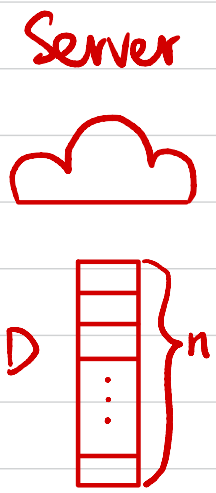
# CSCI 1515 Applied Cryptography

This Lecture:

- Private Information Retrieval (Continued)
- Bootstrapping SWHE to FHE
- Practical Constructions of Block Cipher

# Private Information Retrieval (PIR)





WANT: D[i]

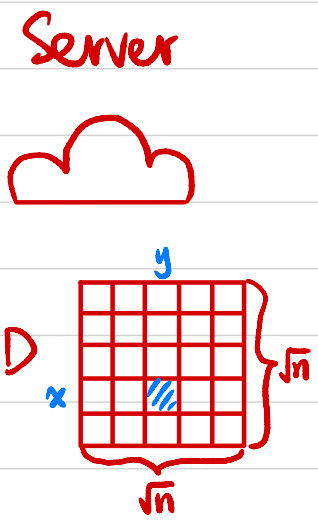
While hiding i against Server

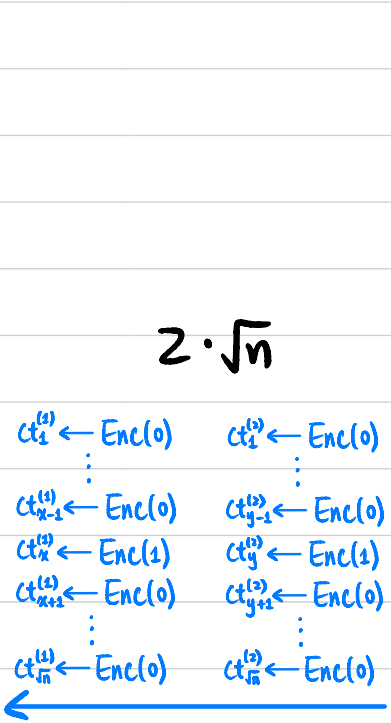
Trivial Solution:

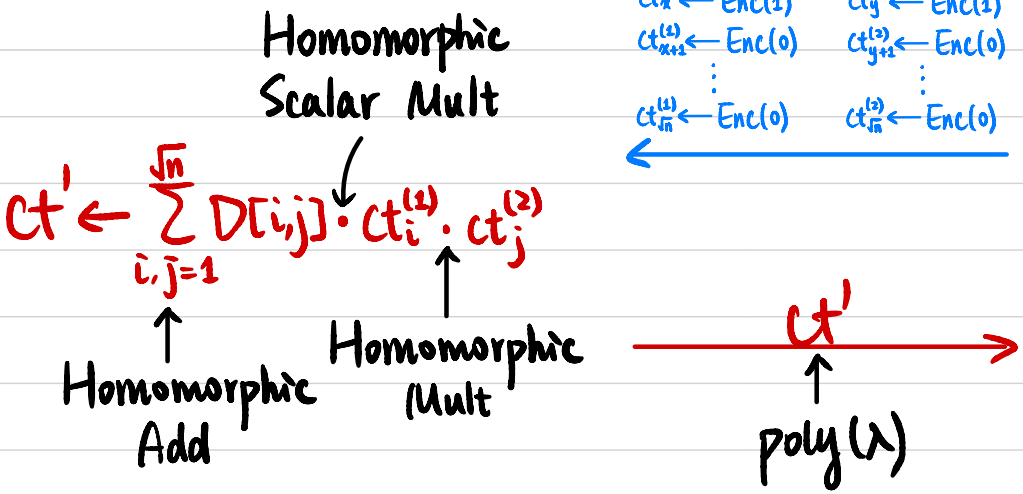

Communication complexity $O(n)$

Goal: Communication complexity $o(n)$

# Private Information Retrieval (PIR)




D[x,y] = Dec(ct')

Extend to dimension d ? Communication $d \cdot \sqrt[d]{n}$

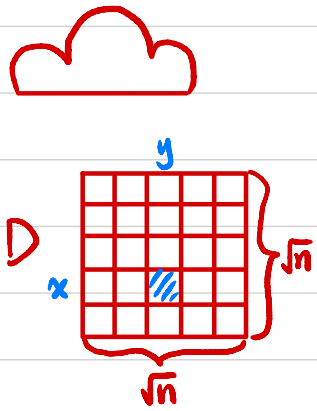
# Homomorphic Mult = (d-1)·n

# Homomorphic Scalar Mult = n

# Homomorphic Add = n

# PIR from Additive HE

Server




Client

$ct_1 \leftarrow Enc(0)$

$\vdots$

$ct_{x-1} \leftarrow Enc(0)$

$ct_x \leftarrow Enc(1)$

$ct_{x+1} \leftarrow Enc(0)$

$\vdots$

$ct_{\sqrt{n}} \leftarrow Enc(0)$

WANT: $D[x,y]$

While hiding $(x,y)$ against Server

Homomorphic Scalar Mult

$\forall j \in [\sqrt{n}],\ ct'_j \leftarrow \sum_{i=1}^{\sqrt{n}} D[i,j] \cdot ct_i$

