Problem 1

We are given the following linear program:

$$
\begin{array}{lrrrrl}
\max & -x_1 & -4x_2 & -3x_3 & -3x_4 & \\
\text{s.t.} & -x_1 & -x_2 & & +x_4 & \leq 1 \\
 & x_1 & -x_2 & -x_3 & -x_4 & \leq 2 \\
 & x_1 & +x_2 & & -x_4 & \leq -1 \\
 & -x_1 & +x_2 & +x_3 & +x_4 & \leq -2 \\
 & \multicolumn{5}{l}{x_1, x_2, x_3, x_4 \geq 0}
\end{array}
$$

Add slack variables (s_1, s_2, s_3, s_4) to convert the problem to standard form, and solve it using the dual simplex algorithm.

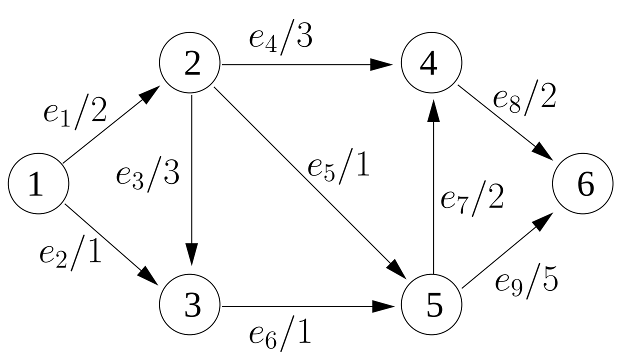
Problem 2

Use the dual simplex algorithm to solve the following linear problem:

$$
\begin{array}{lrrl}
\max & -x_1 & -2x_2 & \\
\text{s.t.} & -3x_1 & +x_2 & \leq -1 \\
 & x_1 & -x_2 & \leq 1 \\
 & -2x_1 & +7x_2 & \leq 6 \\
 & 9x_1 & -4x_2 & \leq 6 \\
 & -5x_1 & +2x_2 & \leq -3 \\
 & 7x_1 & -3x_2 & \leq 6 \\
 & & & x_1, x_2 \geq 0
\end{array}
$$

Problem 3

We will solve the shortest-path problem using the primal-dual algorithm. Consider the following directed graph that has $n = 6$ nodes and $m = 9$ edges.



We can model the shortest-path problem from node 1 to node 6, as a linear program as follows. Let $A \in \mathbb{R}^{n \times m}$ be the matrix where rows correspond to nodes and columns correspond to edges, and whose entry a_{ij} equals

$$a_{ij} = \begin{cases} 1, & \text{if edge } e_j \text{ begins at node } i, \\ -1, & \text{if edge } e_j \text{ ends at node } i, \\ 0, & \text{otherwise.} \end{cases}$$

Let $\hat{A}$ be the $(n-1) \times m$ matrix that equals A with the last row truncated. The vector $d \in {\mathbb{R}_+}^m$ contains the distances of the edges (in the picture, the label of each edge is e_j/d_j). A path p in the graph is given by a vector $x \in \mathbb{R}^m$, where $x_j = 1$ if $e_j \in p$, and $x_j = 0$ otherwise. Then the solution to the shortest-path problem from node 1 to node 6 is given by the solution of the following linear problem:

$$\begin{array}{ll} \min & d^T x \\ \text{s.t.} & \hat{A}x = (1, 0, 0, \ldots, 0)^T \\ & x \geq 0. \end{array}$$

a) Give an explanation why the above problem solves the shortest-path problem. You don't have to give a proof, and you don't have to argue why the solution is going to be integral (unless you want to).

b) Solve the linear problem by applying the primal-dual algorithm.