SAAs (After the EPIC Auction Design Recipe)

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We apply the recipe for designing EPIC indirect mechanisms to simultaneous ascending auctions assuming bidders with additive valuations.

1 SAAs with Additive Valuations: Incentives

In the last lecture, we derived a recipe for designing EPIC indirect auctions:

1. Design an allocation rule that is welfare maximizing, assuming sincere bidding.
2. Show that sincere bidding yields VCG payments.
3. Argue that no inconsistent strategy is a profitable deviation: i.e., none yields greater utility than sincere bidding for any bidder.

We apply this recipe in upcoming lectures to design EPIC ascending auctions for different valuation classes (specifically, unit demand and diminishing marginal values). But first, we use it to establish that k parallel English auctions are indeed EPIC up to $k\epsilon$ assuming additive valuations and, in general, heterogeneous goods.

1. Assuming sincere bidding, a single English auction is welfare maximizing up to ϵ, because allocating to the highest bidder implies allocating to the bidder with the highest value, up to ϵ.

   - If, when the auction ends, exactly one bidder remains, that bidder's value is the highest, so the good is allocated to the highest bidder.
   - If, however, the auctions ends without any remaining bidders, then the highest two or more values are all within ϵ of one another, so the good might not be allocated to a bidder with the highest value, but it is still allocated to a bidder whose value is within ϵ of the highest value.

   Hence, assuming additive valuations and sincere bidding, it holds that k parallel English auctions are welfare maximizing up to $k\epsilon$.

2. Assuming sincere bidding, the final price p^* in a single English auction is the second-highest value up to ϵ, because the bidders with the second-highest value up to ϵ are the last to drop out, meaning their values lie somewhere in the range of $[p^* - \epsilon, p^*]$. As

the second-highest value is the VCG payment, the final price in a single English auction is the VCG payment up to ϵ.

While VCG payments are generally defined per bidder, in the case of additive valuations, those per-bidder payments reduce to prices per good, as each one is the second-highest value for that good. Moreover, the VCG payment for a bundle of goods is the sum of the second-highest values of all the goods in that bundle. Therefore, assuming additive valuations and sincere bidding, the price of a bundle of goods of size $m \leq k$ in k parallel English auctions is within $m\epsilon$ of the VCG payment.

3. As sincere bidding in k parallel English auctions is welfare maximizing up to $k\epsilon$, and terminates at VCG payments up to $m\epsilon$, it follows that sincere bidding is an EPNE up to $k\epsilon = \max\{k\epsilon, m\epsilon\}$ among consistent strategies.

   It remains to show that no inconsistent strategy is a profitable deviation from sincere bidding up to some additive error: i.e., no inconsistent strategy yields substantially greater utility than sincere bidding for any bidder.[1]

   The proof proceeds by showing that any deviation via an inconsistent strategy can be replicated by a sincere one up to $2k\epsilon$. Then, since sincere bidding is an EPNE up to $k\epsilon$ among (only) consistent strategies, sincere bidding is an EPNE up to $2k\epsilon = \max\{k\epsilon, 2k\epsilon\}$ among both consistent and inconsistent strategies.

[1] Grad Problem