CSCI 1510

- CBC-MAC (continued)
- CCA-Security & Unforgeability
- Authenticated Encryption

Message Authentication Code (MAC)

- **Syntax:**

  A message authentication code (MAC) scheme is defined by PPT algorithms (Gen, Mac, Vrfy):

  $$k \leftarrow Gen(1^n)$$

  $$t \leftarrow Mac_k(m) \qquad m \in \{0,1\}^*$$

  $$0/1 := Vrfy_k(m,t)$$

- **Correctness:** $\forall n$, $\forall k$ output by $Gen(1^n)$, $\forall m \in \{0,1\}^*$

  $$Vrfy_k(m, Mac_k(m)) = 1$$

- **Canonical Verification:**

  If $Mac_k(m)$ is derterministic, then $Vrfy_k(m,t)$ is straightforward.

  $$Mac_k(m) \stackrel{?}{=} t$$

Message Authentication Code (MAC)

Def 1 A message authentication code (MAC) scheme $\Pi = (\mathsf{Gen}, \mathsf{Mac}, \mathsf{Vrfy})$ is existentially unforgeable under adaptive chosen message attack, or EU-CMA-secure, or secure, if $\forall$PPT $\mathcal{A}$, $\exists$ negligible function $\varepsilon(\cdot)$ s.t.

$$\Pr[\mathsf{MacForge}_{\mathcal{A},\Pi} = 1] \le \varepsilon(n).$$

$C(1^n)$		$\mathcal{A}(1^n)$
$k \leftarrow \mathsf{Gen}(1^n)$		
	$\xleftarrow{\ m \in \{0,1\}^*\ }$	
$t \leftarrow \mathsf{Mac}_k(m)$		
	$\xrightarrow{\ t\ }$	
		Output (m^*, t^*)

$Q := \{m \mid m \text{ queried by } \mathcal{A}\}$

$\mathsf{MacForge}_{\mathcal{A},\Pi} = 1$ ($\mathcal{A}$ succeeds) if

① $m^* \notin Q$, and

② $\mathsf{Vrfy}_k(m^*, t^*) = 1$.

Message Authentication Code (MAC)

Def 2 A message authentication code (MAC) scheme $\Pi = (\mathrm{Gen}, \mathrm{Mac}, \mathrm{Vrfy})$ is strongly secure if $\forall$ PPT $\mathcal{A}$, $\exists$ negligible function $\varepsilon(\cdot)$ s.t.

$$\Pr[\mathrm{MacForge}^{s}_{\mathcal{A},\Pi} = 1] \leq \varepsilon(n).$$

$C(1^n)$		$\mathcal{A}(1^n)$
$k \leftarrow \mathrm{Gen}(1^n)$		
	$\xleftarrow{\; m \in \{0,1\}^* \;}$	
$t \leftarrow \mathrm{Mac}_k(m)$		
	$\xrightarrow{\quad t \quad}$	
		Output (m^*, t^*)

$Q := \{(m, t) \mid m$ queried by $\mathcal{A}$, t is the response$\}$

$\mathrm{MacForge}^{s}_{\mathcal{A},\Pi} = 1$ ($\mathcal{A}$ succeeds) if

① $(m^*, t^*) \notin Q$, and

② $\mathrm{Vrfy}_k(m^*, t^*) = 1$.

Thm If $\Pi = (\mathrm{Gen}, \mathrm{Mac}, \mathrm{Vrfy})$ is a secure MAC with canonical verification (Mac is a deterministic algorithm), then Π is also strongly secure.

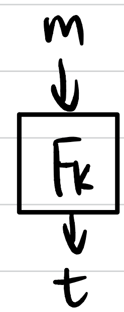
$m^* \neq m$

Fixed-Length MAC

Let $F: \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ be a PRF.

