

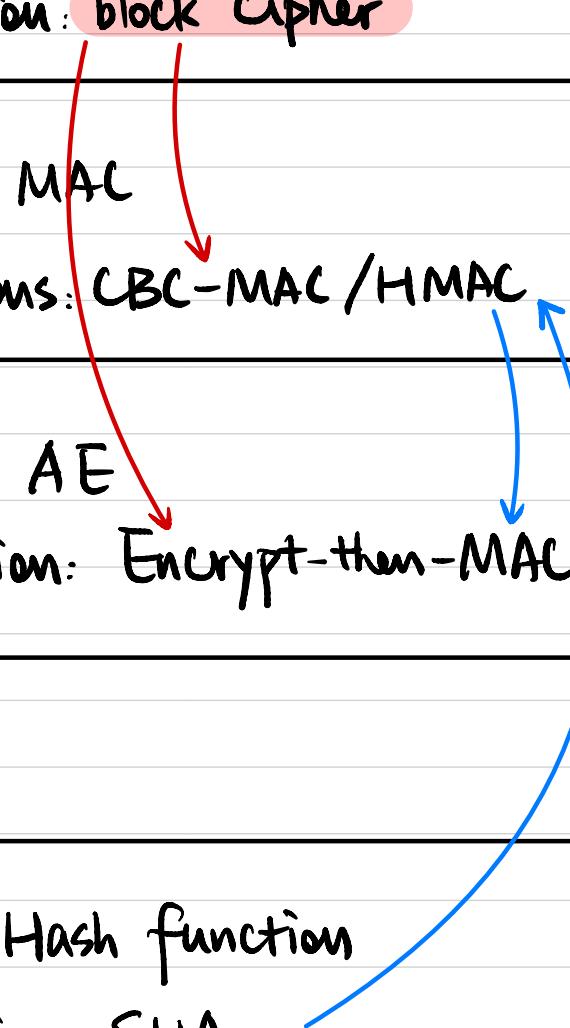
# CSCI 1515 Applied Cryptography

This Lecture:

- Pseudorandom Function (PRF)  
Pseudorandom Permutation (PRP)
- Block Cipher and Modes of Operation
- CBC-MAC

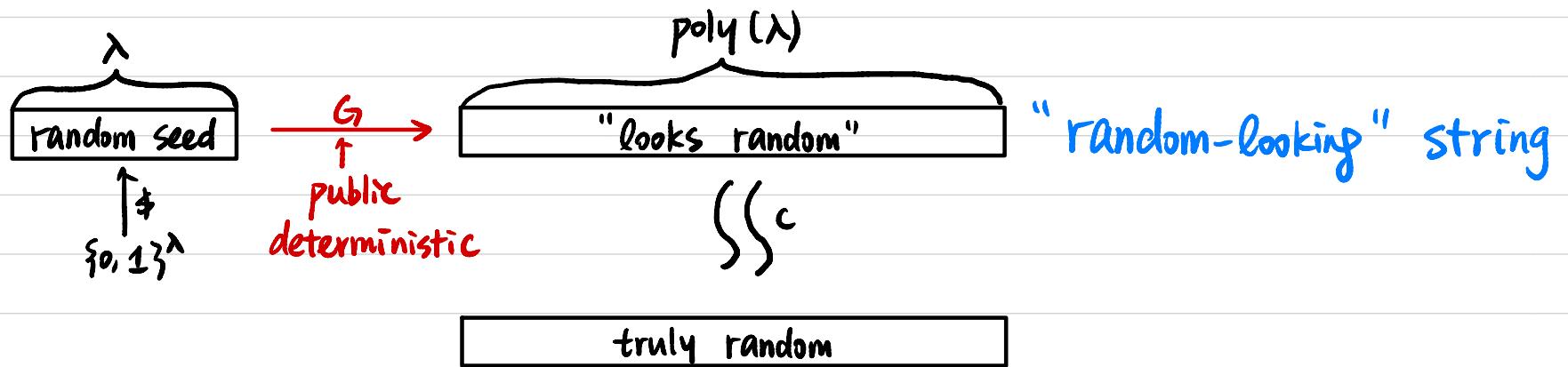
# Summary

	Symmetric-Key	Public-Key
Message Secrecy	Primitive: SKE Construction: block Cipher	Primitive: PKE Constructions: RSA / ElGamal
Message Integrity	Primitive: MAC Constructions: CBC-MAC / HMAC	Primitive: Signature Constructions: RSA / DSA
Secret & Integrity	Primitive: AE Construction: Encrypt-then-MAC	
Key Exchange		Construction: Diffie-Hellman
Important Tool	Primitive: Hash function Construction: SHA	



