Problem 1

Formulate the Knapsack Problem mathematically.

Problem 2

The company "Olivander, Inc." produces several lengths of wands. Wands are cut from ten inch long mahogany staffs that Mr. Olivander orders from a supplier. Mr. Olivander would like to produce 120 wands of length 4", 60 of length 3", and 80 of length 2".

a) Mathematically formulate the optimization problem of finding the minimum number of staffs that Mr. Olivander needs to buy if he only considers losing at most 10% of a staff when cutting it down to smaller lengths.

b) How many mahogany staffs does he need to buy? Provide two different optimal solutions (with the minimal number of staffs to buy) telling him how he can cut down the staffs so that he meets his demands.

c) Mr. Olivander now wants to produce 120 seven inch wands, 75 three inch wands, and 160 two inch wands, but he can only buy stock wands 11 inches long. Can you come up with a good solution for how he should cut the wands? How many staffs does he have to buy, and how does he need to cut them? Can you prove your solution optimal?

Problem 3

Find all solutions of the following linear system: (Hint: if you don't remember how to find the null space, you should review your linear algebra)

$$\begin{aligned} 2x_1 + 4x_2 - 2x_3 + 4x_4 - 2x_5 &= -4 \\ 4x_1 + 3x_2 + x_3 - 2x_4 + x_5 &= 2 \\ 3x_1 + 2x_2 + x_3 - 2x_4 + x_5 &= 2 \\ x_1 + 2x_2 - x_3 + 3x_4 + x_5 &= 1 \end{aligned}$$

Problem 4

Olivander Inc. also creates furniture, in particular chairs and tables. For each chair that the company creates, the company can make a profit of \$45, while for every table there is a profit of \$80. In order to create a chair, 5 square feet of mahogany and ten man hours are needed. In order to create a table, 20 square feet of mahogany and fifteen man hours are required. The total available stock of mahogany is 400 square feet, while the total man hours available are 450.

a) Give a mathematical formulation of the problem of maximizing the total profit.

b) Sketch graphically the feasible region.

c) Use the illustration to deduce the optimal solution to the problem.