CS149

Problem 1

Show that the following matrix is totally unimodular.

		1 1 1 1 1	1 1 1	1 1 1 1 1	1 1 1 1	1 1 1 1
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	-1 -1 -1 -1 -1 -1	1 1 1	1 1 1	1 1 1	1 1	1 1 1

Problem 2

- a) Prove that a node-arc incidence matrix is TU. (Recall that a node-arc incidence matrix is a matrix where, for a directed graph, each column corresponds to an edge, and each row to a vertex. For each vertex, there is a −1 in the column of every edge entering that vertex, and a 1 for every edge leaving that vertex.)
- b) Prove that if A is a TU matrix, and we perfom one pivot step on A to get A', A' is TU.

Problem 3

You are asked to provide a one-to-one matching between five professors and five graduate students for the upcoming seminar. Everybody speaks different language and the communication barrier between pairs is given below, the higher the number the more difficult to communicate is. A zero means that they are unable to work together. You should pair one professor with one student such that total language overhead is minimized.

Draw the graph that visualizes the problem. Write down the LP. Think about the Dual, explore the Dual solution.

gr1 gr2 gr3 gr4 gr5pr1 4 4 \inf inf 8 pr2 inf 5 \inf 3 inf \inf 9inf pr3 2555pr4 6 \inf inf pr5 inf 4 inf 8 inf