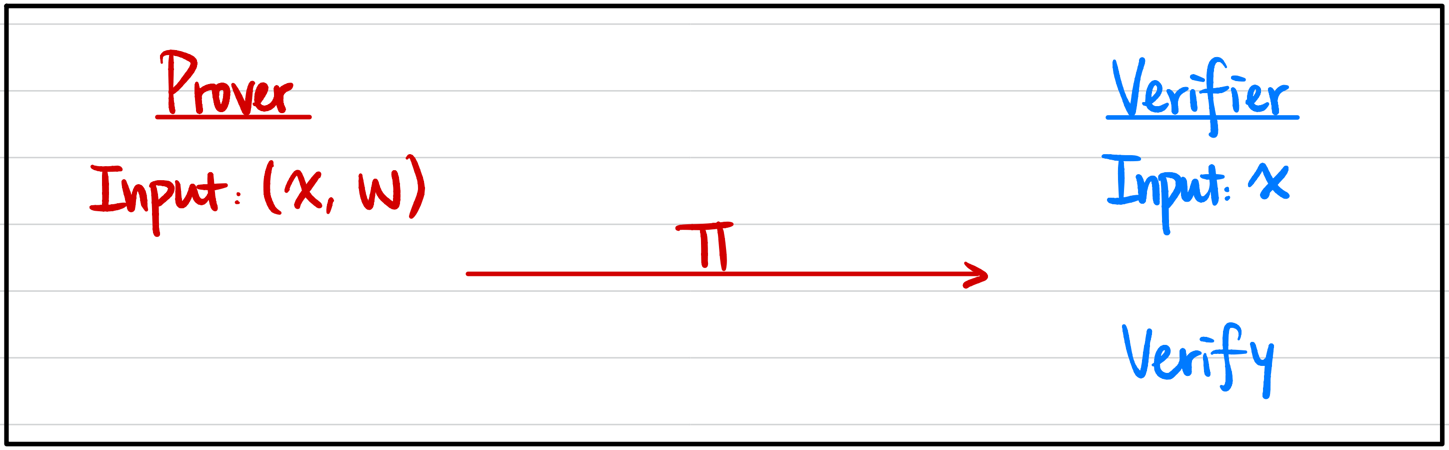
# Proof Systems for Circuit Satisfiability

NP relation $R_{L_C} = \{ (x, w) : C(x, w) = 1 \}$

| | NP | Σ-Protocol | (Fiat-Shamir)<br>NIZK |
|---|---|---|---|
| | $P(x,w) \xrightarrow{w} V(x)$ | $P(x,w) \begin{array}{c}\longrightarrow\\ \longleftarrow\\ \longrightarrow\end{array} V(x)$ | $P(x,w) \xrightarrow{\pi} V(x)$ |
| Zero-Knowledge | NO | YES | YES |
| Non-Interactive | YES | NO | YES |
| Communication | $O(\lvert w \rvert)$ | $O(\lvert C \rvert \cdot \lambda)$ | $O(\lvert C \rvert \cdot \lambda)$ |
| V's computation | $O(\lvert C \rvert)$ | $O(\lvert C \rvert)$ | $O(\lvert C \rvert)$ |

Can we have communication complexity & verifier's computational complexity sublinear in $|C|$ & $|w|$ ?