## Pseudorandom Function (PRF)

### Pseudorandom Generator (PRG)



Pseudorandom Function (PRF): "random-looking" function

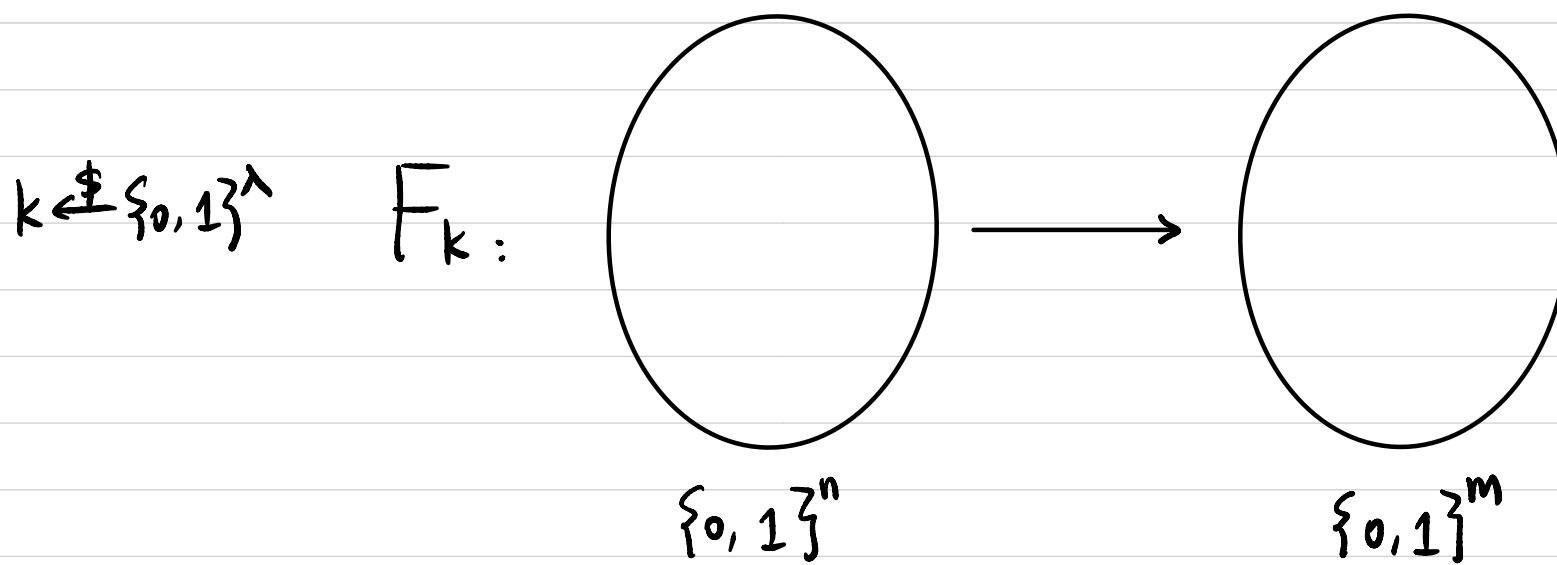
## Pseudorandom Function (PRF)

Keyed Function  $F: \{0,1\}^\lambda \times \{0,1\}^n \rightarrow \{0,1\}^m$

$F(k, x) \rightarrow y$

↑  
key  
↑  
input  
↑  
output

deterministic  
poly-time

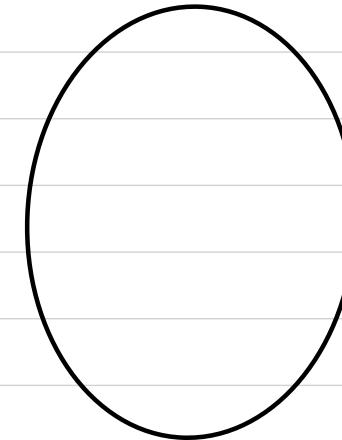
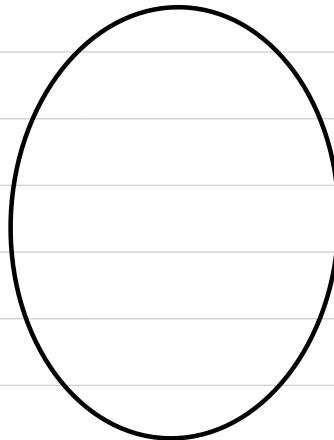


"looks like a random function"

## Pseudorandom Function (PRF)

$$k \xleftarrow{\$} \{0,1\}^\lambda$$

$F_k :$



How many possible  $F_k$ 's ?

