Problem 1

a) Give the recurrence relation for the following problem: Given a sequence of numbers, find the longest strictly increasing subsequence. The subsequence does not have to be contiguous.

b) Give the longest strictly increasing subsequence for the following sequence and show your dynamic programming table.

3 0 1 5 2 7 4 5 0 3 2

Problem 2

The Mayor of Providence has begun an urban renewal project to showcase Providence to the world. He has demolished several old buildings and wants you to build a tower out of their remains. The tower must be as tall as possible so everybody knows what a great Mayor he is. Luckily for you, the buildings have crumbled into n different blocks, where the dimensions of block i are given by the 3-tuple (w_i, ℓ_i, h_i) which stand for the width, length, and height of the block respectively. Place the blocks on top of each other to build the tallest tower possible. You may not rotate blocks in any direction, and in order for a block i to be placed on top of a block j, $w_i < w_j$, $\ell_i < \ell_j$ must hold.

a) Give the recurrence relation of the dynamic program and the pseudo code to fill the DP table to build the Mayor his tower.

b) What is the run time of your DP?

Problem 3

Assume you are given an ARP problem consisting of a listing of overlapping programs each with a start time s_i, end time e_i, profit p_i for program i, where there are n total programs, and a low recording capacity C.

a) Give the recurrence relation for a dynamic program solving the ARP problem and point out which cell has the optimal value. (Hint: Sort programs by their ending times)

b) What is the run time of this algorithm?