# Succinct Non-Interactive Argument

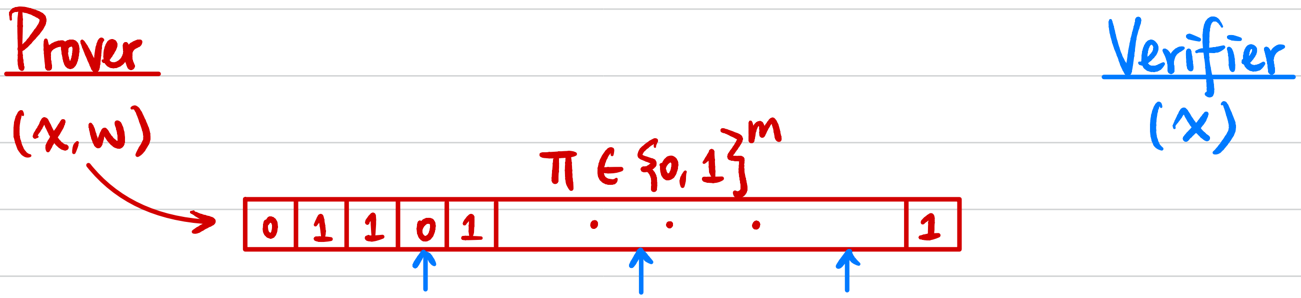


- **SNARG:** Succinct Non-Interactive Argument
- **SNARK:** Succinct Non-Interactive Argument of Knowledge
- **zk-SNARG / zk-SNARK:** SNARG / SNARK + Zero-Knowledge

- **Succint:** $|\pi| = \mathrm{poly}(\lambda, \log|C|)$

  Verifier runtime $\mathrm{poly}(\lambda, |x|, \log|C|)$
- **Argument:** In Soundness / Proof of Knowledge: $\forall$ PPT $P^*$

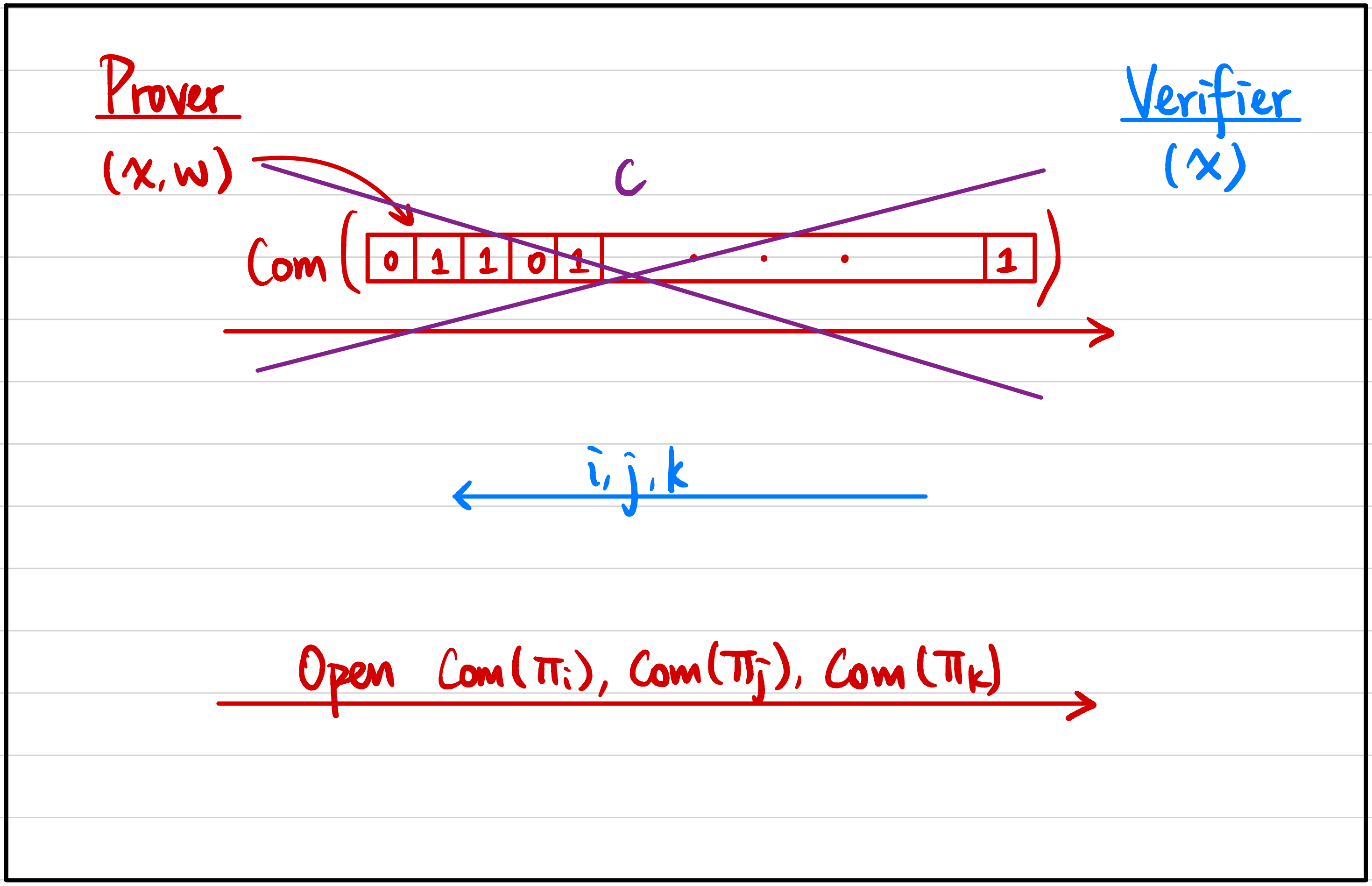
# Probabilistically Checkable Proof (PCP)



