Problem 1

Provide all the solutions (in the vector form that we did in class and recitation) of the system $\mathbf{Ax} = \mathbf{b}$, when

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & 2 & 1 & 0 & 2 & 0 \\ -6 & 0 & 1 & 0 & 1 & -7 & 0 \\ -5 & 1 & 0 & 0 & 0 & 4 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 4 \\ 2 \\ 2 \\ -5 \end{bmatrix}.$$

Problem 2

Consider the following linear program:

$$\begin{aligned} \max \quad & \mathbf{x_1} + \mathbf{x_2} \\ \text{s.t.} \quad & 2\mathbf{x_1} - \mathbf{x_2} \leq 4 \\ & \mathbf{x_1} + \mathbf{x_2} \leq 6 \\ & -\mathbf{x_1} + \mathbf{x_2} \leq 2 \\ & \mathbf{x} \geq \mathbf{0}. \end{aligned} \tag{1}$$

Recall from lecture that it has the corresponding standard form:

$$\begin{aligned} \min \quad & -\mathbf{x_1} - \mathbf{x_2} \\ \text{s.t.} \quad & \mathbf{s_1} + 2\mathbf{x_1} - \mathbf{x_2} = 4 \\ & \mathbf{s_2} + \mathbf{x_1} + \mathbf{x_2} = 6 \\ & \mathbf{s_3} - \mathbf{x_1} + \mathbf{x_2} = 2 \\ & \mathbf{x} \geq \mathbf{0}, \mathbf{s} \geq 0 \end{aligned} \tag{2}$$

- a) Determine the basic solutions corresponding to the bases $B_1 = \{s_1, s_2, s_3\}$, $B_2 = \{s_1, x_1, x_2\}$, $B_3 = \{s_2, s_3, x_1\}$ and say whether or not they are feasible.
- b) For each basis, compute A_B and ${A_B}^{-1}$.
- c) Determine the vectors that give all the solutions to the homogeneous system $\mathbf{Ay} = \mathbf{0}$ for each of the bases obtained in part a (the $\mathbf{y}$'s that we computed in class/recitation).
- d) Make a 2D sketch of each of the problems and draw the vectors you found in part b.
- e) Find the optimal solution to the system and prove its optimality.

Problem 3

Prove that if $K \subseteq \mathbb{R}^n$ is convex and $a^1, \ldots, a^m \in K$, then $\kappa(a^1, \ldots, a^m) \subseteq K$.

Problem 4

Given the set $\{x : |x_i - y_i| < \epsilon, i = 1, 2, \ldots, n\}$ with some real $\epsilon > 0$ and $y \in \mathbb{R}^n$, what are the extreme points? Provide a proof.