Homomorphic Add

$ct'_1, \cdots, ct'_{\sqrt{n}}$

$D[x,y] = Dec(ct'_y)$

# Application: Secure 2PC ?

Alice

$C(x, y)$

Bob

Input: $x$

Input: $y$

$ct \leftarrow Enc(y)$

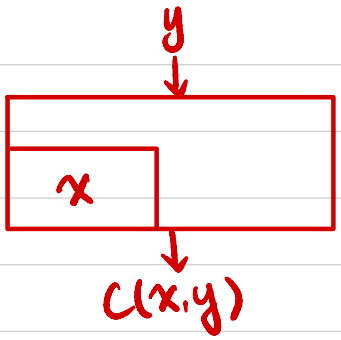
$ct$

$ct' \leftarrow Eval(f, ct)$

$ct'$

$f_x(y) = C(x, y)$

$f(y) \leftarrow Dec_{sk}(ct')$




Function privacy ?

Noise flooding

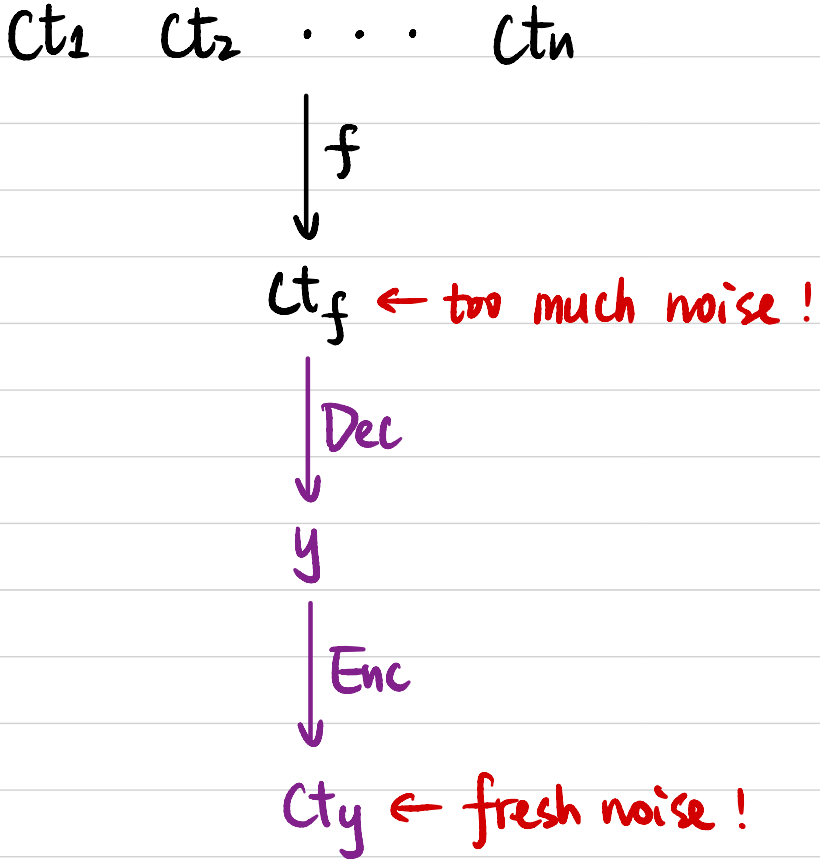
# FHE Constructions

**Step 1:** Somewhat Homomorphic Encryption (SWHE)

- over Integers
- from LWE (GSW)
- from RLWE (BFV)

**Step 2:** Bootstrapping

## Step 2: Bootstrapping



# Leveled FHE

$(pk_1, sk_1)$ $\quad Ct_1 \quad Ct_2 \quad \cdots \quad Ct_n$

$f$

$Ct_f$ ← too much noise !

$=$

$1001011 \cdots 0$ $\quad$ $sk_1$

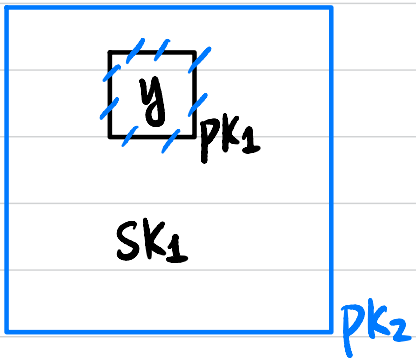
$=$

$01101 \cdots 1$

$\ell$ $\quad$ $k$

$(pk_2, sk_2)$ $\quad Enc_{pk_2}$ $\quad Enc_{pk_2}$

$Ct_1^{(2)} \quad Ct_2^{(2)} \quad \cdots \quad Ct_\ell^{(2)} \quad \tilde{Ct}_1^{(2)} \quad \cdots \quad \tilde{Ct}_k^{(2)}$




$f^{(2)} = Dec_{sk_1}(Ct_f)$

$Ct_{f^{(2)}} = Enc_{pk_2}(y)$

One more operation ADD & MULT

## Step 2: Bootstrapping

Leveled FHE: $pk_1$, $pk_2$, $pk_3$, . . . , $pk_n$

$Enc_{pk_2}(sk_1)$ $Enc_{pk_3}(sk_2)$ $Enc_{pk_n}(sk_{n-1})$

FHE: $pk$. $Enc_{pk}(sk)$

"circular secure" assumption

# Block Cipher

$$F: \{0,1\}^{\lambda} \times \{0,1\}^{n} \to \{0,1\}^{n}$$

$\lambda$: key length

$n$: block length

It is assumed to be a pseudorandom permutation (PRP).