## PCP Theorem (Informal):

Every NP language has a PCP where the verifier reads only a constant number of bits of the proof.

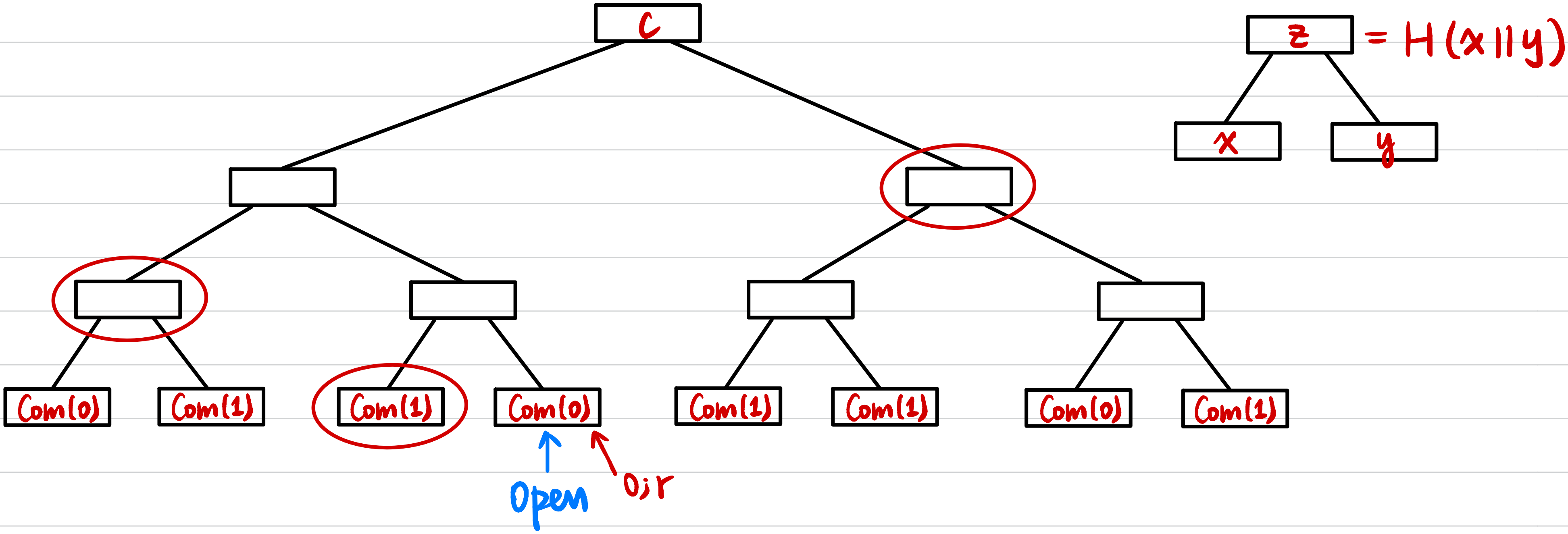
# First Attempt



Is it succint ?