Construct a MAC scheme:

- $\text{Gen}(1^n)$: sample $k \xleftarrow{\$} \{0,1\}^n$, output k.
- $\text{Mac}_k(m)$: $m \in \{0,1\}^n$
  output $t := F_k(m)$
- $\text{Vrfy}_k(m,t)$: $F_k(m) \stackrel{?}{=} t$



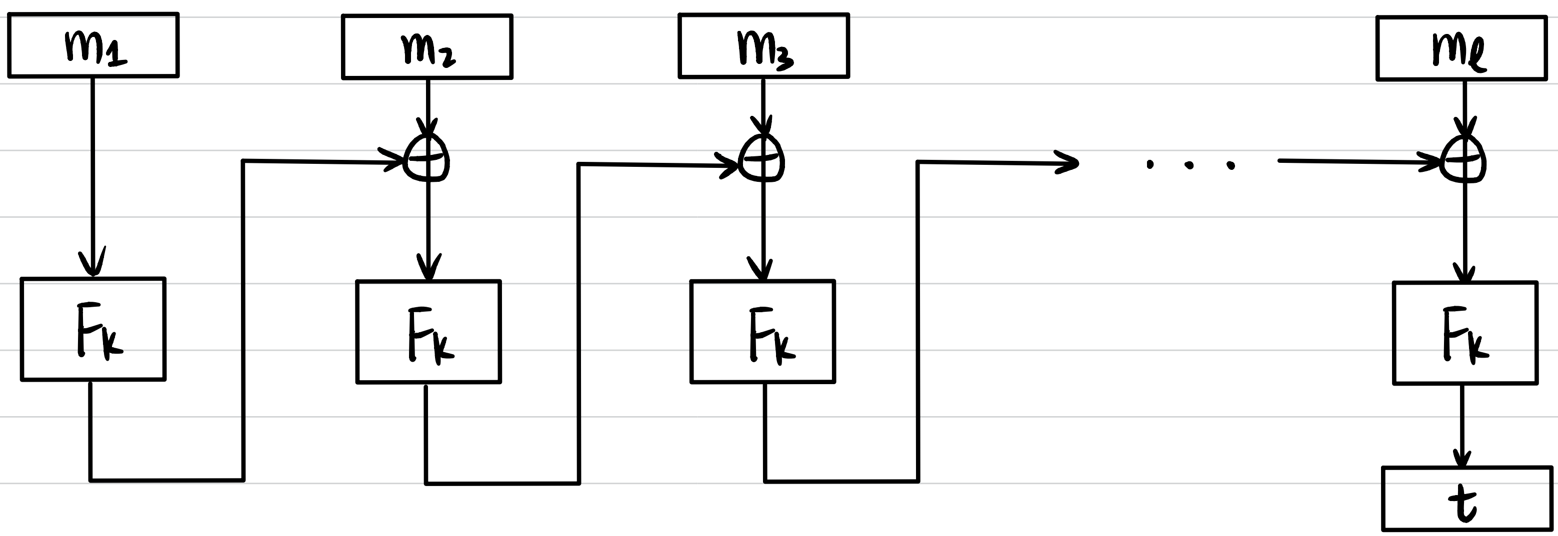
<u>Thm</u> If F is a PRF, then $\Pi = (\text{Gen}, \text{Mac}, \text{Vrfy})$ is a secure MAC scheme for fixed-length messages of length n.

CBC-MAC (for fixed-length messages)

Let $F: \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ be a PRF.

Construct a MAC scheme for messages of length $\ell(n) \cdot n$:

- $Gen(1^n)$: sample $k \xleftarrow{\$} \{0,1\}^n$, output k.
- $Mac_k(m)$: $m \in \{0,1\}^{\ell(n) \cdot n}$ $\quad m = m_1 \| m_2 \| \cdots \| m_\ell \quad m_i \in \{0,1\}^n$