**Construction:** Advanced Encryption Standard (AES)

- $\lambda = 128/192/256$, $n = 128$
- Standardized by NIST in 2001
- Competition 1997-2000

Before AES: Data Encryption Standard (DES)

- $\lambda = 56$, $n = 64$

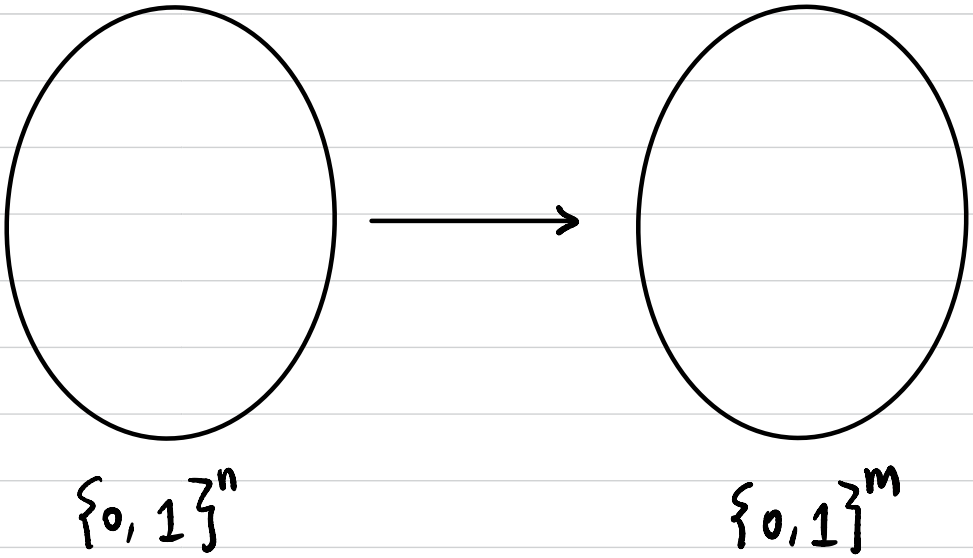
# Pseudorandom Function (PRF)

Keyed Function $F: \{0,1\}^\lambda \times \{0,1\}^n \to \{0,1\}^m$

$F(k, x) \to y$

- $F$: deterministic poly-time
- $k$: key
- $x$: input
- $y$: output

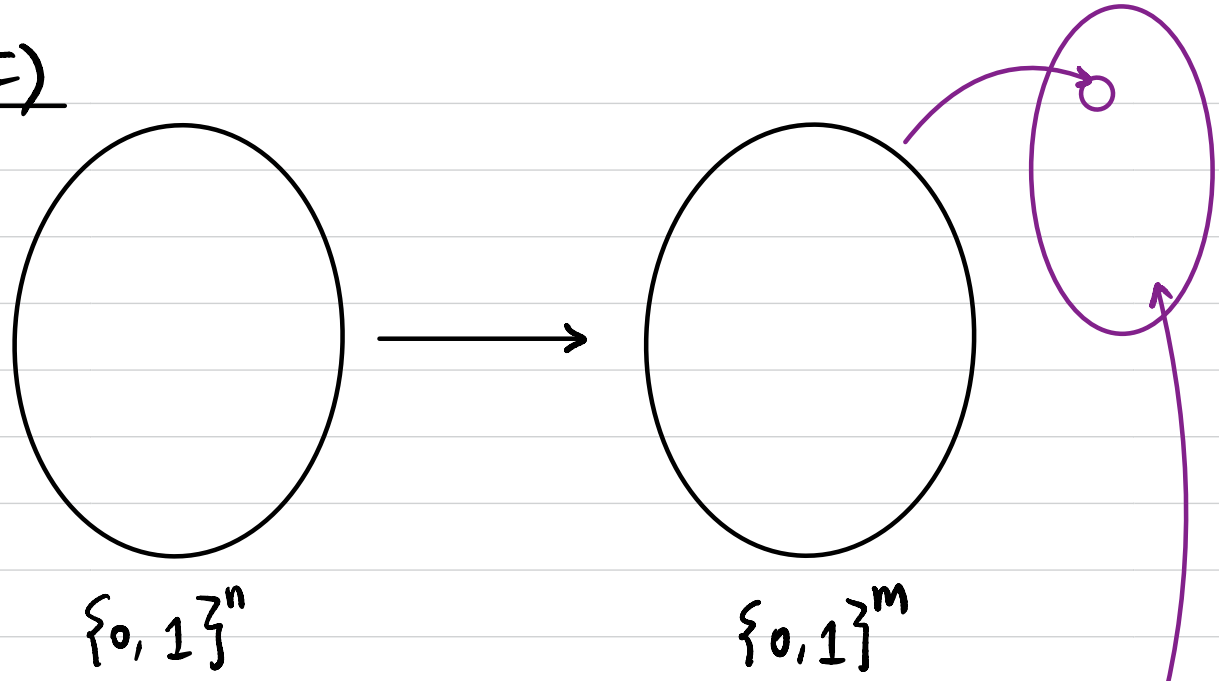
$k \xleftarrow{\$} \{0,1\}^\lambda$ $\quad F_k:$