Is it ZK ?

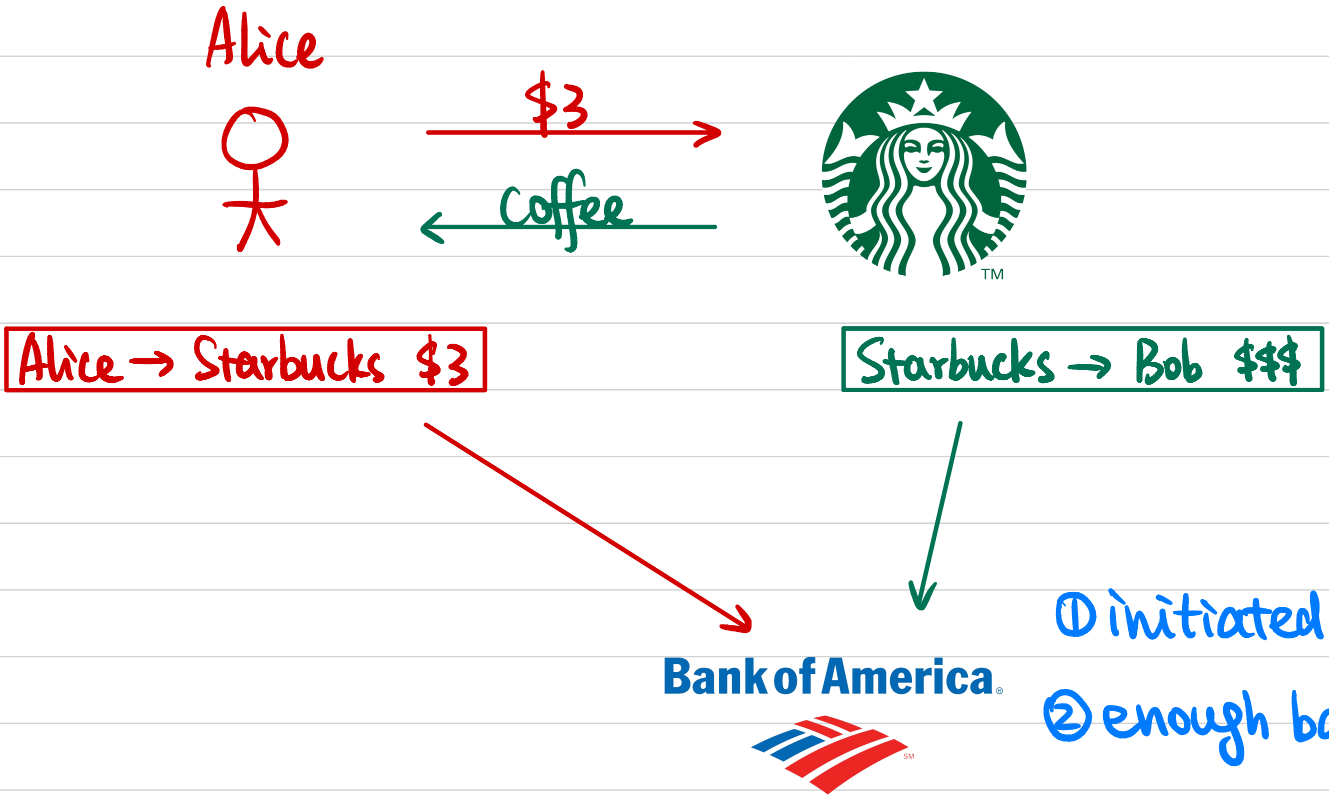
# Merkle Tree



How to open Commitment ?

Why hiding & binding ?