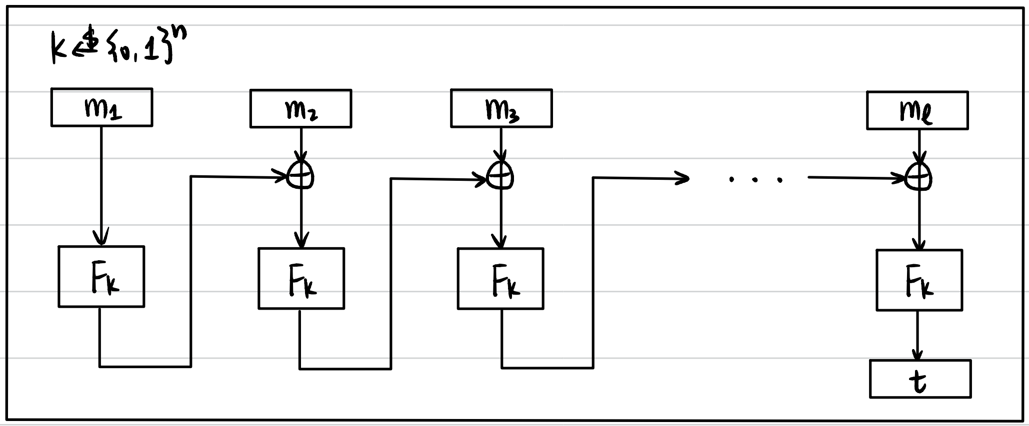
- $Vrfy_k(m,t)$: $Mac_k(m) \stackrel{?}{=} t$

<u>Thm</u> If F is a PRF, then CBC-MAC is a secure MAC scheme for fixed-length messages of length $\ell(n) \cdot n$.

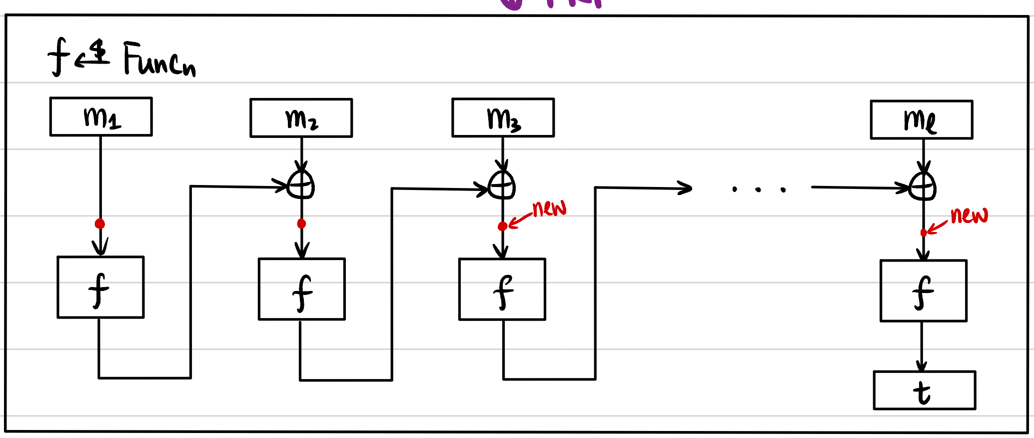
Thm If F is a PRF, then CBC-MAC is a secure MAC scheme for fixed-length messages of length $\ell(n)\cdot n$.

Proof Sketch $\quad \mathrm{Mac}: \{0,1\}^n \times \{0,1\}^{\ell(n)\cdot n} \to \{0,1\}^n$

Suffices to show Mac is a PRF.