"looks like a random function"

# Pseudorandom Function (PRF)

$k \xleftarrow{\$} \{0,1\}^\lambda$ $\quad F_k:$



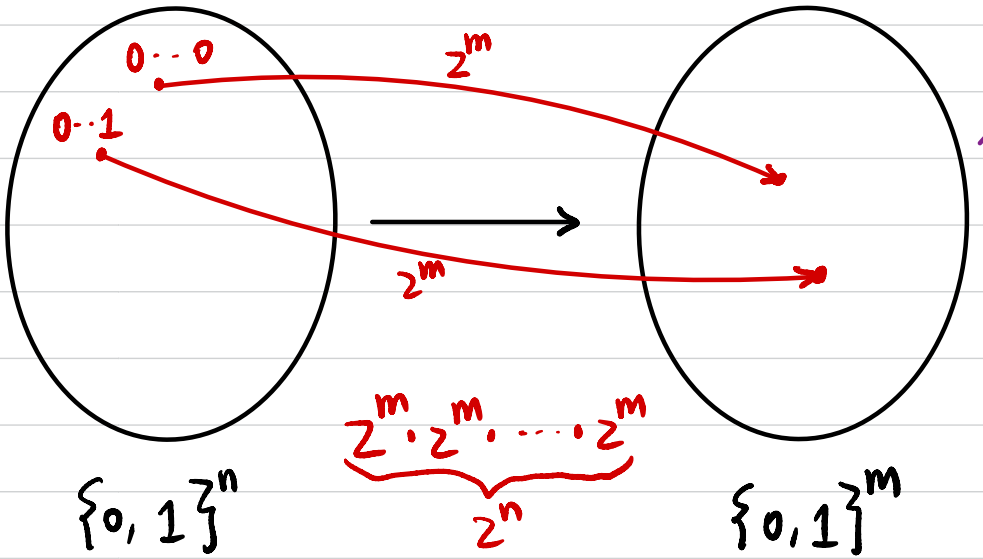

How many possible $F_k$'s ?

$2^\lambda$

$\{\{C$ ( not knowing k)

$f \xleftarrow{\$} \{ F \mid F: \{0,1\}^n \to \{0,1\}^m \}$

$f:$

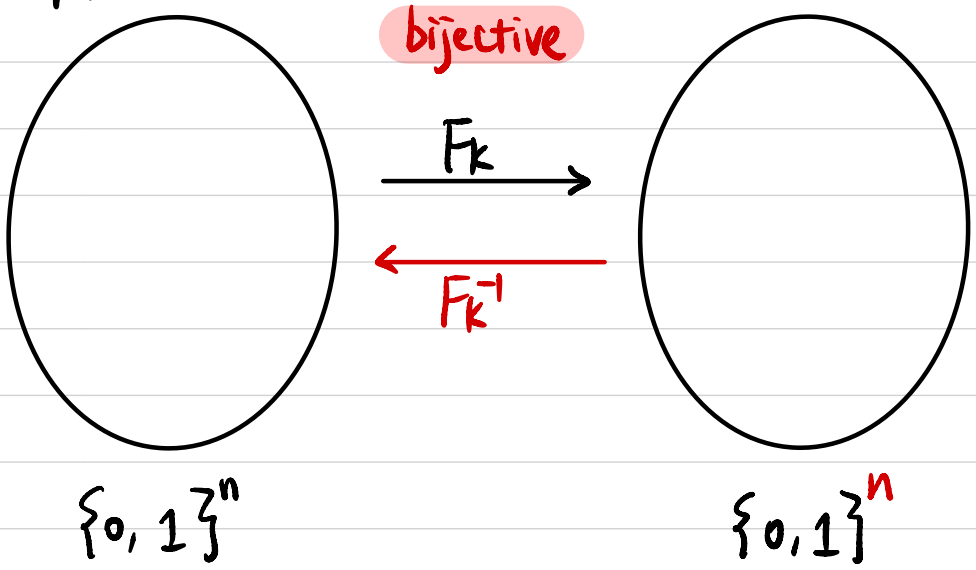



How many possible f's ?

