Chapter 1, Paragraph 4 – Lecture Notes

Bland's Anticycling Algorithm (Slide 9)

What does it mean to choose $B(i)$ minimal?
Suppose we we have a tableau

$$
\begin{array}{cccccccc|c}
\vdots & 27 & \vdots & 129 & \vdots & 503 & & & \vdots \\
\hline
\vdots & 0 & \vdots & 0 & \vdots & 1 & & & \vdots \\
\vdots & 0 & \vdots & 0 & \vdots & 0 & & & \vdots \\
\vdots & 0 & \vdots & 1 & \vdots & 0 & & & \vdots \\
\vdots & 1 & \vdots & 0 & \vdots & 0 & & & \vdots
\end{array}
$$

What does B look like?

$$B = (503, 27, 129)$$

$B(i)$ is minimal with $i = 2, B(i) = 27$
Remember,

$$\lambda = \min\left\{\frac{x_{B(i)}}{-y_{B(i)}}|y_{B(i)} < 0\right\} = \min\left\{\frac{\bar{b}_i}{\bar{a}_i^t}|\bar{a}_i{}^t > 0\right\}$$

An observation : consider the tableau

$$
\begin{array}{cccccccc|c}
\vdots & 27 & 129 & \vdots & t & 503 & q & & \vdots \\
\hline
\vdots & 0 & 0 & \vdots & \leq 0 & 1 & 0 & & 0 \\
\vdots & 0 & 0 & \vdots & \oplus & 0 & 1 & & 0 \\
\vdots & 0 & 1 & \vdots & \leq 0 & 0 & 0 & & 0 \\
\vdots & 1 & 0 & \vdots & \leq 0 & 0 & 0 & & 0
\end{array}
$$

If we choose variable q to leave the basis, with the right-hand-side equal to zeroes, it implies that the element in nonbasic column t in the row where q contains a 1 is positive and all other elements are ≤ 0.

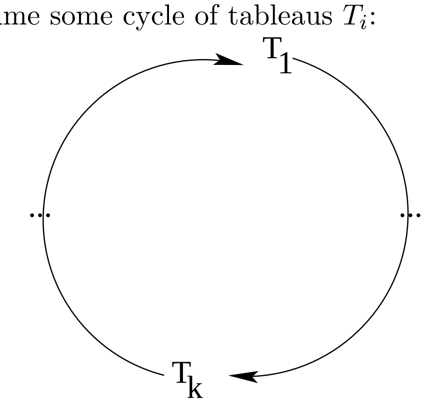
Assume there is a cycle among columns c_2, c_4, and q in our tableau:

	$\vdots$	c_2	$\vdots$	c_4	$\vdots$	q	$\vdots$
1	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
2	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	1
3	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0
4	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	0

Then we can remove all rows and columns not involved in the cycle while still maintaining a cycle:

	c_2	c_4	q	$\vdots$
1	$\vdots$	$\vdots$	$\vdots$	0
3	$\vdots$	$\vdots$	$\vdots$	0
4	$\vdots$	$\vdots$	$\vdots$	0

Now assume some cycle of tableaus T_i:



Where q enters the basis after tableau T_1 , and q leaves the basis after tableau T_k .

$$
\begin{array}{c|c}
 & \overbrace{\hphantom{\geq 0 \ldots \geq 0 \ldots \ominus}}^{c^T} \\
 & \hfill q \\
 & \geq 0 \ldots \geq 0 \ldots \ominus & \vdots \\
\hline
 & \vdots & \vdots \\
T_1 : & \vdots & \vdots \\
(q\ enters) & \vdots & \vdots \\
 & \vdots & \vdots
\end{array}
$$

$$\begin{array}{llcccc|cl}
 & & & p & & q & & \\
 & \bar{c}^T & & \ominus & & 0 & \vdots & \\
\hline
 & & \vdots & \leq 0 & \vdots & \vdots & \vdots & \\
T_k : & & \vdots & \leq 0 & \vdots & \vdots & \vdots & \\
(q\ leaves) & & \vdots & \oplus & \vdots & 1 & \vdots & r \\
 & & \vdots & \leq 0 & \vdots & \vdots & \vdots &
\end{array}$$

Due to Bland's Anticycling restrictions, we know that for these 2 stages in the cycle, the following 4 statements hold:

$$(1)\ \forall i < q,\ c_i \geq 0$$
$$(2)\ c_q < 0$$
$$(3)\ \bar{c}_p < 0,\ \bar{a}_r^p > 0$$
$$(4)\ \forall i \neq r,\ \bar{a}_i^p \leq 0$$

Given our cycle, and the Bland's rule restrictions, the following must hold:

We choose y as: $y_j = \begin{cases} -\bar{a}_i^p & \text{if } \bar{B}(i) = j \\ 1 & \text{if } j = p \\ 0 & \text{if } j \neq p \end{cases}$

(a)

$$c^T y = \bar{c}_p \leq 0$$

(b)

$$\begin{aligned} 0 \leq c^T y &= \sum_{j=1}^{q} c_j y_j \\ &= \sum_{i=1}^{m} -\bar{a}_i^p c_{\bar{B}(i)} + c_p \\ &= \Big(\sum_{i=1, i \neq r}^{m} -\bar{a}_i^p c_{\bar{B}(i)}\Big) - \bar{a}_r^p c_{\bar{B}(r)} + c_p \end{aligned}$$

$c_p \geq 0$, from (1) and the fact that $p < q$
$c_{\bar{B}(r)} = c_q < 0$ from (2) and $\bar{a}_r^p > 0$ from (3). Thus $-\bar{a}_r^p c_{\bar{B}(r)} > 0$
For $i \neq r, c_{\bar{B}(i)} \geq 0$ from (1) and $\bar{a}_i^p \leq 0$ from (4).

Thus the entire sum is ≥ 0 and we have reached a contradiction between (a) and (b).

Further notes about Bland's rule:
(1) Bland's anticycling will not avoid getting into a degeneracy, but once at a degenerate corner is guaranteed to escape.
(2) In practice one finds the use of "mixed pivot rules" – If your objective function is not improving after some number iterations, use Bland's Anticycling rules until improvement is seen, otherwise use whatever pivot rule you prefer.