↕ PRF



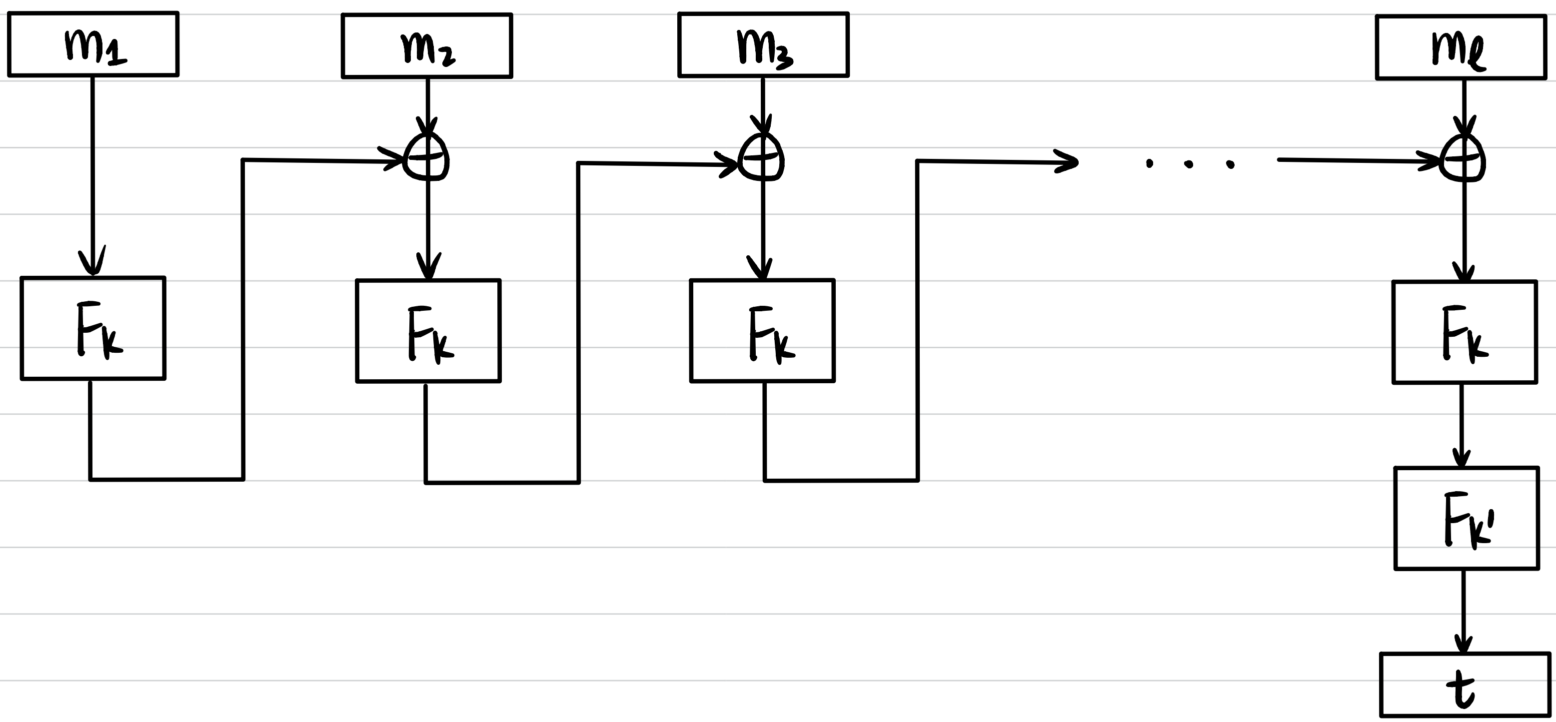
↕ statistical

$g \xleftarrow{\$} \{ h \mid h: \{0,1\}^{\ell(n)\cdot n} \to \{0,1\}^n \}$

$t := g(m_1 \| \cdots \| m_\ell)$

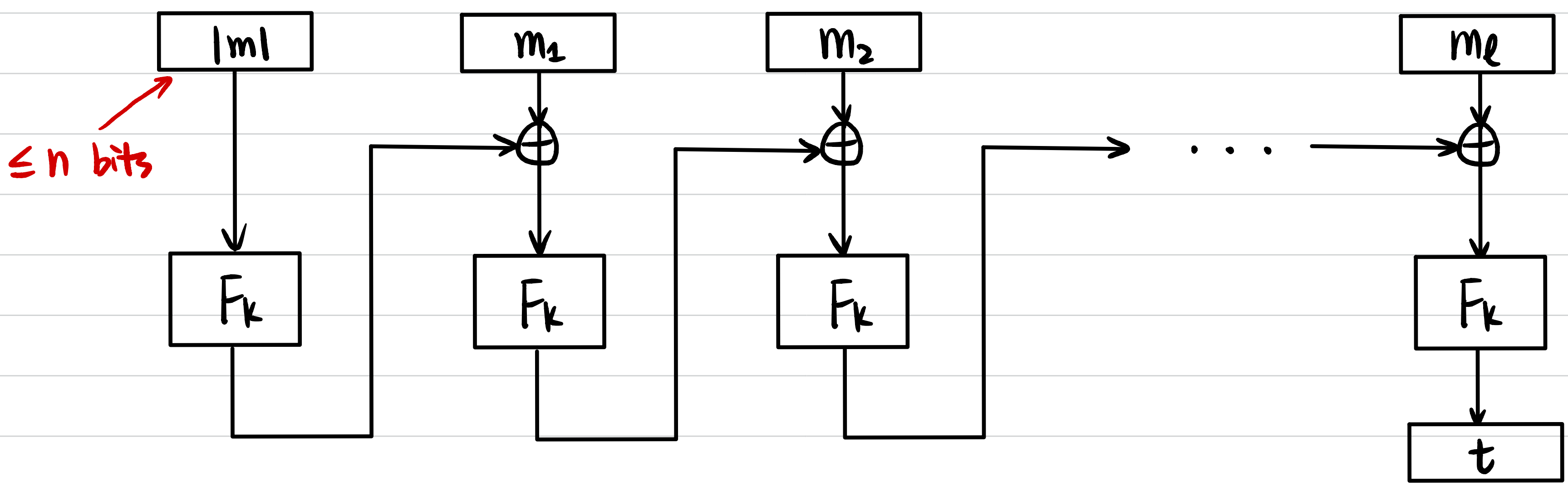
MAC for messages of arbitrary length (multiple of n)

