Spectrum Auctions

CSCI 1440/2440

2025-11-05

We introduce several alternative combinatorial auction designs, other than the classic VCG mechanism. We cite unsuccessful examples of real-world wireless spectrum auctions as evidence that sequential and simultaneous Vickrey auctions suffer from immediate deficiencies. Simultaneous ascending auctions, in contrast, offer more promise.

1 *Combinatorial Auctions*

A **combinatorial auction** is an auction for multiple goods, where bidders value goods in combination: i.e., bundles, or subsets, of goods. Moreover, these values do not generally decompose "nicely."

Given a set of goods G, we write $v_i(X)$ to denote i's value for bundle $X \subseteq G$. Note that in the special case of a singleton bundle $\{j\}$, we simply write $v_i(j)$ to denote i's value for good j.

Two special (and relatively nice) cases of combinatorial valuations are **additive** and **unit demand** valuations. Given a bundle of goods $X \subseteq G$, bidder i's valuations are additive iff

$$v_i(X) = \sum_{j \in X} v_i(j),$$

and unit demand iff

$$v_i(X) = \max_{j \in X} v_i(j).$$

Other (less nice, but prevalent) combinatorial valuations of interest include **complementary** and **substitutable** valuations. Given two bundles of goods $X, Y \subseteq G$, bidder i's valuations satisfy the former iff they are superadditive:

$$v_i(X \cup Y) \geq v_i(X) + v_i(Y),$$

and the latter if they are subadditive:

$$v_i(X \cup Y) \leq v_i(X) + v_i(Y).$$

Examples of complements include left and right shoes. The value of the pair far exceeds the sum of the individual values. Examples of substitutes include an iPhone and and Android. Presumably you don't have a real need for one, if you already have the other.

Even in the context of combinatorial auctions, we continue to assume valuations are normalized (i.e., $v_i(\emptyset) = 0$, for all bidders $i \in [n]$) and monotone (i.e., there is free disposal).

2 *Combinatorial Auction Designs*

Recall the VCG mechanism, which generalizes the second-price (Vickrey) auction to the multi-parameter setting. This mechanism suffers from several anomalies, which you explored in your homework. In addition, when valuations have exponential complexity, the communication complexity of VCG is likewise exponential, and solving for a welfare-maximizing allocation is NP-hard.

Having noted the shortcomings of VCG, and the prevalence of combinatorial valuations, we would be remiss if we did not explore alternative combinatorial auction designs. Two obvious alternatives include sequential and simultaneous sealed-bid auctions. That is, it might make sense to auction off goods separately, either one after another or all at the same time, using, say, standard Vickrey auctions. These two n-dimensional designs eliminate the communication complexity of VCG, although it may be NP-hard for bidders to solve the requisite **value** or **demand** queries, in which bidders are asked how much they value a (set of) bundle(s) (the former), or which bundles they value at given prices (the latter).

Imagine, for example, a set of goods of size m, and a set of bidders with unit-demand valuations. Let's consider running m sequential Vickey auctions in such an environment. What can possibly go wrong?[1] (Think for a moment before reading on.)

[1] Robert Weber. Multiple-object auctions. *Discussion Paper #496*, August 1981

The problem with this arrangement is, as the auctions proceed, more and more bidders win goods, so competition in later auctions decreases, causing prices to likewise decrease. Consequently, the high-value bidders face a strategic decision—how to bid so that they win a good eventually, but not when prices are still high.

Another possible auction design is simultaneous Vickrey auctions. Here, unit-demand bidders (among other bidders with combinatorial valuations) face the **exposure** problem. They are forced to bid in all auctions, as they have only one shot at winning the goods, but unit-demand bidders—who demand exactly one good—would not want to bid too high on too many goods, and risk winning more than one, or too low on too few, and risk winning none at all.

In sum, unlike many single-parameter auction designs, which we have seen are truthful, bidders with combinatorial valuations face strategic decisions in both sequential and simultaneous sealed-bid auctions. This situation is undesirable from the point of view of mechanism design, because it can make it difficult to predict the outcome of the mechanism, and hence optimize it as desired.

3 Case Study: Spectrum Auctions

In 1959, the (eventual) Nobel laureate Ronald Coase lamented the power of the Federal Communications Commission (FCC) to control the operators of radio and television spectrum. He argued that welfare would be better served if the spectrum were instead allocated to those who valued it most.[2] In 1994, a variant of Coase's proposal was finally adopted; the FCC sold spectrum licenses via auctions that year, and raised over $600 million for the U.S. Treasury Department. That said, the early spectrum auction designs were far from perfect.

[2] Ronald Coase. The Federal Communications Commission. *Law and Economics*, 2:1–40, 1959

For example, in one of the earliest auctions in New Zealand in 1990, revenue was projected to be $250 million, but the auction realized only $36 million. The goods on offer were essentially identical spectrum licenses, and the mechanism was simultaneous Vickrey auctions. For one license, the highest bid was $100,000, while the second highest bid was $6; for another license, the highest bid was $7 million, while the second highest bid was $5,000. These bids were all public information, revealing how much money was left on the table. In a later rendition, New Zealand instead ran simultaneous first-price auctions. The auction still did not meet its revenue projections, but the outcome was less embarrassing.

In 2000, Switzerland auctioned off licenses for three blocks of spectrum sequentially, again using Vickrey auctions. The first two licenses, which were identical (28 MHz blocks), sold for rather different prices: 121 million Swiss francs and 134 million Swiss francs, respectively.[3] But the third auction, for a larger block of spectrum (58 MHz), was the most problematic—it sold for only 55 milion!

[3] A uniform price auction, as is used in US Treasury Bill sales for example, would have avoided this outcome.