$\beth^\lambda$

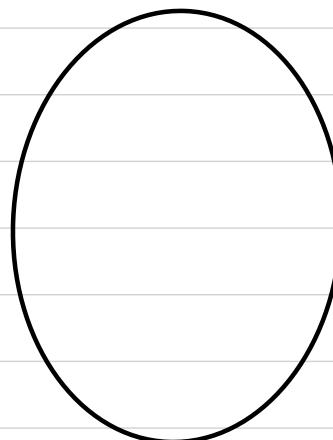
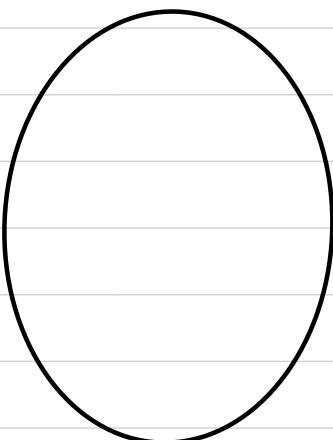
$$\{0,1\}^n$$

$$\{0,1\}^m$$

$\beth^c$  (not knowing  $k$ )

$$f \xleftarrow{\$} \{F \mid F : \{0,1\}^n \rightarrow \{0,1\}^m\}$$

$f :$



How many possible  $f$ 's ?

$$(\beth^m)^{\beth^n}$$

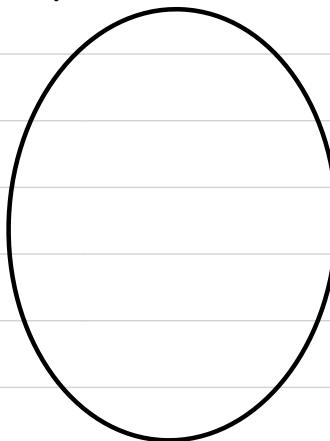
$$\{0,1\}^n$$

$$\{0,1\}^m$$

# Pseudorandom Permutation (PRP)

$$k \leftarrow \{0, 1\}^\lambda$$

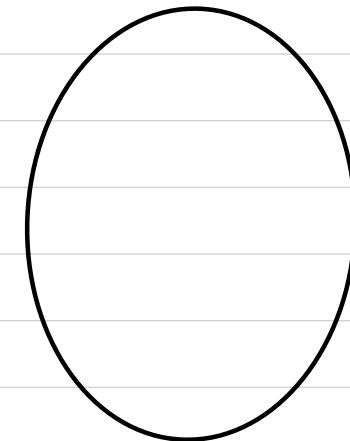
$F_k :$



bijective

$$F_k$$

$$F_k^{-1}$$



How many possible  $F_k$ 's?

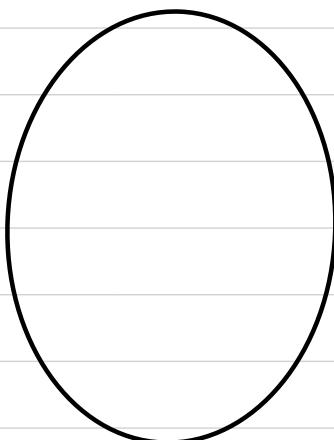
$$2^\lambda$$

$$\{0, 1\}^n$$

$$\{0, 1\}^n$$

$$f \leftarrow \{ F \mid F : \{0, 1\}^n \rightarrow \{0, 1\}^n, \\ F \text{ is bijective} \}$$

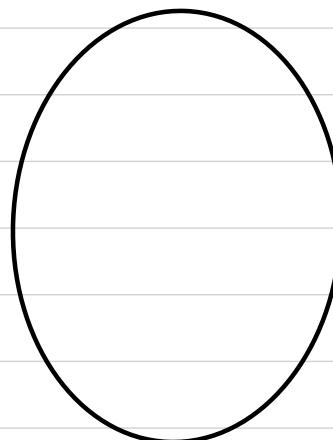
$f :$



bijective

$$f$$

$$f^{-1}$$



How many possible  $f$ 's?

$$2^n!$$

$$\{0, 1\}^n$$

$$\{0, 1\}^n$$

## Block Cipher

$$F: \{0, 1\}^\lambda \times \{0, 1\}^n \rightarrow \{0, 1\}^n$$

$\lambda$ : key length

$n$ : block length

It is assumed to be a pseudorandom permutation (PRP).

### Construction: Advanced Encryption Standard (AES)

- $\lambda = 128/192/256$ ,  $n = 128$
- Standardized by NIST in 2001
- Competition 1997–2000

### Before AES: Data Encryption Standard (DES)

- $\lambda = 56$ ,  $n = 64$

## Block Cipher Modes of Operation

$$F: \{0, 1\}^\lambda \times \{0, 1\}^\lambda \rightarrow \{0, 1\}^{\lambda \leftarrow 128}$$

It is assumed to be a pseudorandom permutation (PRP).

