Problem 1

We want to buy some quantity of meat and bread, in order to satisfy some dietary needs. A kg of meat contains 100g of carbs, 100g of fat and 175g of proteins. A kg of bread contains 250g, 25g, and 150g, respectively. The daily needs are 500g of carbs, 300g of fat, and 600g of proteins. A kg of meat costs \$30, while a kg of bread costs \$25, and we want to cover all the dietary needs with the smallest possible cost.

a) Formulate the above problem as a program in the standard form, and solve it using the dual simplex algorithm. Sketch the feasible region, mark the intermediate basic solutions, and the vectors y pointing from one basic solution to the next.

b) Now formulate the dual program, and apply the regular simplex algorithm. Again sketch the feasible region (in this case the region corresponding to the dual problem) and the intermediate basic solutions (you don't need to draw the y's in this case).

Problem 2

Solve the following problem

$$
\begin{array}{lrrrrrrl}
\min & 3x_1 & +x_2 & +x_3 & +2x_4 & +x_5 & +x_6 & \\
\text{s.t.} & x_1 & & +3x_3 & +x_4 & -5x_5 & +4x_6 & = 7 \\
 & & x_2 & -2x_3 & -x_4 & +4x_5 & -3x_6 & = -3 \\
 & & & & & & 0 \le x_1 & \le 8 \\
 & & & & & & 0 \le x_2 & \le 6 \\
 & & & & & & 0 \le x_3 & \le 4 \\
 & & & & & & 0 \le x_4 & \le 15 \\
 & & & & & & 0 \le x_5 & \le 2 \\
 & & & & & & 0 \le x_6 & \le 10.
\end{array}
$$

Problem 3

Prove Farkas' Lemma:
We are given $a_i \in \mathbb{R}^n, 1 \le i \le m$, and a vector $c \in \mathbb{R}^n$. We define the set

$$\Gamma = \left\{ x \in \mathbb{R}^n : x = \sum_{i=1}^{m} \pi_i a_i, \pi_i \ge 0 \right\}.$$

Prove that if (for all $y \in \mathbb{R}^n$ with $y^T a_i \ge 0$ (for all i) it holds that $y^T c \ge 0$), then and only then $c \in \Gamma$.