Approach 1: MAC of CBC-MAC

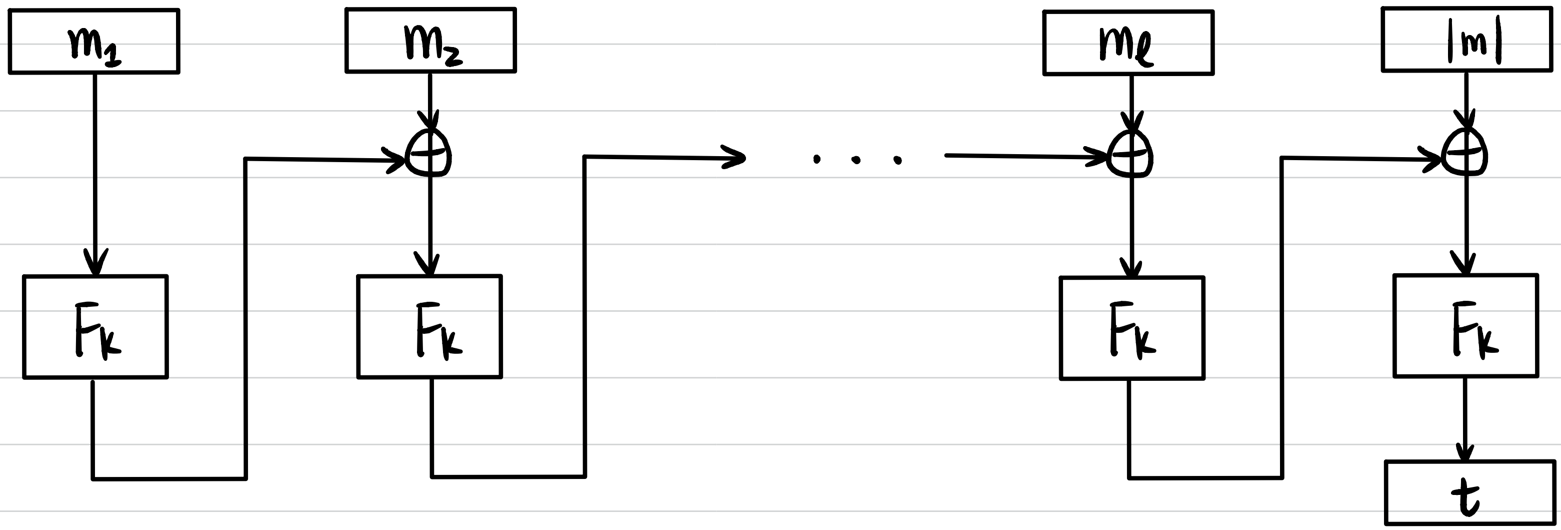


MAC for messages of arbitrary length (multiple of n)