Construct an SKE scheme from F for arbitrary-length messages.

- $k \leftarrow \{0, 1\}^\lambda$

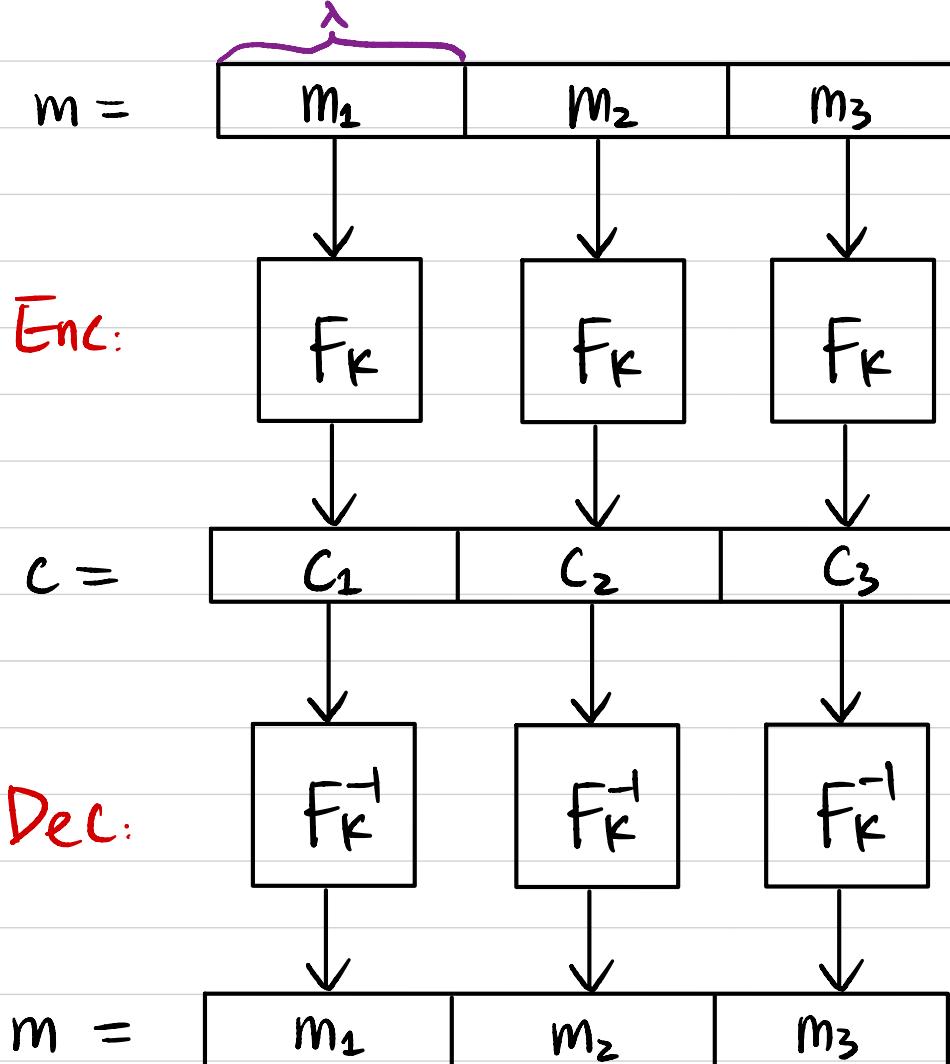
- $\text{Enc}_k(m)$

$$|m| = \alpha \cdot \lambda \quad (\text{If not, pad it to a multiple of } \lambda)$$

