PRIMAL DUAL ALGORITHM

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Example:

Primal $\min 2x_1 + 2x_2 + 3x_3$

s.t. $x_1 + x_3 \geq 1$

$x_2 + x_3 \geq 2$

$x_{1,2,3} \geq 0$

After introducing slack and artificial variables for greater than constraints, we have:

$$x_1 + x_3 - s_1 + a_1 = 1$$

$$x_2 + x_3 - s_2 + a_2 = 2$$

$$a_1, a_2, s_1, s_2 \geq 0$$

Initial tableau is:

a_1	a_2	s_1	s_2	x_1	x_2	x_3	RHS
0	0	0	0	2	2	3	0
1	0	-1	0	1	0	1	1
0	1	0	-1	0	1	1	2

And we have:

c^T = [0 0 2 2 3] "coefficients of s's and x's in row(0)"

π^T = (0,0) "bfs for dual of P"

$\pi^T \cdot A$= [0 0 0 0 0]

Now, we need to decide which of the columns are admissible.

$$J(\pi) = \{ j \mid (c^T - \pi^T A) \cdot j = 0 \}$$

Let's compare c^T: [0 0 2 2 3] and $\pi^T \cdot A$: [0 0 0 0 0]. We see that s1 and s2 are equal on both sides, hence admissible.

$$J(\pi) = \{s_1, s_2\}$$

Restricted primal can now be formulated using the admissible columns:

Restricted Primal **min $a_1 + a_2$**

s.t $a_1 - s_1 = 1$

$a_1 - s_2 = 2$

$a_1, a_2, s_1, s_2 \geq 0$

Initial tableau is:

a_1	a_2	s_1	s_2	RHS
1	1	0	0	0
1	0	-1	0	1
0	1	0	-1	2

Notice that, the tableau is not in proper form. We need to eliminate 1's from row(0) to obtain the identity.

a_1	a_2	s_1	s_2	RHS
0	0	1	1	-3
1	0	-1	0	1
0	1	0	-1	2

Now, the tableau is in proper form and also in the final state. But RHS is -3, so artificials are not zero.

$$\pi^r = [1\ 1] - [0\ 0] = [1\ 1]$$

Update π: $\pi^{new} = \pi + \lambda\pi^r$

In order to find λ, we will only consider j's that are $\not\ni J(\pi)$ and $A_j \cdot \pi^r > 0$

$J \not\ni J(\pi)$ is $\{x_1, x_2, x_3\}$ and

$A_{x1}\,\pi^r \rightarrow [1\ 0] \cdot [1\ 1] > 0$

$A_{x2}\,\pi^r \rightarrow [0\ 1] \cdot [1\ 1] > 0$

$A_{x3}\,\pi^r \rightarrow [1\ 1] \cdot [1\ 1] > 0$

We only consider for j: x_1, x_2, x_3

$$\lambda \text{ is } \min\left\{\frac{cj - \pi \cdot Aj}{Aj \cdot \pi r}\right\} = \min\left\{\frac{2-0}{1}, \frac{2-0}{1}, \frac{3-0}{2}\right\} \rightarrow \lambda = 3/2$$

Update π: $\pi^{new} = \pi + \lambda\pi^r$

$\pi^{new} = (0, 0) + 3/2 \cdot (1, 1) = (3/2, 3/2)$

Now, we need to decide which of the columns are admissible.

$$J(\pi) = \{ j \mid (c^T - \pi^T A) \cdot j = 0 \}$$

So we compare c^T [0 0 2 2 3] and $\pi^T \cdot A = (3/2, 3/2) \cdot A$ = [-3/2, -3/2, 3/2, 3/2, 3]. We see that x3 equals on both sides, hence admissible.

$$J(\pi) = \{x_3\}$$

We write the restricted primal using the admissible columns:

Restricted Primal $\quad$ **min $a_1 + a_2$**

s.t $\quad$ **$a_1 +_3 = 1$**

$a_2 + x_3 = 2$

$a_1, a_2, x_3 \geq 0$

Initial tableau is:

a_1	a_2	x_3	RHS
1	1	0	0
1	0	1	1
0	1	1	2

Notice that, the tableau is not in proper form. We need to eliminate 1's from row(0) to obtain the identity.

a_1	a_2	x_3	RHS
0	0	-2	-3
1	0	1	1
0	1	1	2

We do one simplex iteration:

a_1	a_2	x_3	RHS
2	0	0	-1
1	0	1	1
-1	1	0	1

Now, the tableau is in the final state. But RHS is -1, so artificials are not zero.

$$\pi^r = [1\ 1] - [2\ 0] = [-1\ 1]$$

Update π: $\quad \pi^{new} = \pi + \lambda \pi^r$

In order to find λ, we will only consider j's that are $\not\ni J(\pi)$ and $A_j \cdot \pi^r > 0$

$J \not\ni J(\pi)$ is $\{s_1, s_2, x_1, x_2\}$ and

$A_{x1}\ \pi^r \rightarrow [1\ 0] \cdot [-1\ 1] < 0$ "we do not consider"

$A_{x2}\ \pi^r \rightarrow [0\ 1] \cdot [-1\ 1] > 0$

$A_{s1}\ \pi^r \rightarrow [-1\ 0] \cdot [-1\ 1] > 0$

$A_{s2}\ \pi^r \rightarrow [0\ -1] \cdot [-1\ 1] < 0$ "we do not consider"

We only consider for j: s_1, x_2

$$\lambda \text{ is } min\left\{\frac{cj-\pi\cdot Aj}{Aj\cdot\pi r}\right\} = min\left\{\frac{0-(-\frac{3}{2})}{1}, \frac{2-\frac{3}{2}}{1}\right\} \rightarrow \lambda = 1/2$$

Update π: $\pi^{new} = \pi + \lambda\pi^r$

$\pi^{new} = (3/2, 3/2) + 1/2 \cdot (-1, 1) = (1, 2)$

Now, we need to decide which of the columns are admissible.

$$J(\pi) = \{ j \mid (c^T - \pi^T A)\cdot j = 0 \}$$

So we compare c^T [0 0 2 2 3] and $\pi^T \cdot A = (1/2, 1/2)\cdot A$ = [-1, -2, 1, 2, 3]. We see that x2 and x3 are equal on both sides, hence admissible.

$$J(\pi) = \{ x_2, x_3\}$$

We write the restricted primal using the admissible columns:

Restricted Primal **min a1 + a2**

s.t a1 + x3 = 1

a2 + x2 + x3 = 2

a1, a2, x2, x3 ≥ 0

Initial tableau is:

a_1	a_2	x_2	x_3	RHS
1	1	0	0	0
1	0	0	1	1
0	1	1	1	2

Notice that, the tableau is not in proper form. We need to eliminate 1's from row(0) to obtain the identity.

a_1	a_2	x_2	x_3	RHS
0	0	-1	-2	-3
1	0	0	1	1
0	1	1	1	2

We do one simplex iteration:

a_1	a_2	x_2	x_3	RHS
1	1	0	0	0
1	0	0	1	1
-1	1	1	0	1

Now, the tableau is in the final state. And RHS is 0, so RP is feasible. Hence we are done!

Dual solution is $\pi^T = [1, 2]$.

Primal solution is $x_2 = 1$, $x_3 = 1$ (from the last tableau) and $x_1 = 0 \rightarrow x^{T=} [0, 1, 1]$.