Problem 1

Here we will see an example where Bland's rule is needed to avoid cycles. We are given the tableau:

$-3/4$	20	$-1/2$	6	0	0	0	3
$1/4$	-8	-1	9	1	0	0	0
$1/2$	-12	$-1/2$	3	0	1	0	0
0	0	1	0	0	0	1	1

Perform the Simplex algorithm twice, each time using a different tie-breaking rule:

a) Always select the non-basic variable with the most negative $\overline{c_j}$ to enter the basis. In case of a tie, select the basic variable with the smallest subscript to leave the basis. Notice that we arrive on a cycle.

b) Apply Bland's anti-cycling tie-breaking rule. Notice that we manage to avoid the cycle.

Problem 2

For this problem, you **may not** use any proofs relating to Simplex or Duality (Strong or Weak). This problem is intended to motivate these ideas and so should not depend on them. It is not intended to be difficult, and a short 2-3 sentence answer will suffice for each part.

a)

$$\begin{array}{lrrrrl} \text{min} & 2x_1 & +3x_2 & -5x_3 & +x_4 & \\ \text{s.t.} & 2x_1 & +2x_2 & -6x_3 & +\frac{1}{2}x_4 & \geq 5 \\ & & & & x & \geq 0. \end{array}$$

Show that 5 is a lower bound for our objective function.

b)

$$\begin{array}{lrrrrl} \min & 3x_1 & +5x_2 & -x_3 & +7x_4 & \\ \text{s.t.} & 12x_1 & +19x_2 & -4x_3 & +x_4 & \geq 16 \\ & & & & x & \geq 0. \end{array}$$

Show that 4 is a lower bound for our objective function.

c)

$$\begin{array}{lrrrrl} \min & 2x_1 & +5x_2 & -5x_3 & +7x_4 & \\ \text{s.t.} & x_1 & +6x_2 & & +7x_4 & \geq 5 \\ & x_1 & -x_2 & -5x_3 & & \geq 3 \\ & & & & x & \geq 0. \end{array}$$

Show that 8 is a lower bound for our objective function.

Problem 3

The CEO of Sawmill Inc. asks to see next months log hauling schedule to his two sawmills. He wants to make sure he keeps a steady, adequate flow of logs to his sawmills to capitalize on the good lumber market. Secondary, but still important to him, is to minimize the cost of transportation. The average travel cost is \$2 per mile for both loaded and empty trucks. The logging supervisor estimated the number of truckloads of logs coming off each harvest site daily. The number of truckloads varies because terrain and cutting patterns are unique for each site. Finally, the sawmill managers have estimated the truckloads of logs their mills need each day. All these estimates are summarized on the following table.

Logging site	Distance to mills (miles) Mill A	Mill B	Max truckloads/day per logging site
1	8	15	20
2	10	17	30
Mill demand (truckloads/day)	30	20	

a) Formulate the above problem as a linear program.

b) Solve the LP using the 2-phase Simplex algorithm.

Problem 4

Consider the following two-person zero-sum game: Each player holds 5 cards labeled 1 through 5. Each round both players choose a card privately and then reveal their cards simultaneously. If they choose the same card then player 1 gets the value shown on the card. If they show different cards and the two values sum to an even number, then player 1 gets 2 points. If they sum to an odd number then player 2 gets the maximum of the 2 values.

a. Write down the payoff matrix for this game (from player 1's perspective).

b. Write down an LP formulation for computing player 1's optimal strategy.

c. Use CPLEX to solve your formulation. You can find the department distribution of CPLEX as well as documentation under /com/cplex. You can either run CPLEX on a .LP file or using the Concert C++ callable library. Please submit the solution as well as either your .LP file or C++ code.