- $\text{Dec}_k(c)$

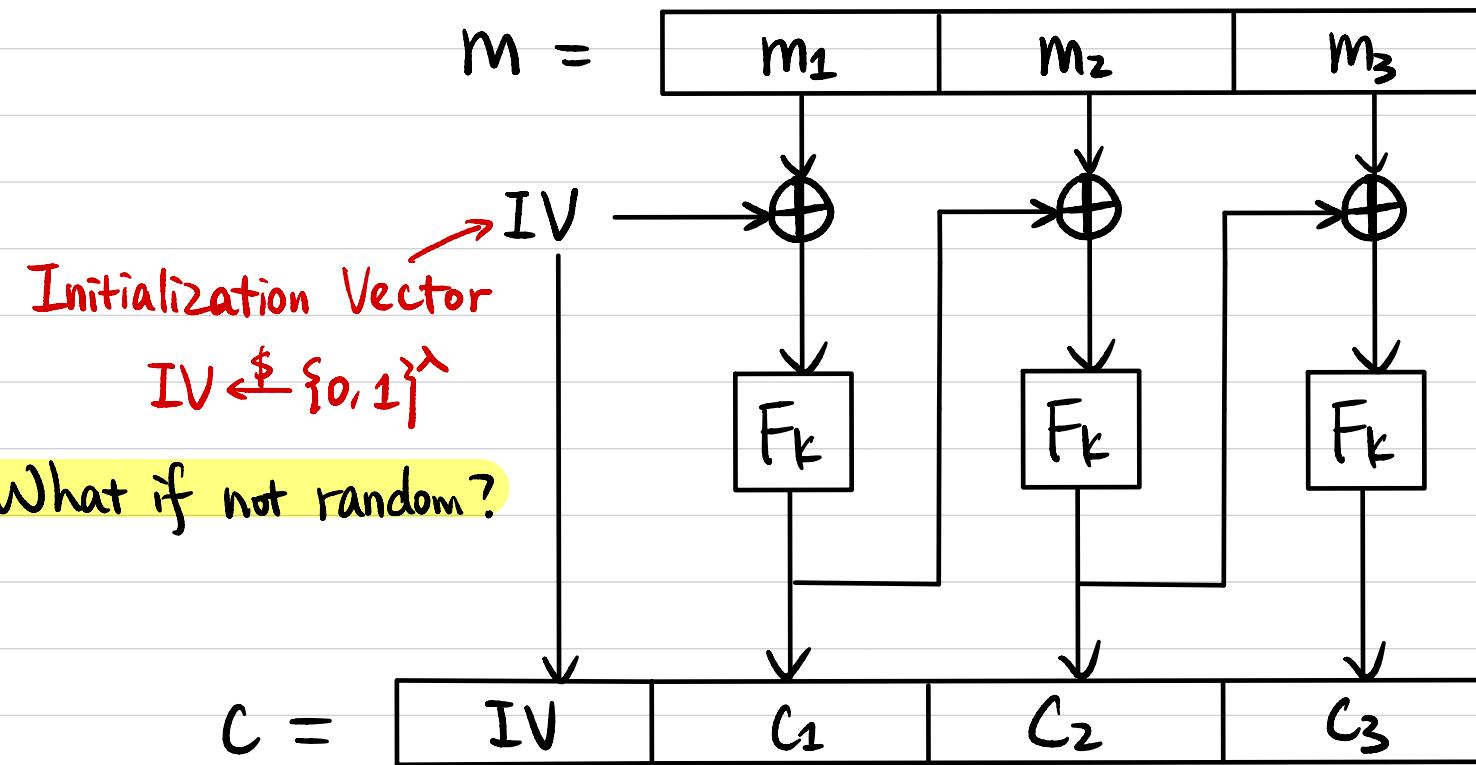
Goal: CPA (Chosen Plaintext Attack) Security

# Electronic Code Book (ECB) Mode



CPA Secure ?