# Transactions in Real Life



① initiated by sender

② enough balance in sender's account

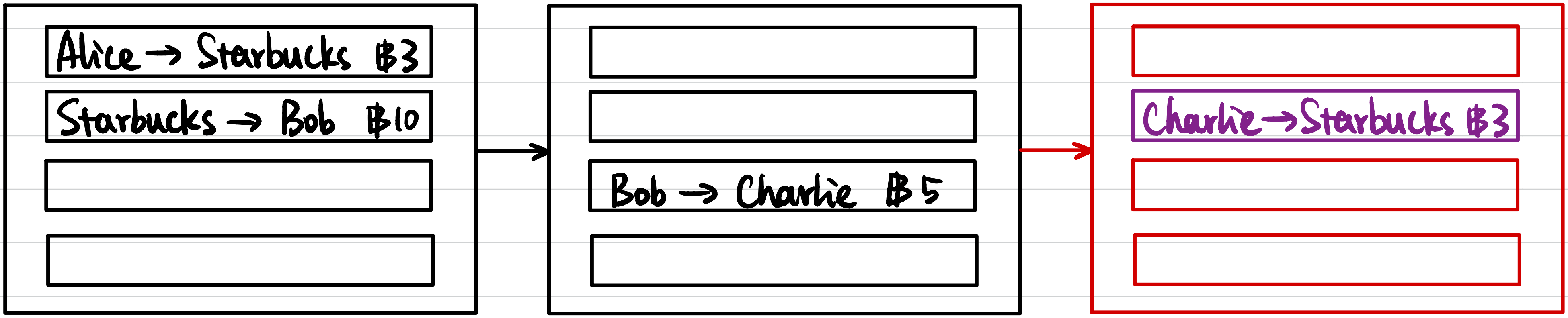
A trusted party that maintains a private ledger

Alice → Starbucks $3

Starbucks → Bob $$$

. . .

# Blockchain



- Public ledger that everyone can view & verify
- Maintained by "miners" in a distributed way

Step 1: Charlie wants to make a transaction Charlie→Starbucks ₿3

↳ broadcasts it to the entire network

Step 2: All miners collect all transactions in the network

- Verify validity
  - ① initiated by sender ← How?
  - ② enough balance in sender's account
- Agree on next block ← How?

Step 3: Repeat

# Transaction Authentication

Alice: $(vk_A, sk_A) \leftarrow KeyGen(1^\lambda)$

Bob: $(vk_B, sk_B) \leftarrow KeyGen(1^\lambda)$

Charlie: $(vk_C, sk_C) \leftarrow KeyGen(1^\lambda)$

Starbucks: $(vk_S, sk_S) \leftarrow KeyGen(1^\lambda)$

$\boxed{\text{Bob} \rightarrow \text{Charlie ₿5}}$ :

$m_1 = (vk_B, vk_C, 5)$ $\quad \sigma_1 \leftarrow Sign_{sk_B}(m_1)$

$\boxed{\text{Charlie} \rightarrow \text{Starbucks ₿3}}$ :