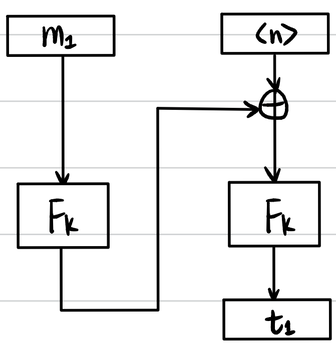
Approach 2: CBC-MAC on $|m| \| m$

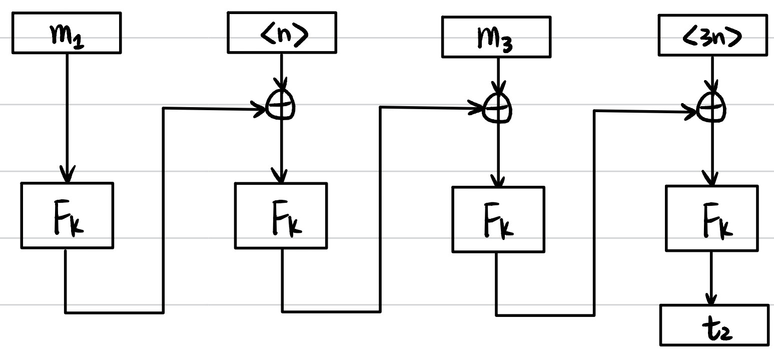


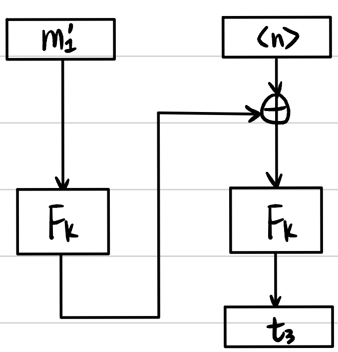
Exercises

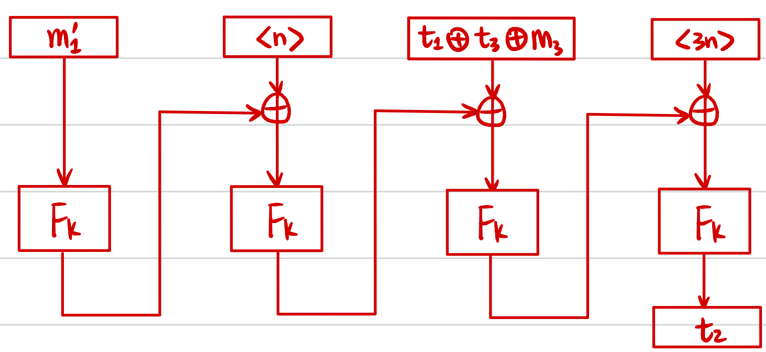


Show this is not a secure MAC for messages of arbitrary length (multiple of n).

