SAAs (Before the EPIC Auction Design Recipe)

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We begin our study of simultaneous ascending auctions in the multi-parameter setting with arguably the simplest of all combinatorial valuations, additive valuations. In this lecture, we are concerned with establishing incentive guarantees.

1 SAAs with Additive Valuations: Incentives

Our goal in this series of notes is to establish that simultaneous ascending auctions (SAAs) yield strong incentive guarantees. To proceed, we must articulate these incentive goals, in the context of indirect mechanisms. We thus begin by putting forth an analog of truthful bidding in indirect mechanisms.

Recall that indirect auctions can invoke value queries or demand queries, the former asking bidders for a cardinal value (i.e., a bid above the current ask price), and the latter asking them for a set (i.e., their preferred bundles). These two types of queries give rise to two analogs of truthfulness, both of which we call sincere.

Definition 1.1. In an ascending auction that repeatedly queries bidders, a bidder's strategy is called **sincere** if she replies to all queries, and do so in a way that truthfully reflects her valuations.

For example, a sincere bidder in an ascending auction for a single good would answer "yes" to the demand query "Would you like the good for $10?" iff her value for the good were at least $10. Similarly, a sincere bidder in an analogous auction, but with value queries, would bid at least $10 in response to the query "Do I hear $10?"

Definition 1.2. An indirect mechanism is DSIC if sincere bidding is a dominant strategy. Likewise, an indirect mechanism is EPIC or BIC, if sincere bidding is an EPNE or BNE, respectively.

Recall that DSIC is a particularly strong property of an auction. Indeed, the English auction is not DSIC. Here is a counterexample that establishes this claim.

Example 1.3. Consider an English auction with two bidders, you and an "unusual" bidder. The unusual bidder employs the following (somewhat) unusual strategy:[1] If you bid before the price reaches $10, he will bid the price up to $1000. If you don't bid before the price reaches $10, he will bid up to $10, and then quit. Given this opposing strategy, it is in your interest to start your bidding at $10, as long as your value v is greater than $10, and earn utility $v - 10$.

[1] This strategy is not entirely unusual. It is possible that early bidding in an English auction can create excitement that causes bidders to bid up prices to surprisingly high values.

Not only is the English auction not DSIC, it is also not EPIC.[2] The English auction is also not DSIC "up to (the price increment) ϵ."[3] It is, however, EPIC up to ϵ,[4] because any benefits that can be accrued by bidding insincerely are necessarily less than ϵ.

[2] Problem 1, Part 1

[3] Example 1.3

[4] Problem 1, Part 2

Definition 1.4. A strategy is an ϵ-best response to another strategy if it is a best response with an additive boost of size ϵ. More formally, a strategy $s_i \in S_i$ is an ϵ-best response to $\mathbf{s}_{-i} \in S_{-i}$ if $u_i(s_i, s_{-i}) + \epsilon > u_i(s_i', s_{-i})$, for all $s_i' \in S_i$.

The Japanese auction, which imposes an activity rule and poses demand queries instead of value queries (as in the English auction), is also not DSIC,[5] but it is in fact DSIC up to ϵ.[6]

[5] Problem 2, Part 1

[6] Problem 2, Part 2

Knowing that the Japanese auction is DSIC up to ϵ, we might hope that k parallel Japanese auctions might likewise be DSIC up to $O(\epsilon)$ (e.g., $k\epsilon$), assuming additive bidder valuations (and, in general, heterogenous goods), so that the bidders' interests across auctions are decoupled. But alas they are not.

For starters, since we know that a single Vickrey auction is DSIC, let's investigate k parallel Vickrey auctions.

Example 1.5. Suppose goods a and b are being auctioned off in two parallel Vickrey auctions, and assume two bidders, i and j. Let $v_i(a) = v_i(b) = 2$, and $v_j(a) = v_j(b) = 1$. Suppose bidder j declares that he will bid \$1 million on both goods unless bidder i forfeits a, in which case he will forfeit b (which wasn't rightly his to win, anyway). Given this opposing strategy, it is in bidder i's interest to (non-truthfully) make no attempt to procure a so that she will at least win b, obtaining a utility of 1, rather than 0.

Hence, we observe that k parallel Vickrey auctions are not DSIC. The reason for this is that bidders' strategies (bidder j's strategy in this example) can create dependencies across auctions, even in the very special case of additive valuations, where they are independent. As SAAs only give bidders more power to create such dependencies, simultaneous Japanese auctions are likewise not DSIC.

At this point, we might decide that it is time to throw in the towel. But as it happens there are other fish in the sea: i.e., there are other equilibrium concepts besides DSIC that are worthy of our attention.

Specifically, we now set our sights on designing EPIC auctions, another worthy goal, which we can (at least sometimes) achieve. In EPIC auctions, sincere bidding is an equilibrium (i.e., a best response to itself), regardless of others' valuations.

Remark 1.6. Though weaker than DSIC, EPIC is still a worthwhile goal. Recall, for example, that we have derived only a BNE for first-price auctions, not an EPNE. In particular, the BNE of a symmetric first-price auction assuming uniformly distributed valuations involves shading bids by a constant $(n-1/n)$. This strategy is optimal (i.e., a best response), in expectation over all other-bidder values. But it is not optimal for an agent to shade in this way *ex-post*. In fact, no shading strategy is effective regardless of other-bidders' values. Consequently, there is no non-trivial EPNE in first-price auctions.

Having moved the goalpost, we can repeat the question: are k parallel Vickrey auctions EPIC, assuming additive valuations? That is, if everyone else is bidding truthfully, should bidder i likewise bid truthfully? In this case, it cannot hurt bidder i to do so, just as it cannot hurt her to do so in a single Vickrey auction, assuming everyone else is bidding truthfully.

As DSIC is in general stronger than EPIC, a single Vickrey auction is EPIC as well as DSIC. But the two concepts actually coincide in direct mechanisms! Why? Because reports are just another valuation, and sincere bidding must be a best response with respect to all possible alternative reports, which is the same as all possible alternative valuations in direct mechanisms.

Seek and you shall find. Indeed, k parallel English auctions are EPIC up to $O(\epsilon)$, assuming additive valuations. We lay the groundwork for a rigorous proof of this claim in the next lecture,[7] after presenting a recipe for the design of EPIC ascending auctions.

[7] Grad problem

To summarize:

- The English auction is not DSIC.[8] The English auction is *not* even DSIC up to ϵ.[9]
- The Japanese auction, which forbids re-entry (once a bidder stops bidding, he can never bid again), is also *not* DSIC.[10] The Japanese auction is, however, DSIC up to ϵ.[11]
- The English auction is *not* EPIC (or even BIC).[12] It is, however, EPIC up to ϵ.[13]
- Assuming additive valuations, neither k parallel English nor Japanese auctions for heterogeneous goods are DSIC, even up to $O(\epsilon)$.[14]
- Assuming additive valuations, k parallel English and k parallel Japanese auctions for heterogeneous goods are EPIC up to $O(\epsilon)$.[15]

[8] Example 1.3
[9] Problem 1, Part 1
[10] Problem 2, Part 1
[11] Problem 2, Part 2
[12] Example 1.3
[13] Problem 1, Part 2
[14] Example 1.5 plus the Revelation Principle
[15] Grad problem