# Cipher Block Chaining (CBC) Mode

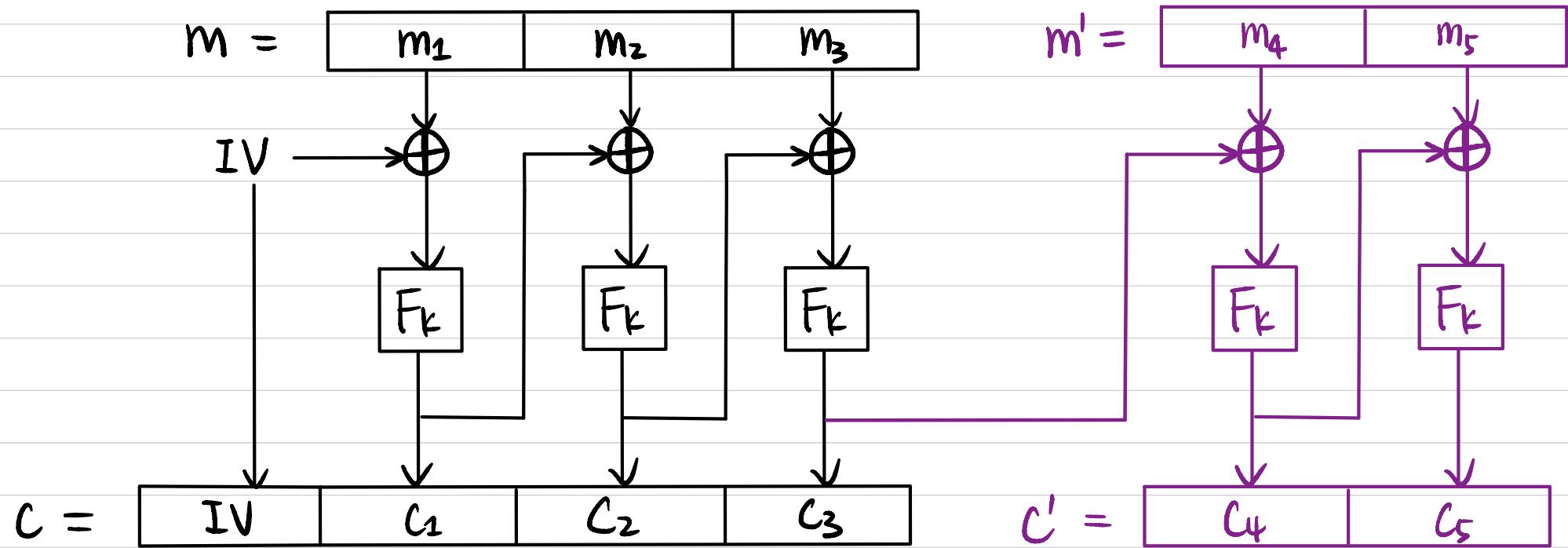


How to decrypt?

CPA Secure?

Can we parallelize the computation?

## Chained Cipher Block Chaining (CBC) Mode

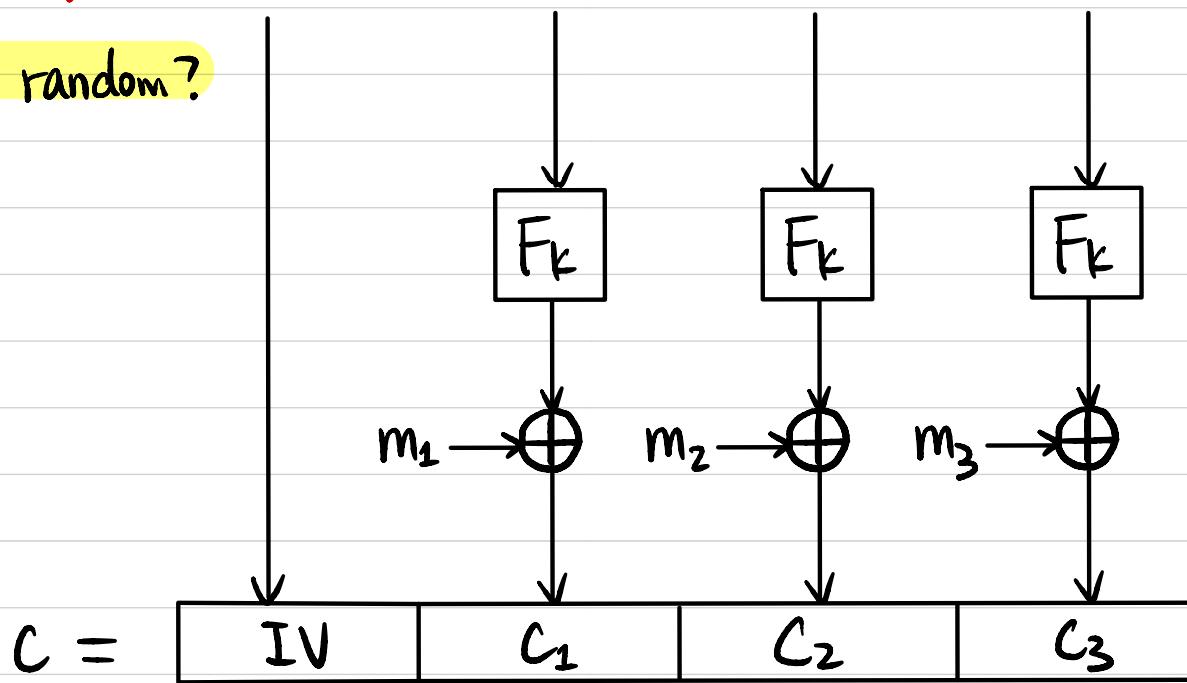


CPA Secure ?

## Counter (CTR) Mode

$$\{0,1\}^k \xrightarrow{\$} IV$$

What if not random?



How to decrypt?

CPA Secure?

"Stateful" CTR Mode?