$(2^m)^{2^n}$

# Pseudorandom Permutation (PRP)

$k \xleftarrow{\$} \{0,1\}^\lambda$ $\quad F_k :$



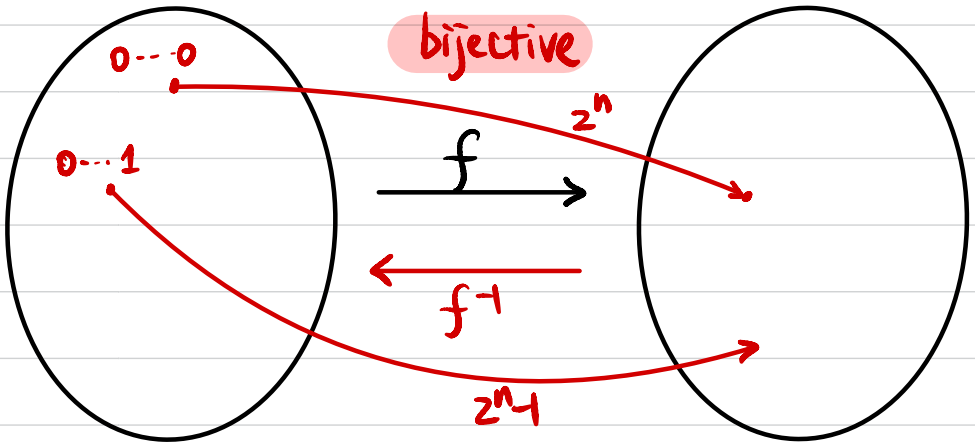

How many possible $F_k$'s ?

$2^\lambda$

$\{\{C$ (not knowing k)

$f \xleftarrow{\$} \{ F \mid F: \{0,1\}^n \to \{0,1\}^n,$ F is bijective $\}$

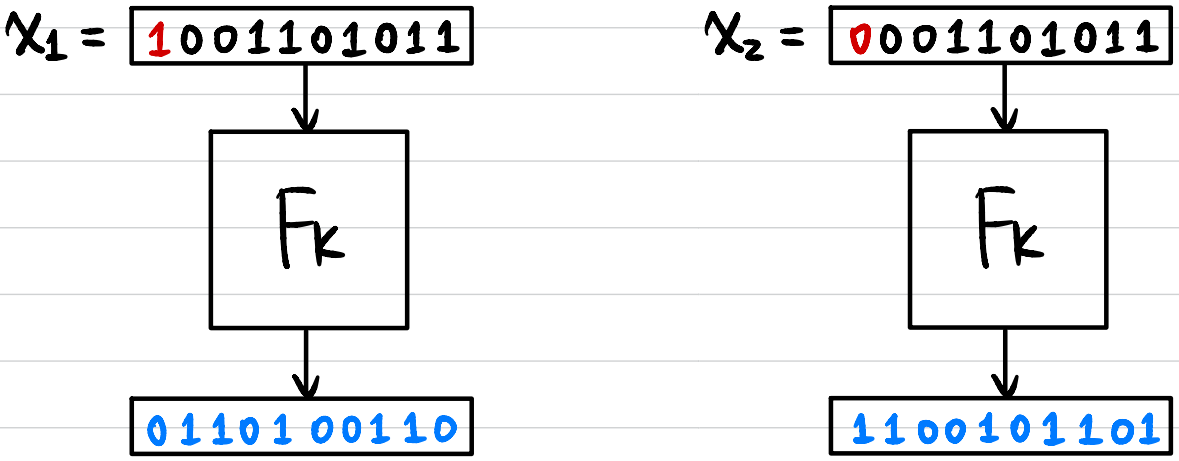
$f :$




How many possible f's ?