$m_2 = (vk_C, vk_S, 3)$ $\quad \sigma_2 \leftarrow Sign_{sk_C}(m_2)$

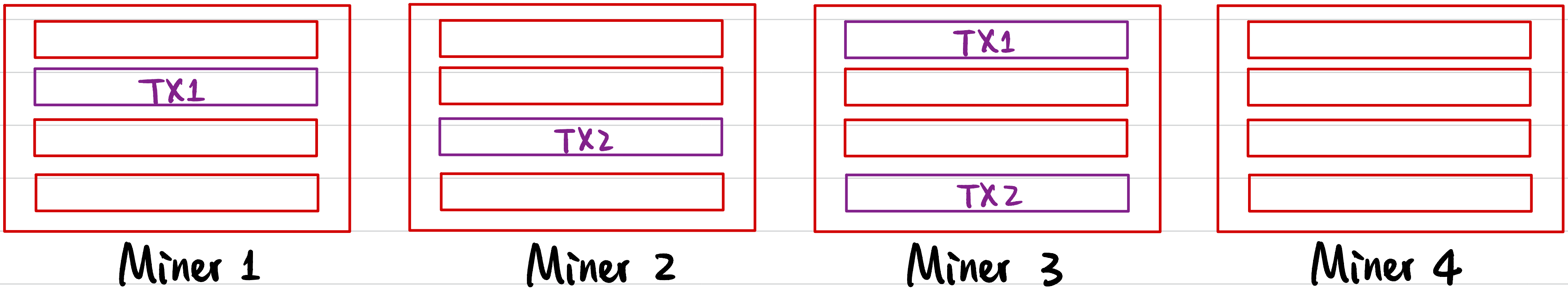
# Consensus Protocol

TX1 = $\boxed{\text{Charlie} \rightarrow \text{Starbucks ₿3}}$ :

$$m_2 = (vk_C\ ,\ vk_S\ ,\ 3) \qquad \sigma_2 \leftarrow \text{Sign}_{sk_C}(m_2)$$

TX2 = $\boxed{\text{Charlie} \rightarrow \text{Alice} \ \ ₿4}$ :

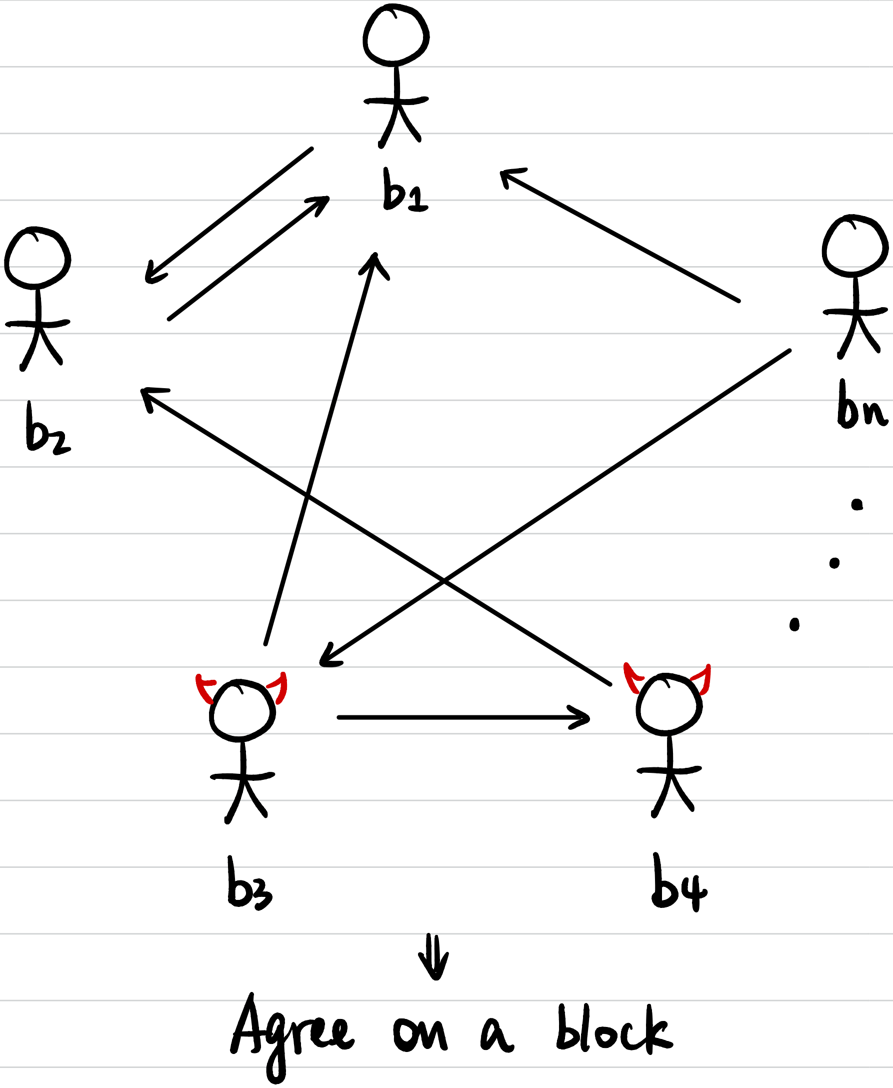
$$m_3 = (vk_C\ ,\ vk_A\ ,\ 4) \qquad \sigma_3 \leftarrow \text{Sign}_{sk_C}(m_3)$$