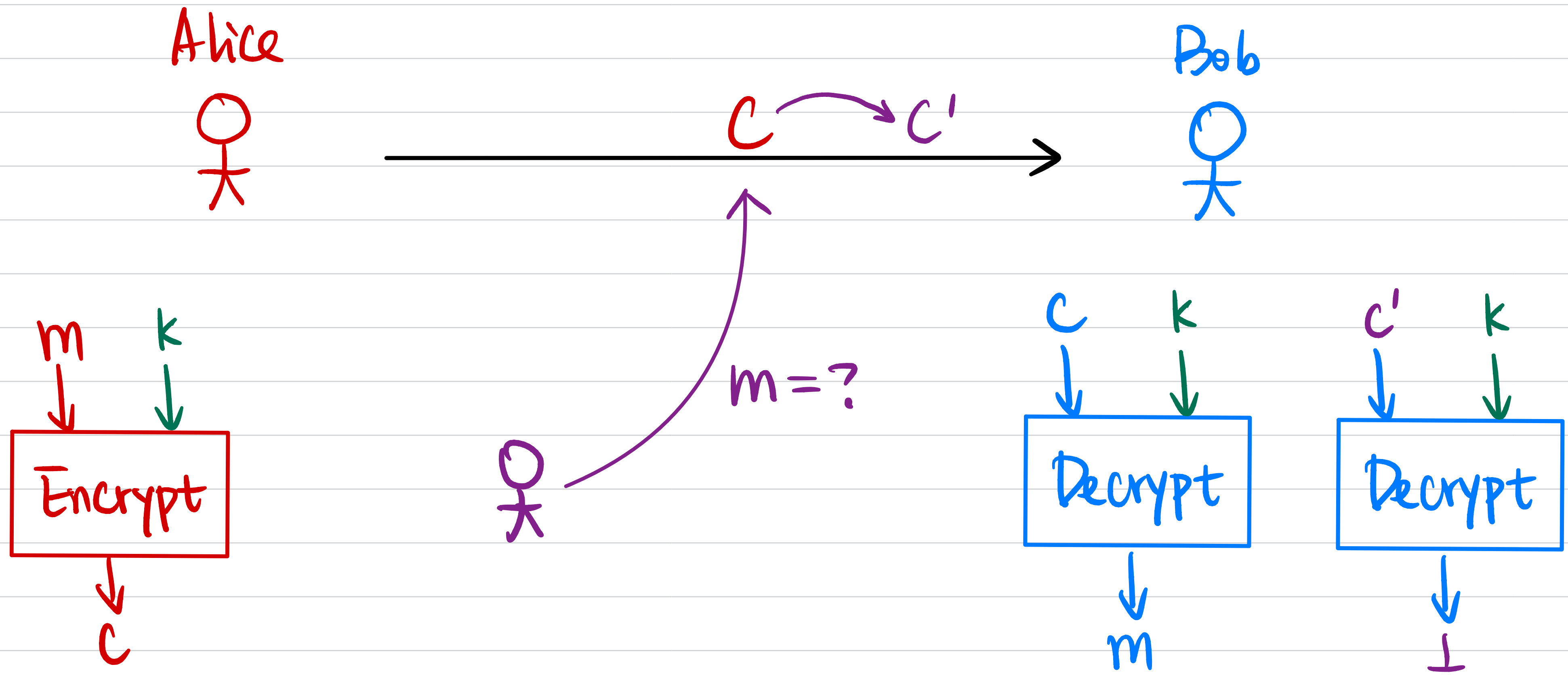








Authenticated Encryption

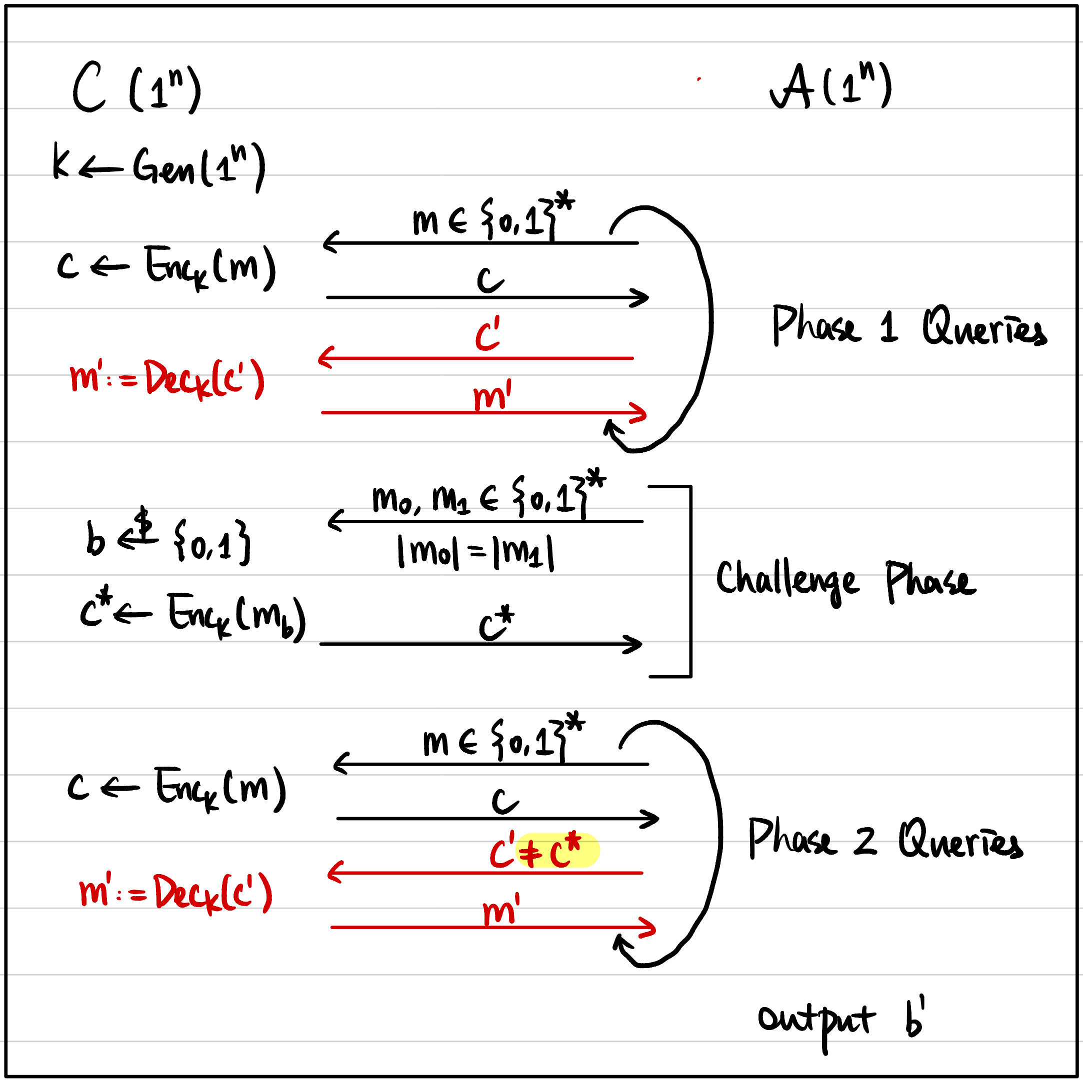


Security Guarantees:

- Message Secrecy: CCA Security
- Message Integrity: Unforgeability

Chosen Ciphertext Attack (CCA) Security

Def A symmetric-key encryption scheme $(\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ is secure against chosen ciphertext attacks, or CCA-secure, if $\forall$ PPT $\mathcal{A}$, $\exists$ negligible function $\varepsilon(\cdot)$ s.t. $\Pr[b=b'] \le \frac{1}{2} + \varepsilon(n)$



Unforgeability

Def A symmetric-key encryption scheme $\Pi = (\mathrm{Gen}, \mathrm{Enc}, \mathrm{Dec})$ is unforgeable if $\forall$ PPT $\mathcal{A}$, $\exists$ negligible function $\varepsilon(\cdot)$ s.t. $\Pr[\mathrm{EncForge}_{\mathcal{A},\Pi} = 1] \le \varepsilon(n)$.

$C(1^n)$ $\qquad\qquad\qquad\qquad$ $\mathcal{A}(1^n)$

$k \leftarrow \mathrm{Gen}(1^n)$

$\xleftarrow{\;m \in \{0,1\}^*\;}$

$c \leftarrow \mathrm{Enc}_k(m)$

$\xrightarrow{\;c\;}$

output c^*

$Q := \{m \mid m \text{ queried by } \mathcal{A}\}$