$2^n!$

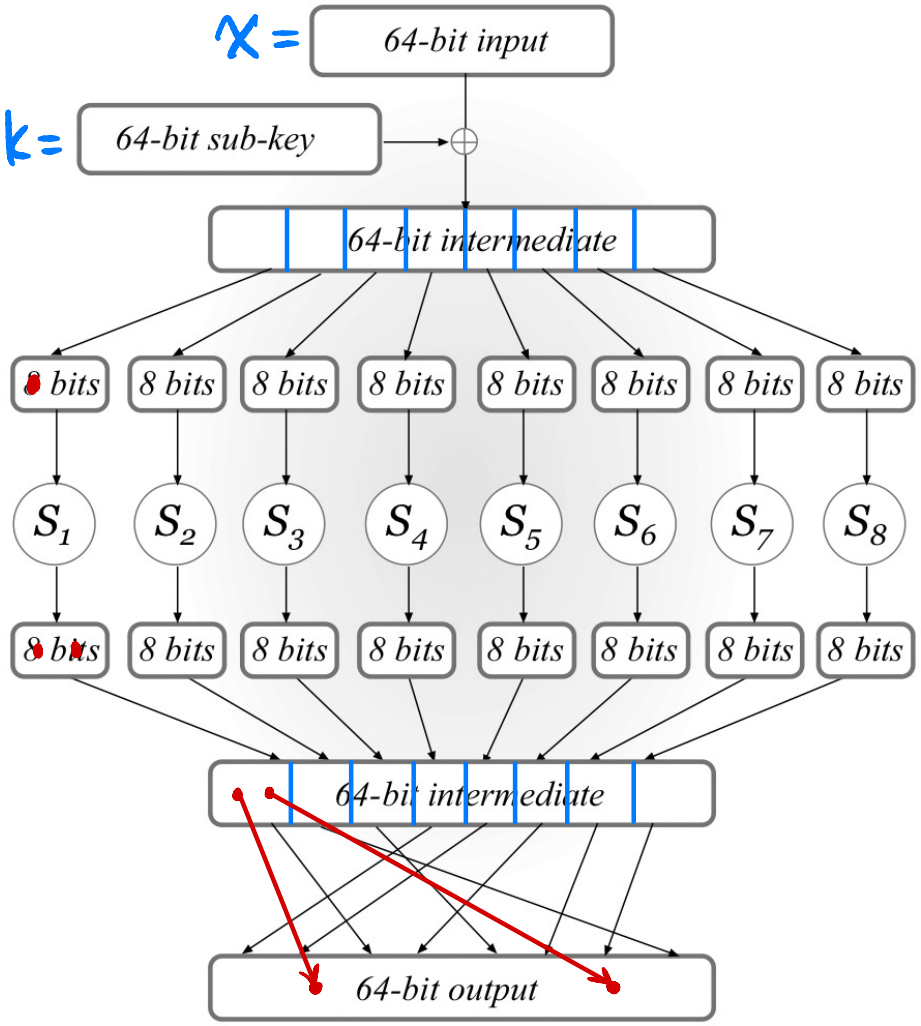
# Substitution-Permutation Network (SPN)




**Design Principle:** "Avalanche Effect"

A one-bit change in the input should "affect" every bit of the output.

# Substitution-Permutation Network (SPN)




A single round of SPN

"Confusion-Diffusion Paradigm"

**Step 1:** Key Mixing

$x := x \oplus k$

**Step 2:** Substitution (Confusion Step)

$S_i : \{0,1\}^8 \rightarrow \{0,1\}^8$ (S-box)

Public permutation / one-to-one map

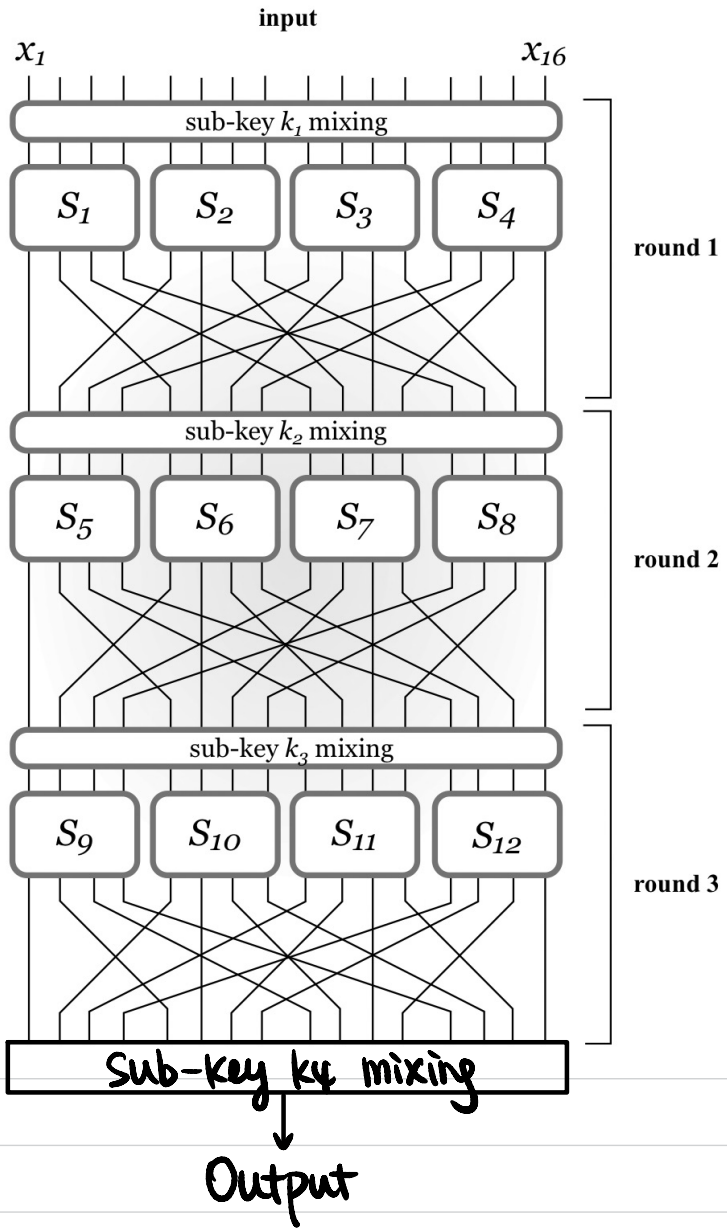
1-bit change of input
→ at least 2-bit change of output