Having observed the spectacular failures of both sequential and simultaneous sealed-bid auctions in practice, economists turned to **simultaneous ascending auctions (SAAs)** as a possible spectrum auction design.[4] Ascending auctions afford an opportunity for *price discovery*, which, when run simultaneously, can help bidders hone in on their utility-maximizing bundles. SAAs also afford an opportunity for incremental *value discovery*. Bidders need not precisely evaluate all bundles, only those which they can procure at reasonable prices. Indeed, from the earliest use of SAAs to allocate wireless spectrum, near equal selling prices were obtained among near substitutes.[5]

[4] *Sequential* ascending auctions suffer from the same shortcomings as sequential sealed-bid auctions.

[5] Paul Milgrom. Game theory and the spectrum auctions. *European Economic Review*, 42(3):771–778, 1998

While exposure is somewhat mitigated in SAAs, as compared to simultaneous sealed-bid auctions, it remains an issue. Imagine two goods, A and B, and two bidders, 1 and 2, and with valuations:

- Bidder 1: $v_1(A) = v_1(B) = 0$ and $v_1(\{A \cup B\}) = 3$
- Bidder 2: $v_2(A) = v_2(B) = 2$ and $v_2(\{A \cup B\}) = 2$

If both bidders bid sincerely (i.e., in a way that is consistent with their valuations), then the prices of both goods rise together—as the allocation cycles among bidder 2 winning good A and bidder 1 winning good B; bidder 1 winning both goods; and bidder 1 winning good A and bidder 2 winning good B—until the prices reach 1.5 and $1.5 + \epsilon$, at which point bidder 1 drops out (since the price of the bundle $\{A, B\}$ exceeds 3), and the auction ends with each bidder winning one of the goods. Indeed, bidder 1 suffers from the exposure problem in this example.

SAAs also suffer from a second anomaly, namely the potential for **strategic demand reduction**. Imagine two bidders, 1 and 2, and two goods, A and B, with the following valuations:

- Bidder 1: $v_1(A) = v_1(B) = 2$ and $v_1(\{A \cup B\}) = 3$
- Bidder 2: $v_2(A) = v_2(B) = 1$ and $v_2(\{A \cup B\}) = 1$

If bidder 1 bids sincerely (i.e., in a way that is consistent with his valuation), he wins both goods, each at a price $1 + \epsilon$, earning utility $3 - 2(1 + \epsilon) = 1 - 2\epsilon$. On the other hand, if he bids only on one good, so that both auctions can clear at a price near ϵ, he earns utility $2 - \epsilon > 1 - 2\epsilon$. Thus, bidders are sometimes incentivized to reduce their demand.

In spite of this potential for strategic demand reduction, the FCC (and analogous institutions in many other nations, most notably the UK) have adopted SAAs as the standard format by which to sell spectrum licenses. But, as is typical, the devil is in the details. One important design choice is the type of query. **Value** queries ask bidders their values for bundles of goods (much like a sealed-bid auction would), while **demand** queries ask bidders which set (or sets) of goods they prefer, given good prices.

In one SAA design, the simultaneous multi-round auction (SMRA), a generalization of the English auction, participants bid on all goods simultaneously, responding to value queries, with prices rising until quiescence, at which point all auctions close simultaneously.

An alternative called the combinatorial clock auction (CCA),[6] comprises two phases; the first uses SAAs to facilitate per-good price discovery, while the second is a sealed-bid format, with bids placed on bundles, so as to mitigate exposure. The SAA design uses demand queries, so that bidders report a set[7] of goods—one of her preferred bundles. Prices then increase on all goods that are overdemanded until quiescence. After this first round, there is a second round, in which bidders place sealed bids on bundles, with knowledge of the prices realized in the first. The second round resembles VCG, but aims to avoid some of its issues—particularly, low-revenue outcomes.

Both of these auction designs are enhanced with activity rules to encourage all bidders to participate in the price (and value) discovery

[6] Lawrence M. Ausubel, Peter Cramton, and Paul Milgrom. The clock-proxy auction: A practical combinatorial auction design. In Yoav Shoham, Peter Cramton, and Richard Steinberg, editors, *Combinatorial Auctions*. MIT Press, 2005

[7] Depending on the auction's **bidding language**, the auction may accept bids on more than one set of goods: e.g., the bid XOR(A, B, C) indicates that the bidder is interested in at most one of the bundles A, B, or C.

process. Among other things, these rules are designed to preclude temporarily exiting and later re-entering the auction, thereby limiting bidders' abilities to strategically withhold information.

While the CCA has been in use by the FCC since 2008, laboratory experiments have established that SMRA, which is simpler than CCA, yields both greater efficiency and greater revenue than CCA.[8] Consequently, we adopt a variant of SMRA (essentially, simultaneous eBay auctions) for our course final project.

[8] Martin Bichler, Jacob K. Goeree, Stefan Mayer, and Pasha Shabalin. *Spectrum Auction Design: Simple Auctions For Complex Sales*, pages 672–686. Cambridge University Press, 2017

References

[1] Lawrence M. Ausubel, Peter Cramton, and Paul Milgrom. The clock-proxy auction: A practical combinatorial auction design. In Yoav Shoham, Peter Cramton, and Richard Steinberg, editors, *Combinatorial Auctions*. MIT Press, 2005.

[2] Martin Bichler, Jacob K. Goeree, Stefan Mayer, and Pasha Shabalin. *Spectrum Auction Design: Simple Auctions For Complex Sales*, pages 672–686. Cambridge University Press, 2017.

[3] Ronald Coase. The Federal Communications Commission. *Law and Economics*, 2:1–40, 1959.

[4] Paul Milgrom. Game theory and the spectrum auctions. *European Economic Review*, 42(3):771–778, 1998.

[5] Robert Weber. Multiple-object auctions. *Discussion Paper #496*, August 1981.