$m^* := \mathrm{Dec}_k(c^*)$

$\mathrm{EncForge}_{\mathcal{A},\Pi} = 1$ ($\mathcal{A}$ succeeds) if

① $m^* \notin Q$, and

② $m^* \neq \perp$

Def A symmetric-key encryption scheme is authenticated encryption if it is CCA-secure and unforgeable.

Exercises

Is the CPA-secure encryption from PRF CCA-secure? Unforgeable?

$Enc_k(m)$: $m \in \{0,1\}^n$

$r \xleftarrow{\$} \{0,1\}^n$

output $c := \langle r, F_k(r) \oplus m \rangle$

Not CCA-secure: $\quad C \qquad\qquad \mathcal{A}$

$\xleftarrow{m_0,\ m_1}$

$\xrightarrow{c^* = \langle r^*,\ s^* = F_k(r^*) \oplus m_b \rangle}$

$\xleftarrow{c' = \langle r^*,\ s' \rangle}$

$\xrightarrow{m' = F_k(r^*) \oplus s'}$ $\Rightarrow$ derive $F_k(r^*)$ $\Rightarrow$ derive m_b

Not unforgeable: $\mathcal{A}$ can output an arbitrary $2n$-bit string.

Intuitions

Can we have an encryption scheme that is unforgeable but not CCA-secure?

$ct \to ct'$ encrypting the same message

But hard to generate a new ct encrypting a new message

Can we have an encryption scheme that is CCA-secure but not unforgeable?