**Step 3:** Permutation (Diffusion Step)

$P : [64] \rightarrow [64]$

Public mixing permutation
↓
affect input to multiple S-boxes next round

# Substitution-Permutation Network (SPN)




## 3-round SPN:

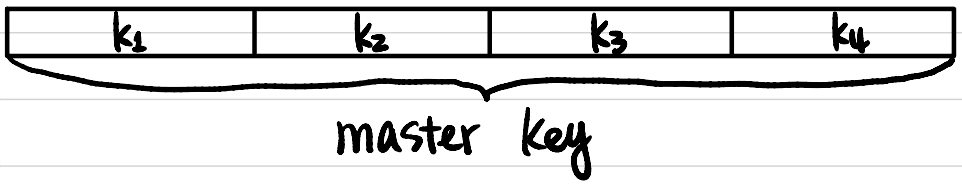
3-round
- key mixing
- substitution
- permutation

1 final-round key mixing

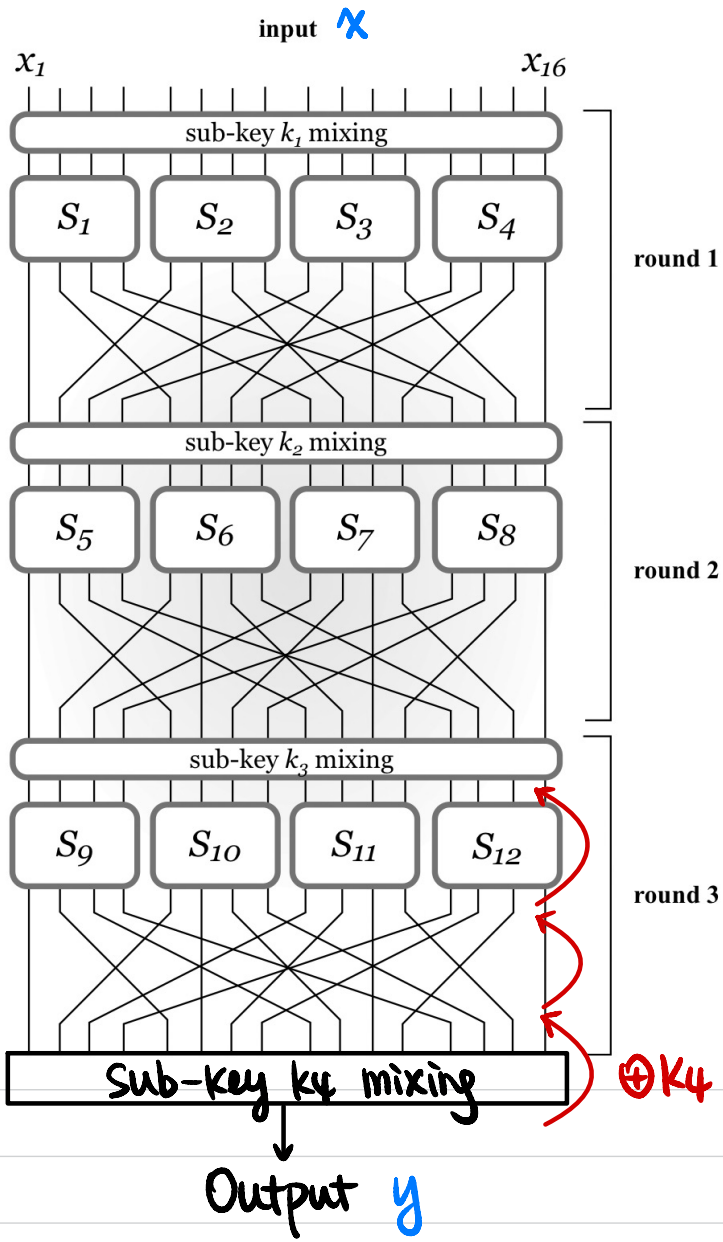
## Key Schedule:

How we derive sub-keys from master key.

## Example:



# Substitution-Permutation Network (SPN)




An SPN is invertible given the master key.