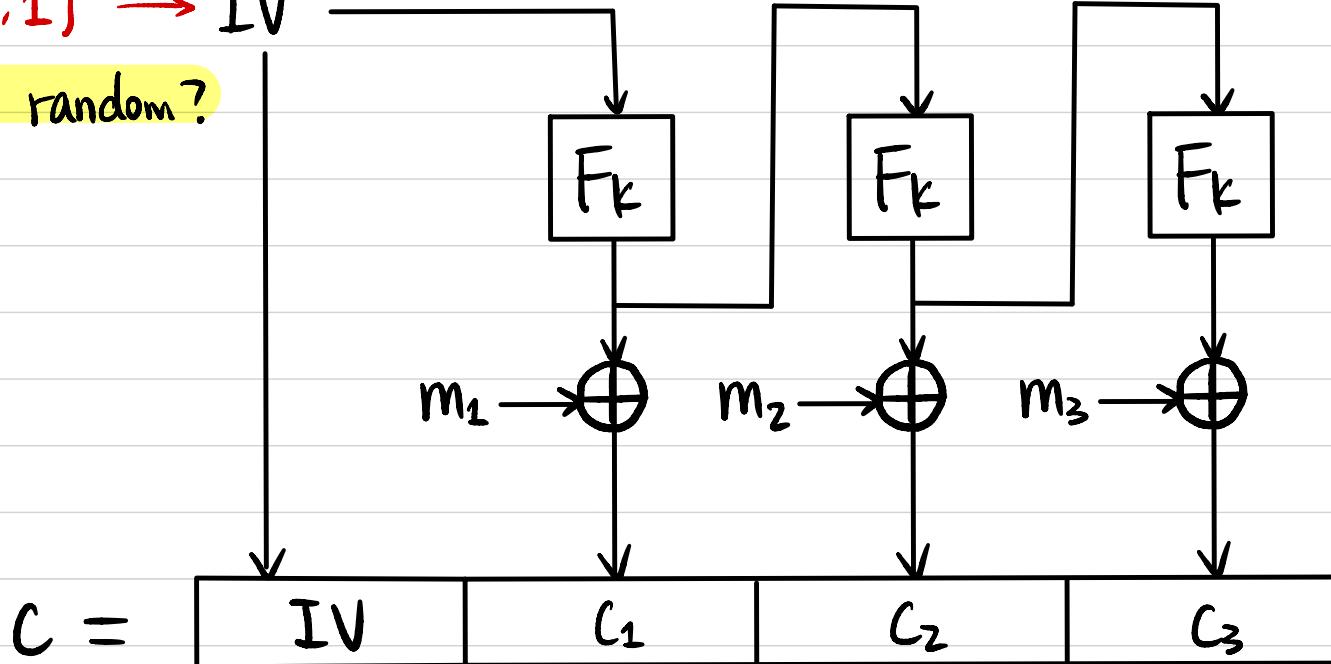
Can we parallelize the computation?

PRG from PRF

## Output Feedback (OFB) Mode

$\{0,1\}^\lambda \xrightarrow{\$} IV$

What if not random?



How to decrypt?

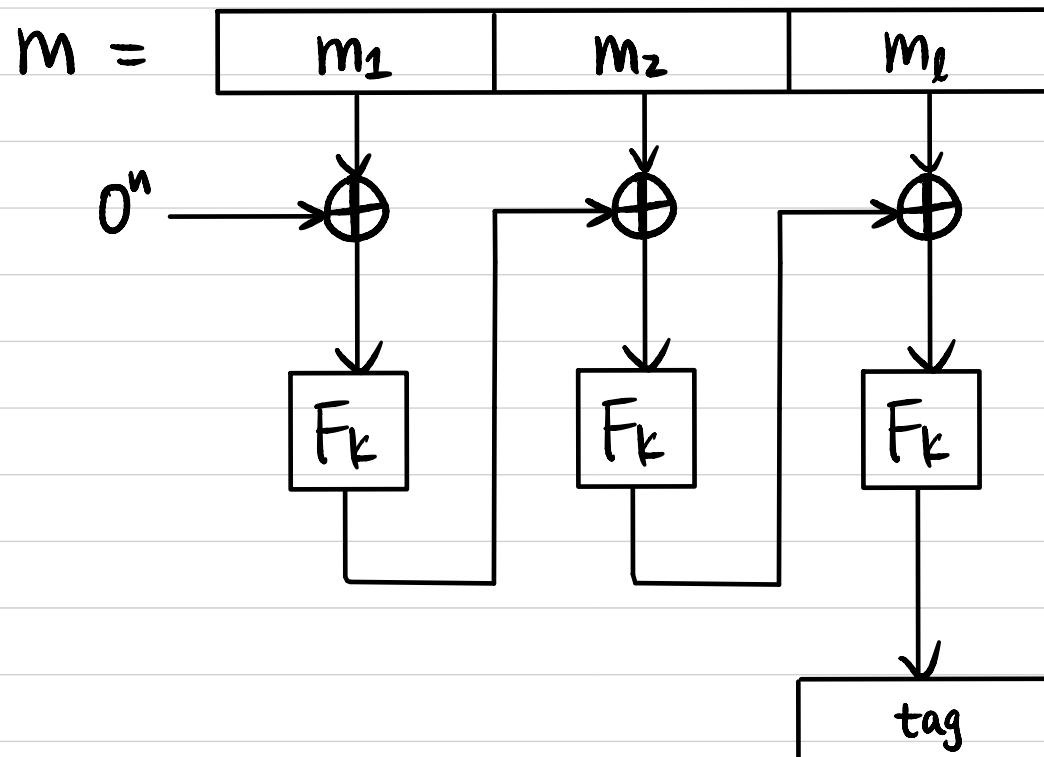
CPA Secure?