WANT : ① All miners agree on the same block

② New block is valid

# Byzantine Agreement



**Byzantine Fault Tolerance (BFT) Protocol:**

If $n \overset{\text{necessary}}{\geq} 3t+1$,

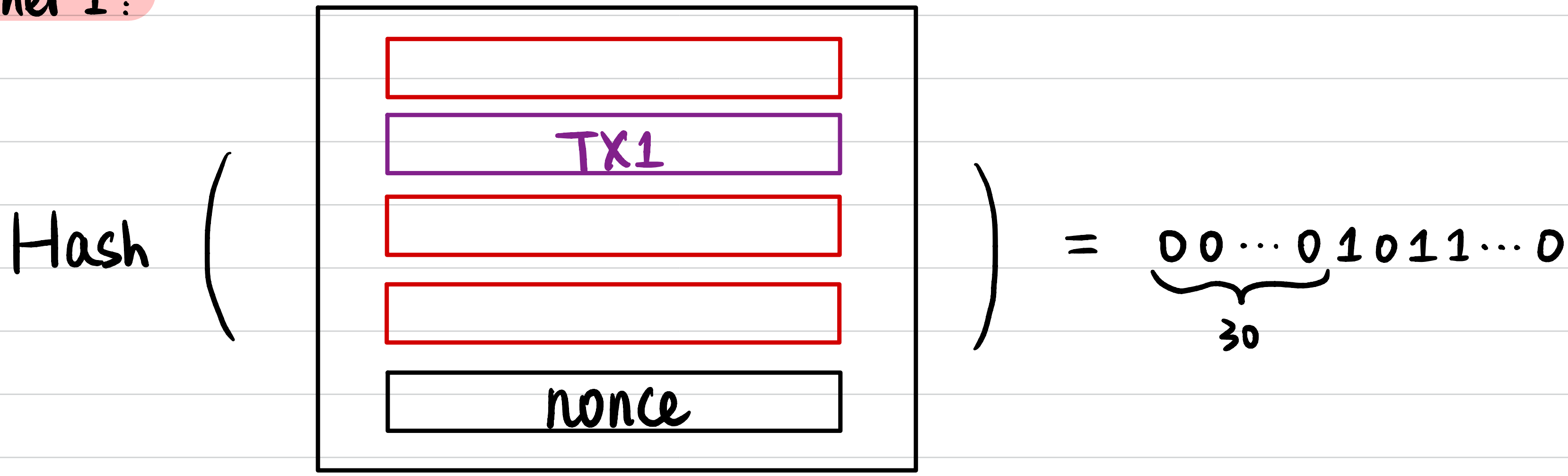
then it's possible to reach consensus.

Assume $t<n/3$, then agree on a valid block.

Any problem?

# Proof of Work (PoW)

Miner 1 :