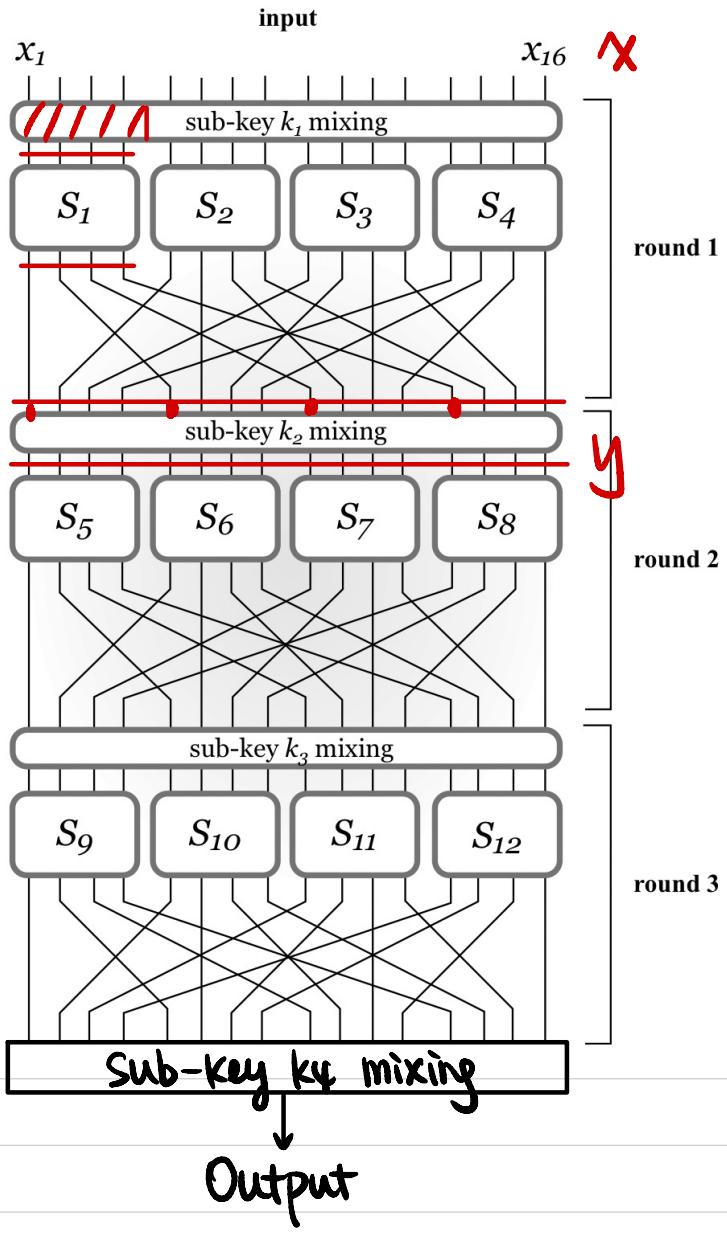
⇓

permutation

How to compute $F_k^{-1}(y)$ ?

Why do we need a final key mixing step ?

# Attacks on Reduced-Round SPN




**1-round SPN w/o final key mixing?**

$(x, y) \Rightarrow$ derive $k_1$

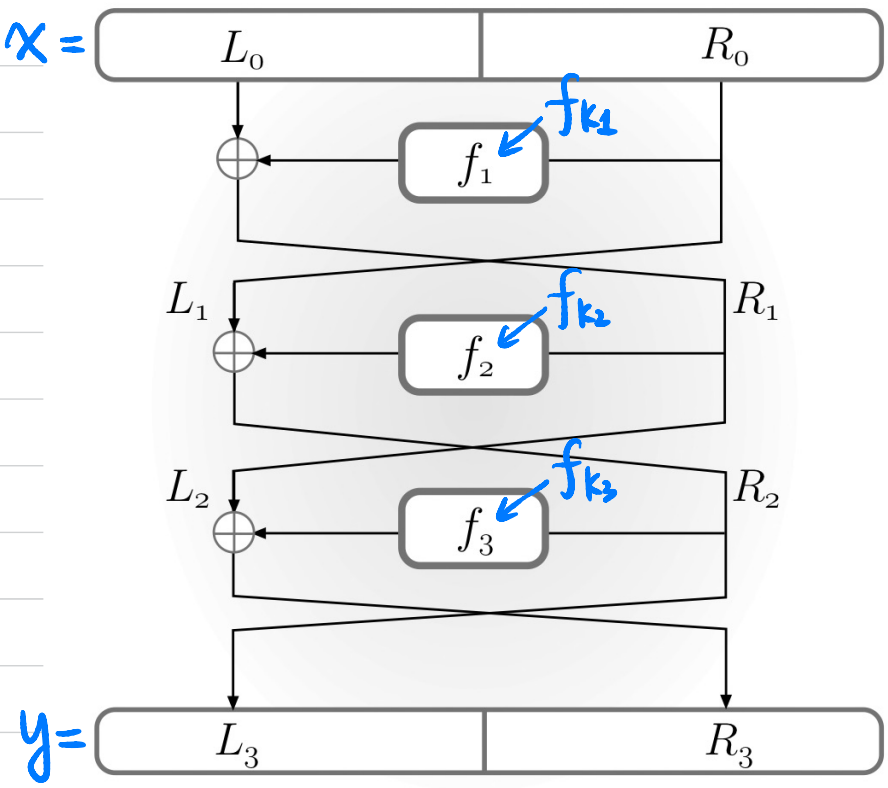
**1-round SPN w/ final key mixing?**

Enumerate all possible $k_2 \Rightarrow$ derive $k_1$

$O(2^{16})$

Enumerate block-by-block: $O(2^4 \cdot 4)$

# Feistel Network




## 3-round Feistel Network

$f_{k_i}: \{0,1\}^{n/2} \to \{0,1\}^{n/2}$

↑ round function

How to compute $F_k^{-1}(y)$ ?

## Attacks on reduced-round Feistel Network