"Stateful" DFB Mode?

Can we parallelize the computation?

PRG from PRF

## CBC-MAC

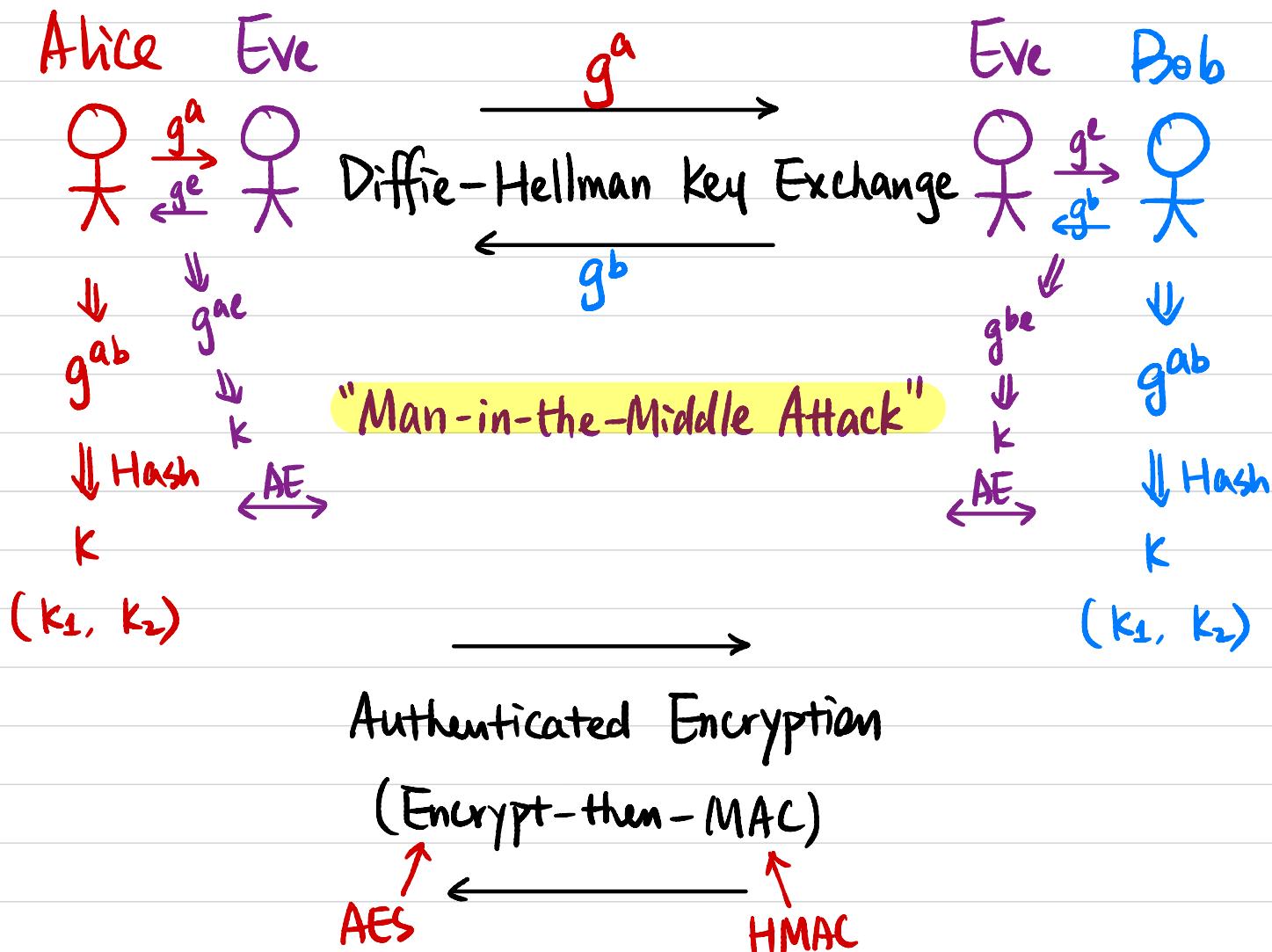


How to verify ?

CMA (Chosen Message Attack) Secure ?

- Fixed-length messages of length  $l \cdot n$
- Arbitrary-length messages

## Putting it All Together: Secure Communication



Any security issue?