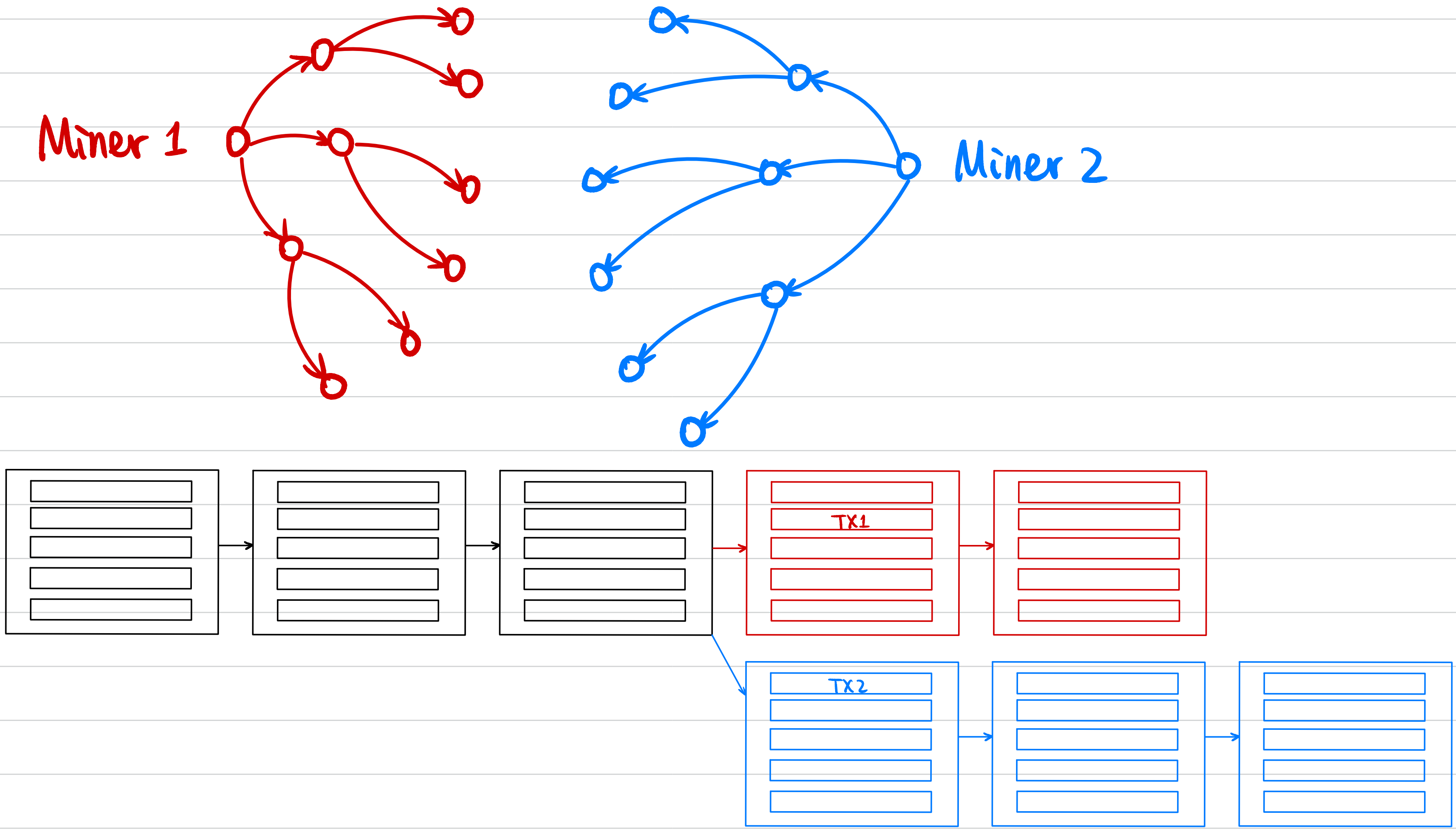


Find nonce s.t. Hash(block) has ≥ 30 leading 0's.

Consensus Protocol :

Whoever first finds a block that hashes to a value w/ ≥ 30 leading 0's, that block becomes the next block.

# Proof of Work (PoW)



Longest Chain Rule: Always adopt the longest chain.

Assuming honest majority of computation power, the longest chain is always valid.

# Blockchain

- Efficient verification of sufficient balance: Merkle Tree

- Settlement of a transaction:
  Included in a block which is $\geq 6$ blocks deep ($\sim 1$ hr)

- Dynamically adjust # leading 0's s.t. each block takes ~10min to mine
  Last 1 hr: $> 6$ blocks: increase # leading 0's
  $< 6$ blocks: decrease # leading 0's

- Miners' motivation:
  - transaction fee
  - new coin generated in each block goes to miner

- Extensions
  - Proof of Stake (PoS)
  - Anonymous transactions (zk-SNARGs)
  - Smart Contracts
  - Public Bulletin Board